Algebraic & Geometric Topology 15 (2015) 667–690

Finite knot surgeries and Heegaard Floer homology

MARGARET I DOIG

It is well known that any 3–manifold can be obtained by Dehn surgery on a link, but not which ones can be obtained from a knot or which knots can produce them. We investigate these two questions for elliptic Seifert fibered spaces (other than lens spaces) using the Heegaard Floer *correction terms* or *d–invariants* associated to a 3–manifold Y and its torsion Spin^c structures. For $\pi_1(Y)$ finite and $|H_1(Y)| \leq 4$, we classify the manifolds which are knot surgery and the knot surgeries which give them; for $|H_1(Y)| \leq 32$, we classify the manifolds which are surgery and place restrictions on the surgeries which may give them.

57M25; 57R65

1 Introduction

In the 1960s, Wallace [44] and Lickorish [22] showed that any oriented 3–manifold can be constructed by *Dehn surgery*[1] on a link in S^3. Soon after, Moser asked which manifolds can be constructed by surgery on a knot [25]. One may also ask which knots give each manifold. We begin to answer these two questions for elliptic (or spherical) manifolds other than lens spaces, that is, those with finite but noncyclic fundamental group.

We know that S^3 only comes from trivial surgeries (Gordon and Luecke [17]) and $S^1 \times S^2$ arises only from 0–surgery on the unknot; see Gabai [13]. On the other hand, lens spaces can come from torus knots [25] but may also arise from integral surgery on some hyperbolic knots; see Culler et al [6]. Berge [1] proposed a comprehensive list of such surgeries using *primitive/primitive* knots, which is now referred to as the Berge conjecture and is listed as Problem 1.78 in Kirby [21]. Ozsváth and Szabó [34] gave a necessary condition on the Alexander polynomial of a knot with a lens space surgery and verified Berge's list for $p \leq 1500$, and Greene [18] verified that any lens space which is surgery on a nontrivial knot is achieved by some knot on the list. (He did not

[1] To perform *p/q–[Dehn] surgery* on a knot K embedded in S^3, remove an open neighborhood $N(K)$ homeomorphic to a solid torus and replace it by identifying a meridian of the solid torus with $p\mu + q\lambda$ in the knot complement. Here, μ and λ are oriented curves on $\partial\overline{N(K)}$ where μ bounds a disk in $\overline{N(K)}$, λ is nullhomologous in $H_1(S^3 - N(K))$, and the geometric intersection number of μ and λ is $+1$.

Published: 22 April 2015 DOI: 10.2140/agt.2015.15.667

verify that all knots giving lens spaces are on the list.) Dean [7] proposed an extension of these results from lens spaces to small Seifert fibered spaces. However, Dean's list is not exhaustive: other hyperbolic surgeries also produce small Seifert fibered manifolds, but all such known manifolds are also given by knots from Dean's list; see Deruelle, Miyazaki and Motegi [8] and Mattman, Miyazaki and Motegi [24].

We address a subset of Dean's case: elliptic (or spherical) manifolds other than lens spaces, ie Seifert fibered manifolds with finite but noncyclic fundamental group. If a surgery gives such a group, we will call it a *finite* and *noncyclic* surgery.

The finite surgeries on torus knots are easy to identify from Moser's classification [25, Propositions 3.1, 3.2 and 4]. Bleiler and Hodgson explicitly listed the finite surgeries on iterated torus knots [2, Theorem 7] on the basis of Gordon's classification [16, Theorem 7.5]; all of the resulting manifolds are also torus knot surgeries. Boyer and Zhang proved that no other satellite knots have finite surgeries [3, Corollary 1.4].

Boyer and Zhang showed that all finite surgeries on hyperbolic knots are integral or half-integral, although it is conjectured that they are integral (see eg [21, Problem 177, Conjecture A]). Additionally, any hyperbolic knot has at most five finite or cyclic surgeries, with at most one nonintegral. Any two such surgeries on the same knot have distance[2] at most 3, and the distance 3 is realized by at most one pair; see Boyer and Zhang [4, Theorems 1.1 and 1.2].

There are a variety of examples of finite surgeries on hyperbolic knots. Fintushel and Stern [11] and Bleiler and Hodgson [2] commented respectively that 17–surgery on the $(-2,3,7)$ pretzel knot and 22– and 23–surgery on the $(-2,3,9)$ pretzel knot are finite (although all three resulting manifolds are also torus knot surgeries), and Mattman et al showed that there are no other finite surgeries on pretzel knots; see [23, Theorem 1.2] and [12, Theorem 1]. It is an interesting question for which p there are finite p/q–surgeries on hyperbolic knots. As Zhang stated in [45, Conjecture $\widehat{I}$] and Kirby formulated in a remark after Problem 3.6(D) in [21], the Poincaré homology sphere (the only manifold with finite $\pi_1 Y$ and $|H_1(Y)| = 1$) has a unique surgery construction. Ghiggini proved the following theorem:

Theorem 1 [14, Corollary 1.7] *The Poincaré homology sphere is (-1)–surgery on the left-handed trefoil (or, reversing orientation, $(+1)$–surgery on the right-handed trefoil)*

$$\Sigma(2,3,5) = S^3_{-1}(T_{3,-2})$$

[2] A surgery coefficient p/q corresponds to a homology class $p\mu + q\lambda$ on $\partial\overline{N(K)}$. The *distance* between two surgery coefficients is the minimum geometric intersection number of two curves representing the corresponding homology classes.

and no other surgery on any knot.[3]

Elliptic spaces fall into a group of manifolds called *L–spaces* whose Heegaard Floer homology is particularly simple; see [34, Proposition 2.3]. If an L–space is given by p/q–surgery on a knot K in S^3, then it obeys the inequality $p/q \geq 2g(K)-1$ and the knot is fibered with one of a very small set of Alexander polynomials. The *correction terms* or *d–invariants* $d(Y,\mathfrak{t})$ take a very nice form for L–space surgeries, and they can be compared to the $d(Y,\mathfrak{t})$ calculated directly from a plumbing graph. We prove:

Theorem 2 *Up to orientation, the only finite, noncyclic surgeries with $p \leq 9$ are*

$$\begin{aligned}
S^3_1(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{5}\right),\\
S^3_2(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}\right),\\
S^3_3(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{3}\right),\\
S^3_4(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{3}\right),\\
S^3_{7/2}(T_{3,2}) = S^3_7(T_{5,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{2}{5}\right),\\
-S^3_8(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{2}{3}\right),\\
S^3_8(T_{5,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{2}{5}\right),\\
S^3_{9/2}(T_{3,2}) = -S^3_9(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{2}{3}\right).
\end{aligned}$$

With the possible exception of $S^3_7(T_{5,2})$ and $S^3_8(T_{5,2})$, there are no other surgeries (up to orientation) giving these manifolds.

The following manifolds cannot be realized as any knot surgery:

$$\begin{aligned}
&\left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{n}\right) \quad \textit{if } n \neq 3,\\
&\left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{2}{n}\right) \quad \textit{if } n \neq 3 \textit{ or } 5.
\end{aligned}$$

Note that there are no elliptic Seifert fibered spaces with $|H_1(Y)| = 5$ or 6; there are unique spaces for each of $|H_1(Y)| = 1, 2, 3, 7$ and 9; and there are infinite families for both $|H_1(Y)| = 4$ and 8. See Theorem 6 below, due to Seifert.

Corollary 3 *Any finite, noncyclic surgery on a hyperbolic knot has surgery coefficient at least 7.*

Any Seifert fibered spaces which are not knot surgeries must be found among the dihedral manifolds, those with $|H_1(Y)|$ a multiple of 4 (see Theorem 6). We will prove the following theorem:

[3] Throughout this paper, we suppress the choice of orientations; unless otherwise stated, surgery coefficients are positive and Seifert fibered descriptions are the canonical ones described in Theorem 6.

Theorem 4 *The following manifolds have unique surgery descriptions:*

$$\begin{aligned}
S^3_4(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{3}\right),\\
-S^3_8(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{2}{3}\right)\\
S^3_{16/3}(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{4}{3}\right),\\
-S^3_{20/3}(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{5}{3}\right)\\
S^3_{28/5}(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{7}{3}\right),\\
-S^3_{32/5}(T_{3,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{8}{3}\right),\\
-S^3_{32/3}(T_{5,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{8}{5}\right).
\end{aligned}$$

The only other dihedral manifolds with $p \leq 32$ which may be surgery are

$$\begin{aligned}
S^3_8(T_{5,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{2}{5}\right),\\
-S^3_{12}(T_{5,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{3}{5}\right),\\
S^3_{12}(T_{7,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{3}{7}\right),\\
-S^3_{16}(T_{7,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{4}{7}\right),\\
S^3_{16}(T_{9,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{4}{9}\right),\\
-S^3_{20}(T_{9,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{5}{9}\right),\\
S^3_{20}(T_{11,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{5}{11}\right),\\
-S^3_{24}(T_{11,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{6}{11}\right),\\
S^3_{24}(T_{13,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{6}{13}\right),\\
S^3_{28/3}(T_{5,2}) = S^3_{28}(K_0) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{7}{5}\right)\\
S^3_{28}(K_1) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{7}{11}\right)\\
-S^3_{28}(T_{13,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{7}{13}\right),\\
S^3_{28}(T_{15,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{7}{15}\right),\\
-S^3_{32}(T_{15,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{8}{15}\right),\\
S^3_{32}(T_{17,2}) &= \left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{8}{17}\right).
\end{aligned}$$

The latter manifolds may also be integral surgery on hyperbolic knots with the same $\Delta_K(T)$ as the knots listed above; see Tables 1 and 4.

Note K_1 is the knot constructed by $(+1)$–surgery on the unknotted component of the $(-2, 3, 10)$ pretzel link [2, Proposition 18]. K_0 may be some knot with symmetrized

Alexander polynomial $T^8 - T^7 + T^5 - T^4 + T^2 - T + 1 \cdots$; the author is not currently aware of any such K_0 with the listed surgery.

Corollary 5 *If $m \le 8$, the following manifolds cannot be realized as knot surgeries:*

$$\left(-1; \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{m}{n}\right) \quad \text{if } n > 2m + 1.$$

We describe Seifert fibered spaces and their nonhyperbolic surgeries in Section 2, we list the necessary prerequisites about L–space surgeries and the d–invariants in Section 3, and we prove Theorems 2 and 4 in Section 4. The first presentation of this work may be found in the author's thesis [10].

Acknowledgements Special thanks go to Zoltán Szabó, who suggested and directed this project with a great deal of kindness and patience, and to Chuck Livingston, Paul Kirk and Dave Gabai, who listened and contributed many helpful suggestions. Thanks also to Stephen Maderak for turning my algorithms into functional code and making it possible to generate gigabytes of examples.

2 Seifert fibered spaces as knot surgeries

Any closed, oriented 3–manifold Y is surgery on some link in S^3; see Lickorish [22] and Wallace [44]. A surgery diagram can be manipulated by the methods of Kirby calculus [20], which alter the diagram but not the diffeomorphism type of the underlying 3–manifold: isotoping by surgery diagrams, stabilizing or destabilizing the manifold by adding or subtracting a (± 1)–framed unknot which can be separated from the rest of the link, and handlesliding one link component over another, replacing L_2 with the band sum of L_1 and L_2. For the last procedure, if n_i is the framing on L_i, then n_2 becomes $n_1 + n_2 + 2lk(L_1, L_2)$. For a presentation of Kirby calculus, including its applications to Dehn surgery, see [15, Chapter 5].

Seifert fibered spaces Seifert fibered spaces were originally defined by Seifert in 1932; see [42], translated by W Heil in [43]. Scott gives a more modern presentation with a slightly expanded definition incorporating the fibered solid Klein bottles mentioned below [41].

A trivial solid torus $\{z \in \mathbb{C} : |z| \le 1\} \times S^1$ may be given the product fibration with fibers $\{z\} \times S^1$. A *fibered solid torus* (or *fibered solid Klein bottle*) is a torus (or Klein bottle) which is finitely covered by the trivial fibered torus, where the covering map preserves fibers. A fibered torus can alternately be constructed by taking the trivial fibered torus, cutting it along $\{z \in \mathbb{C} : |z| \le 1\} \times \{0\}$ and identifying $(z, 0)$ with $(e^{2\pi i q/p} z, 1)$, and a fibered solid Klein bottle can be constructed by taking the same cut fibered torus and identifying $(z,0)$ with $(\bar{z},1)$. The torus then has one *exceptional* (not regular) fiber in the center, and the Klein bottle has a continuous family of exceptional fibers whose union is an annulus.

A *Seifert fibered space* is a manifold foliated by circles so that any circle has a neighborhood which is fiber-isomorphic to a fibered solid torus or Klein bottle. A Seifert fibered space itself can be thought of as a fiber bundle over the orbifold obtained by compressing each fiber to a point (often called the *base orbifold*). Each isolated exceptional fiber corresponds to a cone point on the orbifold and a surface of exceptional fibers corresponds to a reflector line in the orbifold. Each isolated exceptional fiber can be eliminated by some Dehn surgery, and the class of such surgery coefficients is referred to as the fiber's *framing*.

For our purposes, we will need only Seifert fibered spaces with base orbifold S^2 and some number of cone points. Construct such a space by choosing a circle bundle ζ over S^2 and surgering over fibers with framings $-b_i/a_i$ (the negative sign is for historical reasons). It can be described as surgery on a link in S^3 whose components have framing $\{b=c_1(\zeta),-b_1/a_1,\dots,-b_r/a_r\}$. Seifert identified such a manifold with an n–tuple (together with information about the base orbifold which we will exclude)

$$\left(b;\frac{a_1}{b_1},\dots,\frac{a_r}{b_r}\right).$$

For example, the Poincaré homology sphere is $-(-1;\frac{1}{2},\frac{1}{3},\frac{1}{5})$.

The choice of framings is not unique. The b_i, sometimes called the *multiplicities*, are determined, but b and the a_i may be altered by handleslides. For example,

$$\left(-b;\frac{a_1}{b_1},\frac{a_2}{b_2},\frac{a_3}{b_3}\right)\cong\left(-b-1;\frac{a_1}{b_1}+1,\frac{a_2}{b_2},\frac{a_3}{b_3}\right),\tag{1}$$

$$\left(-1;\tfrac{1}{2},\frac{a_2}{b_2},\frac{a_3}{b_3}\right)\cong-\left(-2;1-\tfrac{1}{2},1-\frac{a_2}{b_2},1-\frac{a_3}{b_3}\right).\tag{2}$$

By geometrization [37; 38; 39], the manifolds with finite fundamental group are all Seifert fibered. They fall into five classes depending on whether π_1 is cyclic or is based on one of the four isometries of a sphere. We slightly rephrase Seifert's result:

Theorem 6 (Seifert [42]) *The closed, oriented Seifert fibered spaces with finite but noncyclic fundamental group are exactly those manifolds with base orbifold S^2 and the following presentations:*

(i) **Type I, icosahedral** $(b;\frac{1}{2}a_1,\frac{1}{3}a_2,\frac{1}{5}a_3)$ *with* $H_1(Y)=\mathbb{Z}_m$ *and* $(m,30)=1$.

(ii) **Type O, octahedral** $(b; \frac{1}{2}a_1, \frac{1}{3}a_2, \frac{1}{4}a_3)$ *with* $H_1(Y) = \mathbb{Z}_{2m}$ *and* $(m, 6) = 1$.

(iii) **Type T, tetrahedral** $(b; \frac{1}{2}a_1, \frac{1}{3}a_2, \frac{1}{3}a_3)$ *with* $H_1(Y) = \mathbb{Z}_{3m}$ *and* $(m, 2) = 1$.

(iv) **Type D, dihedral** $(b; \frac{1}{2}a_1, \frac{1}{2}a_2, a_3/b_3)$ *with* $H_1(Y) = \mathbb{Z}_{4m}$ *and* $(m, b_3) = 1$ *(if b_3 is even) or* $H_1(Y) = \mathbb{Z}_2 \times \mathbb{Z}_{2m}$ *with* $(m, 2b_3) = 1$ *(if b_3 is odd).*

Here $|H_1(Y)| = b_1b_2b_3(b + a_1/b_1 + a_2/b_2 + a_3/b_3)$ *and* $(a_i, b_i) = 1$. *Any integer* m *meeting the constraints listed for one of the four types I, O, T or D corresponds (up to orientation) to a unique Seifert fibered space of type I, O or T, or to a unique infinite family of type D indexed by the integer* b_3.

Any choice of b, a_i *and* b_i *meeting the appropriate relative primality conditions gives a Seifert fibered space. For each orientation, we choose a canonical presentation where* $b = -1$, $a_1 = 1$ *and* $a_2 = 1$ *or* 2. *If we allow change of orientation, we can also require* $a_2 = 1$ *and (for type D)* $a_3 > 0$.

Proof Seifert calculates explicit descriptions of the fundamental group and first homology group and then deduces the possible framings; see [42] for the details.

$$\pi_1(Y) = \langle \lambda, \mu, \mu_1, \ldots, \mu_r \mid \mu\mu_1 \cdots \mu_r = 1, [\lambda, \mu_i] = 1, \mu = \lambda^b, \mu_i^{b_i} = \lambda^{a_i} \rangle,$$

and so

$$H_1(Y; \mathbb{Z}) = \frac{\mathbb{Z}m_0 \oplus \cdots \oplus \mathbb{Z}m_r}{(b \cdot m_0 + m_1 + \cdots + m_r = 0,\ a_i \cdot m_0 = b_i \cdot m_i)}.$$

In order to obtain the canonical presentation for a manifold of type D, first turn $(b; \frac{1}{2}a_1, \frac{1}{2}a_2, a_3/b_3)$ into $(b; \frac{1}{2}, \frac{1}{2}, a_3'/b_3)$ using (1). Then adjust b (perhaps changing a_3' but leaving $a_1 = a_2 = 1$) to get $(-1; \frac{1}{2}, \frac{1}{2}, a_3''/b_3)$. If $a_3'' < 0$, then reverse orientation as in (2) to $-(-2; \frac{1}{2}, \frac{1}{2}, (b_3 - a_3'')/b_3) = -(-1; \frac{1}{2}, \frac{1}{2}, -a_3''/b_3)$. For types I, O and T, start by obtaining $(-1; \frac{1}{2}, \frac{1 \text{ or } 2}{3}, a_3/b_3)$. If $a_2 = 2$, reverse orientation to $-(-2; \frac{1}{2}, \frac{1}{3}, 1 - a_3/b_3) = -(-1; \frac{1}{2}, \frac{1}{3}, -a_3/b_3)$.

Given a choice of I, O, T or D and an m that meets the appropriate primality conditions, the b_i are determined, and there are b and a_i as follows. Assume $b = -1$ and $a_1 = a_2 = 1$. For type I with $(m, 30) = 1$, $m \pmod 6 \equiv -5$ or 5. In the former case, set $a_3 = \frac{1}{6}(m+5)$, and then $|H_1(Y)| = |6a_3 - 5| = m$; in the latter, set $a_3 = -\frac{1}{6}(m-5)$, so $|H_1(Y)| = m$. For type O with $(m, 24) = 1$, we have $(m, 3) = 1$, so choose $a_3 = \frac{1}{3}(m+2)$ or $-\frac{1}{3}(m-2)$, whichever is an integer, and then $|H_1(Y)| = |6a_3 - 4| = 2m$. For type T with $(m, 18) = 1$, we have $(m, 3) = 1$, so choose $a_3 = \frac{1}{2}(m+1)$ or $-\frac{1}{2}(m-1)$, whichever is an integer, and then $|H_1(Y)| = |6a_3 - 3| = 3m$. Finally, for type D with $(m, b_3) = 1$, choose $a_3 = m$, so $|H_1(Y)| = 4m$.

Note that the canonical presentations for the two orientations of an I, O or T manifold may be distinguished by whether a_2 is 1 or 2. The two orientations for a D manifold may be distinguished by whether $a_3 = m$ is positive or negative. □

Finite surgeries Many of the elliptic manifolds can be realized as torus knot surgeries.

Theorem 7 [25, Propositions 3.1, 3.2 and 4]

$$S^3_{p/q}(T_{r,s}) = \begin{cases} L_{r,s} \# L_{s,r} & \textit{if } p/q = rs, \\ L_{p,rsq} & \textit{if } p/q = rs \pm 1/q, \\ \left(b; \frac{a_1}{r}, \frac{a_2}{s}, \frac{a_3}{|rsq-p|}\right) & \textit{otherwise, for some choice of } b, a_1, a_2, a_3. \end{cases}$$

Corollary 8 *Every manifold of type I, O or T is surgery on a $T_{n,2}$ torus knot. Of each infinite family of manifolds of type D with the same $|H_1(Y)| = 4m$, only finitely many are surgeries on torus knots, and they are the ones where b_3 divides $2m+1$ or $2m-1$.*

Proof A careful application of Kirby calculus shows that

$$S^3_{p/q}(T_{s,2}) = \left(-1; \frac{1}{2}, \frac{(s-1)/2}{s}, \frac{q}{2sq-p}\right) = -\left(-1; \frac{1}{2}, \frac{(s+1)/2}{s}, \frac{q}{p-2sq}\right),$$
$$S^3_{p/q}(T_{4,3}) = \left(-1; \frac{2}{3}, \frac{1}{4}, \frac{q}{12q-p}\right) = -\left(-1; \frac{1}{3}, \frac{3}{4}, \frac{q}{p-12q}\right),$$
$$S^3_{p/q}(T_{5,3}) = \left(-1; \frac{1}{3}, \frac{3}{5}, \frac{q}{15q-p}\right) = -\left(-1; \frac{2}{3}, \frac{2}{5}, \frac{q}{p-15q}\right).$$

These cases cover all the finite torus knot surgeries, since p/q–surgery on $T_{r,s}$ (if it is not a lens space or sum of lens spaces) has multiplicities $(r, s, |rsq-p|)$. A type I manifold may be surgery on $T_{3,2}$, $T_{5,2}$ or $T_{5,3}$; a type O manifold may be surgery on $T_{3,2}$ or $T_{4,3}$; a type T may be surgery on $T_{3,2}$; and a type D may be surgery on $T_{n,2}$.

By Theorem 6, any I, O or T manifold Y may be written $\pm(-1; \frac{1}{2}, \frac{1}{3}, a_3/b_3)$. A series of blow-ups on the trefoil shows Y is $(6a_3 - b_3)/a_3$–surgery on $T_{3,2}$ (up to orientation).

A manifold of type D with multiplicities $(b_1, b_2, b_3) = (2, 2, n)$ can only be surgery on a knot if $H_1(Y)$ is cyclic, meaning n is odd, and it can only be surgery on $T_{n,2}$ if $2sq - p = \pm 2$. (Note that $q/(2sq-p)$ is a reduced fraction since $(p,q) = 1$.) Then $p = |H_1(Y)|$ and $q = \frac{1}{2n}(|H_1(Y)| \pm 2)$, ie n divides either $\frac{1}{2}|H_1(Y)| + 1$ or $\frac{1}{2}|H_1(Y)| - 1$. □

3 The invariant $d(Y, \mathfrak{t})$

Heegaard Floer homology assigns a set of invariants (in our case, a graded abelian group over $\mathbb{Z}_2$) to a closed, connected, oriented 3–manifold using a Heegaard decomposition of the manifold [32; 33]. A Langrangian Floer homology starts with a $2n$–dimensional symplectic manifold and two n–dimensional Lagrangian submanifolds which meet transversely. The chain complex is a free R–module (for $R = \mathbb{Z}_2$, $\mathbb{Z}$ etc) whose generators come from intersection points of the Lagrangians and whose boundary map counts pseudoholomorphic disks associated to pairs of generators. Heegaard Floer homology is a Floer homology (after the work of Perutz [40]) that defines the symplectic manifold and the Lagrangians using a Heegaard decomposition of a 3–manifold. The generators of its chain complex can be thought of as sets of points on the Heegaard surface and the boundary maps can be analyzed by examining domains in the surface.

Heegaard Floer homology assigns a set of invariants to certain 3–manifolds Y, including rational homology spheres, indexed by their Spin^c structures $\mathfrak{t}$. These invariants are called the *correction terms* or d*–invariants* $d(Y,\mathfrak{t})$. The hat version $\widehat{HF}(Y)$ comes with a relative $\mathbb{Z}$–grading which lifts to an absolute $\mathbb{Q}$–grading for a rational homology sphere (see Theorem 7.1 of [35]); it is defined by requiring that $\widehat{HF}(S^3) \cong \mathbb{Z}$ is supported in degree 0 and that the inclusion map $\widehat{CF}(Y,\mathfrak{t}) \hookrightarrow CF^+(Y,\mathfrak{t})$ preserves degree. Then $d(Y,\mathfrak{t})$ is the minimal grading of any nontorsion class in $HF^+(Y,\mathfrak{t})$ coming from $HF^\infty(Y,\mathfrak{t})$ [29]. If Y is elliptic, all classes in $HF^+(Y)$ come from $HF^\infty(Y)$, and $d(Y,\mathfrak{t})$ is defined for all $\mathfrak{t}$.

L–space surgeries An elliptic Seifert fibered space is an example of an L–space, the Heegaard Floer homology version of a lens space [30]. $\widehat{HF}(Y)$ splits into $\oplus^{\mathfrak{t}}\widehat{HF}(Y,\mathfrak{t})$ over Spin^c structures (equivalence classes of nonzero vector fields $\mathfrak{t}$ that form a torsor over $H^2(Y;\mathbb{Z})$). Lens spaces have the nice property that each generator of $\widehat{HF}(L(p,q))$ falls into a different torsion Spin^c structure ($\mathfrak{t} \in \mathrm{Spin}^c(Y)$ is torsion if $PD(c_1(\mathfrak{t})) \in H_1(Y)$ is torsion). We will call any rational homology sphere with this property an *L–space*. Equivalently, $\widehat{HF}(Y,\mathfrak{t}) \cong \widehat{HF}(S^3)$ for all $\mathfrak{t}$.

Using the surgery exact sequences and absolute grading on $HF^+(Y)$, we can place some restrictions on which knots may have L–space surgeries. Normalize the Alexander polynomial so that

$$\Delta_K(T) = a_0 + \sum_{i=1}^{n} a_i(T^i + T^{-i}).$$

Theorem 9 [34, Corollary 1.3] *If a knot $K \subset S^3$ admits an L–space surgery, then the nonzero coefficients of $\Delta_K(T)$ are alternating $+1$ and -1.*

Ozsváth and Szabó [31] showed that the knot Floer homology $\widehat{HFK}(K,i)$ is $\mathbb{Z}$ in the top grading $i = g(K)$ for any fibered knot, and Ghiggini [14] and Ni [27] and, independently, Juhász [19] showed the converse; since $\Delta_K(T)$ is the graded Euler characteristic of $\widehat{HFK}(S^3,K)$, this means the following:

Corollary 10 [27, Corollary 1.3] *If a knot $K \subset S^3$ admits an L–space surgery, then K is fibered.*

Finally, we recall the following theorem:

Theorem 11 [36, Corollary 1.4] *If a nontrivial knot K admits a positive L–space surgery, then $S^3_{p/q}(K)$ is an L–space if and only if*

$$\frac{p}{q} \geq 2g(K) - 1.$$

These facts lead to another observation which seems to be known among the community but not frequently written down.

Corollary 12 *No nontrivial knot has both positive and negative L–space surgeries. No amphichiral knots have L–space surgeries. In particular, no knot has both positive and negative finite surgeries, and no amphichiral knot has any finite surgeries.*

Proof If K has a positive L–space surgery, then $\tau(K) = \deg(\Delta_K(T)) = g(K)$ [34, Corollary 1.6]. If K has both positive and negative L–space surgeries, meaning both K and its mirror mK have positive L–space surgeries, then $\tau(K) = g(K) = g(mK) = \tau(mK)$, but $\tau(K) = -\tau(mK)$. □

Calculating $d(Y, \mathfrak{t})$ of a knot surgery If $S^3_{p/q}(K)$ is an L–space, $\widehat{HF}(S^3_{p/q}(K))$ and its gradings can be calculated from $\Delta_K(T)$ and p/q:

Theorem 13 *If $0 < q < p$, there is a particular identification of* Spin^c *structures with $\mathbb{Z}_p$ such that:*

(a) [29, Proposition 4.8] *For $0 \leq i < p+q$,*

$$d(S^3_{p/q}(U), i) = -\Big(\frac{pq - (2i+1-p-q)^2}{4pq}\Big) - d(S^3_{q/r}(U), j),$$

where $r \equiv p \bmod q$ and $j \equiv i \bmod q$.

(b) [36, Theorem 1.2] *For $|i| \leq \frac{1}{2}p$,*

$$d(S^3_{p/q}(K), i) - d(S^3_{p/q}(U), i) = -2\sum_{j=1}^{\infty} j a_{c+j},$$

where $c = |\lfloor i/q \rfloor|$ and the a_j are the coefficients of the symmetrized Alexander polynomial.

Calculating $d(Y,\mathfrak{t})$ of a Seifert fibered space It is often possible to calculate the $d(Y,\mathfrak{t})$ algorithmically using plumbing graphs [30].

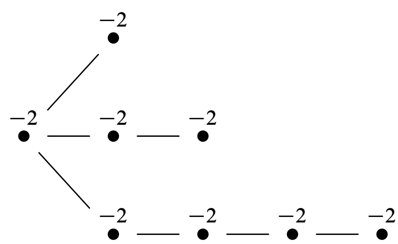
See [26] for a thorough exposition of plumbing graphs.

Let Γ be a tree with vertices $\mathfrak{v}$ which have integer weights $\mathrm{m}(\mathfrak{v})$. The graph Γ describes a 4–manifold $X = X(\Gamma)$: For each vertex, take a disk bundle over the sphere with Euler number $\mathrm{m}(\mathfrak{v})$; for each edge, plumb together the corresponding bundles. The boundary of $X(\Gamma)$ is a 3–manifold we call $Y(\Gamma)$. For example, the lens space $L(7,4)$ may be given as $Y(\Gamma)$ for either graph below, since $[-3,-2,-2]$ and $[-2,3]$ are both continued fraction expansions for $-\frac{7}{3}$:

$$\overset{-3}{\bullet} \text{ --- } \overset{-2}{\bullet} \text{ --- } \overset{-2}{\bullet} \quad = \quad \overset{-2}{\bullet} \text{ --- } \overset{+3}{\bullet}$$

Similarly, we have that the Poincaré homology sphere (with nonstandard orientation) $Y = (-2; \frac{1}{2}, \frac{2}{3}, \frac{4}{5})$ is given by

(3)



where the central vertex is $\mathfrak{v}_1$, the top arm is a single vertex $\mathfrak{v}_2$ of weight $-\frac{2}{1}$, the next arm consists of vertices $\mathfrak{v}_3$, $\mathfrak{v}_4$ labeled from left to right with weights giving the continued fraction expansion of $-\frac{3}{2}$, and the bottom arm is $\mathfrak{v}_5, \dots, \mathfrak{v}_8$ with weights giving the continued fraction expansion of $-\frac{5}{4}$. In general, an elliptic space may be written $Y = (b; a_1/b_1, a_2/b_2, a_3/b_3)$ with $0 < a_i/b_i < 1$ and (perhaps after reversing orientation) $b \leq 0$. If $p/q < 0$, it is possible to chose the $x_i \leq -2$ (choose them to be negative; if a (-1) appears, blow it down). Then $Y = Y(\Gamma)$ for a graph Γ with a central vertex of degree 3 and weight b, and with three arms with vertices of degree ≤ 2 and weights given by the continued fraction expansion of $-b_i/a_i$, chosen so that the weights are ≤ -2. Additionally, if the orientation is chosen so that $e(\Gamma) = b - \sum_{i=1}^{3} a_i/b_i < 0$ (ie $b \leq 0$), then Γ is the dual graph of a good resolution of a singularity and $X(\Gamma)$ is negative definite [26, Corollary 8.3].

An elliptic space Y with the description given above additionally has the property that $\mathrm{m}(\mathfrak{v}) \leq -\deg(\mathfrak{v})$ for each vertex except possibly the central one, as in (3); we will call a vertex violating this property *bad*.

$H_2(X;\mathbb{Z})$ is a lattice freely spanned by the vertices of Γ. Define a matrix Q for the intersection form using Γ: If $\mathfrak{v}$ is a vertex and v the corresponding homology class, $v \cdot v = \mathrm{m}(\mathfrak{v})$; if $\mathfrak{v}$ and $\mathfrak{w}$ are distinct vertices, $v \cdot w = 1$ if there is an edge between $\mathfrak{v}$ and $\mathfrak{w}$ and 0 otherwise. If, as above, $e(\Gamma) < 0$, then Q is negative definite. For Γ in (3), Q is E_8.

The *characteristic vectors* or $\mathrm{Char}(\Gamma)$ are the $V \in H^2(X;\mathbb{Z})$ such that

$$\langle V, w\rangle \equiv w \cdot w \mod 2 \quad \text{for all } w \in H_2(X;\mathbb{Z}).$$

$\mathrm{Char}(\Gamma)$ splits over $\mathrm{Spin}^c(Y(\Gamma))$. Let $\mathrm{Char}_{\mathfrak{t}}(\Gamma)$ be the characteristic vectors such that $V = c_1(\mathfrak{s})$ for some $\mathfrak{s} \in \mathrm{Spin}^c(X(\Gamma))$ with $\mathfrak{s}|_{Y(\Gamma)} = \mathfrak{t}$. It is easy to identify a characteristic vector using Hom duality: For $V \in H^2(X)$, note that $\langle V, w\rangle = PD^{-1}(V)\cdot w = v^T Q w$ for some $v \in H_2(X)$. Then v^T is the Poincaré dual of V, and $v^T Q$ is its Hom dual. V is characteristic exactly when $PD^{-1}(V)\cdot v_i \equiv v_i \cdot v_i \mod 2$, ie the i^{th} coordinate of $v^T Q$ has the same parity as $\mathrm{m}(\mathfrak{v}_i)$ for all vertices $\mathfrak{v}_i$. For example, $\mathrm{Char}(\Gamma)$ of (3) consists of all vectors $v^T Q$ with even coordinates.

$HF^+(-Y,\mathfrak{t})$ can be expressed in terms of $\mathrm{Char}_{\mathfrak{t}}(\Gamma)$. Let

$$\mathcal{T}_0^+ = \mathbb{Z}[U, U^{-1}]/U \cdot \mathbb{Z}[U]$$

as a $\mathbb{Z}[U]$–module with grading so that U^{-d} is homogeneous and supported in degree $2d$ (where $d > 0$). Then $HF^+(-Y,\mathfrak{t})$ is isomorphic to the set of functions

$$\phi\colon \mathrm{Char}_{\mathfrak{t}}(G) \to \mathcal{T}_0^+$$

which preserve the adjunction relations

$$\begin{aligned} U^n \cdot \phi(V + PD(w)) &= \phi(V) && \text{if } n \geq 0,\\ \phi(V + PD(w)) &= U^{-n}\cdot\phi(V) && \text{if } n \leq 0, \end{aligned}$$

where $2n = \langle V, w\rangle + w \cdot w$.

The grading of $HF^+(-Y,\mathfrak{t})$ is induced from the grading on $\mathcal{T}_0^+$ by

$$\deg(\phi) = \deg(\phi(V)) - \frac{V^2 + |\Gamma|}{4}$$

if $\phi(V) \in \mathcal{T}_0^+$ is a nontrivial homogeneous element, where $|\Gamma|$ is the number of vertices in Γ. We could calculate $d(Y,\mathfrak{t})$ by optimizing this grading over $\mathrm{Char}_{\mathfrak{t}}(\Gamma)$, but it would be very labor-intensive. To better study the grading on characteristic vectors, define an operation on $\mathrm{Char}_{\mathfrak{t}}(\Gamma)$ by

$$V \mapsto V + 2PD(v_i) \quad \text{if } \langle V, v_i\rangle = -\mathrm{m}(\mathfrak{v}_i). \tag{4}$$

That is, find $v^T Q$ where $v^T Q$ has $-\mathrm{m}(\mathfrak{v}_i)$ as its i^{th} coordinate and has the same parity as $\mathrm{m}(\mathfrak{v}_j)$ in all other coordinates. This operation changes the i^{th} coordinate to $\mathrm{m}(\mathfrak{v}_i)$ and adds 2 to the j^{th} coordinate if and only if there is an edge between $\mathfrak{v}_i$ and $\mathfrak{v}_j$. This operation does not change the class in $\mathrm{Char}_{\mathfrak{t}}(\Gamma)$, and it does not change the value $V^2 = \langle V, PD^{-1}(V)\rangle$. For the graph Γ in (3), the vector

$$V = (2, 0, 0, 0, 0, 0, 0, 0)$$

satisfies $\langle V, v_i\rangle = -\mathrm{m}(\mathfrak{v}_i)$ for $i = 1$, so the operation gives

$$V + 2PD(v_1) = (-2, 2, 2, 0, 2, 0, 0, 0),$$

and

$$V' = (-2, 2, 2, 0, 2, 0, 0, 0)$$

satisfies the equality for $i = 2,\ 3$ or 5, which gives

$$\begin{aligned} V' + PD(v_2) &= (0, -2, 2, 0, 2, 0, 0, 0),\\ V' + PD(v_3) &= (0, 2, -2, 2, 2, 0, 0, 0),\\ V' + PD(v_5) &= (0, 2, 2, 0, -2, 2, 0, 0). \end{aligned}$$

A *path* of vectors is a sequence $\{V_0, V_1, \dots, V_k\}$ where V_{i+1} is derived from V_i by this operation, and a *full* path is maximal with respect to this operation. For example, $\{(0, 0, 0, 0, 0, 0, 0, 0)\}$ is actually a full path for Γ in (3).

A *nice* characteristic vector obeys

$$\mathrm{m}(\mathfrak{v}_i) \le \langle V, v_i\rangle \le -\mathrm{m}(\mathfrak{v}_i) \quad \text{for all } i, \tag{5}$$

that is, the i^{th} coordinate of $v^T Q$ is between $-\mathrm{m}(\mathfrak{v}_i)$ and $\mathrm{m}(\mathfrak{v}_i)$. There are a finite number of nice characteristic vectors. By [30, Proposition 3.2], every full path of *nice* vectors $\{V_0, V_1, \dots, V_k\}$ obeys the additional property that V_0 and V_k satisfy

$$\mathrm{m}(\mathfrak{v}_i) < \langle V_0, v_i\rangle \le -\mathrm{m}(\mathfrak{v}_i) \quad \text{for all } i, \tag{6}$$

$$\mathrm{m}(\mathfrak{v}_i) \le \langle V_k, v_i\rangle < -\mathrm{m}(\mathfrak{v}_i) \quad \text{for all } i. \tag{7}$$

For example, for Γ in (3), $(2, 2, 0, 0, 0, 0, 0, 0)$ is nice, but $(4, -2, 0, 0, 0, 0, 0, 0)$ is not. For a given vector V, if there is any full path of nice vectors containing V, then all paths containing V have only nice vectors, and all full paths containing V are the same length and start and end at the same V_0 and V_k. For Γ in (3), there is only one full path of nice vectors, and it contains only the vector $(0, 0, 0, 0, 0, 0, 0, 0)$.

Using full nice paths, we may now calculate $d(Y(\Gamma), \mathfrak{s})$ in a reasonably efficient fashion.

Theorem 14 [30, Corollaries 1.5 and 3.2] *Let Γ be a connected tree with at most one bad vertex and $\mathfrak{t} \in \mathrm{Spin}^c(Y(\Gamma))$. Then*

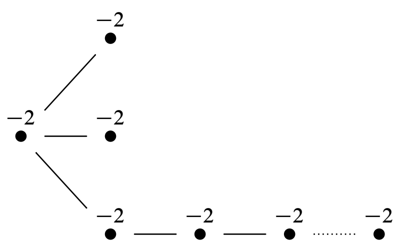
$$d(Y(\Gamma), \mathfrak{t}) = - \max_{V \in \mathrm{Char}_{\mathfrak{t}}(\Gamma)} \frac{V^2 + |\Gamma|}{4}.$$

In fact, this maximum is obtained over the vectors that are part of nice full paths and obey (6) *or, equivalently,* (7).

Recall that V is actually an element in $HF^+(-Y(\Gamma), \mathfrak{t})$, hence the negative sign and the fact that we must calculate the maximum instead of the minimum.

Given a characteristic vector V written as $v^T Q$, it is easy to calculate $d(Y(\Gamma), \mathfrak{t})$ since $V^2 = VQV^T = (v^T Q)Q^{-1}(v^T Q)^T$. In the case of Γ in (3), the vector $v^T Q = (0,0,0,0,0,0,0,0)$ has $V^2 = 0$ and so $d(-Y(\Gamma), \mathfrak{t}_0) = 2$, where $\mathfrak{t}_0$ is the single Spin^c structure.

Example 15 Calculate the correction terms for the first family of dihedral manifolds from Theorem 6, $Y = (-1; \frac{1}{2}, \frac{1}{2}, \frac{1}{n})$. Reversing orientation so that $e(-Y) < 0$, $-Y = -(-2; \frac{1}{2}, \frac{1}{2}, (n-1)/n)$ and Γ is



where $\mathfrak{v}_1$ is the degree-3 vertex on the left, $\mathfrak{v}_2$ is on the top arm, $\mathfrak{v}_3$ is on the middle arm, and $\mathfrak{v}_4, \dots, \mathfrak{v}_{n+2}$ are on the bottom arm.

Equivalently, the 4–manifold has intersection form

$$Q = \begin{bmatrix} -2 & 1 & 1 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & -2 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & -2 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & -2 \end{bmatrix}.$$

There are four full nice paths for Γ; they start at the four vectors V (written in the form $v^T Q$)

$$(0,0,0,0,\dots,0),$$
$$(0,2,0,0,\dots,0),$$
$$(0,0,2,0,\dots,0),$$
$$(0,0,0,0,\dots,2).$$

These vectors have squares

$$0, \quad -(n+2), \quad -(n+2), \quad -4,$$

respectively, and so the correction terms of $Y(\Gamma)$ are

$$-\frac{n+2}{4}, \quad 0, \quad 0, \quad -\frac{n-2}{4}.$$

We list the correction terms for all the dihedral manifolds with $|H_1(Y)| \le 32$ in Table 3.

4 $d(Y,\mathfrak{t})$ as a knot surgery obstruction

To demonstrate some of the techniques that will be used to prove Theorems 2 and 4, we summarize the proof of Theorem 1, due to Ghiggini:

Proof of Theorem 1 [14, Corollary 1.7] By Theorem 6, the Poincaré homology sphere is (up to orientation)

$$Y = -\left(-1; \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{5}\right).$$

Assume that Y or $-Y = S^3_{p/q}(K)$ with $p/q > 0$. Recall $S^3_{p/q}(K) = -S^3_{-p/q}(mK)$ and $d(Y,\mathfrak{t}) = -d(-Y,-\mathfrak{t})$. Then $|H_1(Y)| = p = 1$. Since Y is not a lens space, $g(K) > 0$; since it is an L–space, Theorem 11 says that

$$\frac{1}{q} \ge 2g(K) - 1.$$

Therefore, $p/q = 1$ and $g(K) = 1$.

By Theorem 9,

$$\Delta_K(T) = T - 1 + T^{-1}.$$

By Corollary 10, K is fibered. Therefore, K is the right-handed trefoil $T_{3,2}$ or the left-handed one, $T_{3,-2}$; see eg Burde and Zieschang [5]. By the calculations in the proof of Corollary 8, $S^3_1(T_{3,2}) = -S^3_{-1}(T_{3,-2}) = (-1; \frac{1}{2}, \frac{1}{3}, \frac{1}{5})$, and $S^3_1(T_{3,-2}) = -S^3_{-1}(T_{3,2}) = -(-1; \frac{1}{2}, \frac{1}{3}, \frac{1}{8})$, which is not elliptic by Theorem 6. □

Proof of Theorem 2 Assume Y is Seifert fibered but not a lens space, and $Y = S^3_{p/q}(K)$, where $p/q > 0$. In general, if $S^3_{p/q}(K)$ is an elliptic space, then $S^3_{-p/q}(mK)$ (where mK is the mirror) is elliptic too, but $S^3_{p/q}(mK)$ and $S^3_{-p/q}(K)$ are not; see Corollary 12.

$|H_1(Y)| = 2$ Then $Y = \pm(-1; \frac{1}{2}, \frac{1}{3}, \frac{1}{4})$ and $2/q \geq 2g(K) - 1$, so $p/q = 2$ with $g(K) = 1$ and $\Delta_K(T) = T - 1 + T^{-1}$, and K must be a trefoil. By Corollary 8,

$$S^3_2(T_{3,2}) = (-1; \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}).$$

$|H_1(Y)| = 3$ Then $Y = \pm(-1; \frac{1}{2}, \frac{1}{3}, \frac{1}{3})$. Either $p/q = \frac{3}{2}$ with $g(K) = 1$ and $\Delta_K(T) = T - 1 + T^{-1}$ and K the trefoil, or else $p/q = 3$ with $0 < g(K) \leq 2$. In the latter case, Theorem 9 shows that the symmetrized Alexander polynomial of K may be $\Delta_1(T)$, $\Delta_2(T)$ or $\Delta_{2'}(T)$ (see Table 1 for a number of the Alexander polynomials we will use).

i	$\Delta_i(T)$
1	$T - 1 \cdots$
2	$T^2 - T + 1 \cdots$
$2'$	$T^2 - 1 \cdots$
3	$T^3 - T^2 + T - 1 \cdots$
4	$T^4 - T^3 + T^2 - T + 1 \cdots$
5	$T^5 - T^4 + T^3 - T^2 + T - 1 \cdots$
6	$T^6 - T^5 + T^4 - T^3 + T^2 - T + 1 \cdots$
7	$T^7 - T^6 + T^5 - T^4 + T^3 - T^2 + T - 1 \cdots$
8	$T^8 - T^7 + T^6 - T^5 + T^4 - T^3 + T^2 - T + 1 \cdots$
$8'$	$T^8 - T^7 + T^5 - T^4 + T^2 - T + 1 \cdots$
$9'$	$T^9 - T^8 + T^5 - T^4 + T^3 - T^2 + 1 \cdots$

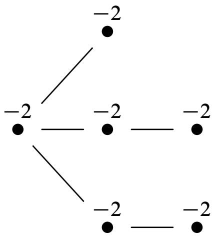
Table 1: The Alexander polynomials $\Delta_i(T)$.

To narrow this down, calculate the corresponding correction terms that would result from $(+3)$–surgery on knots with these Alexander polynomials. By Theorem 13,

$$d(S^3_3(K), i) = d(S^3_3(U), i) - 2\sum_{j=1}^{\infty} j a_{j+c} = \begin{cases} \frac{1}{2} \\ -\frac{1}{6} \end{cases} + \begin{array}{cccc} \Delta_1 & \Delta_2 & \Delta_{2'} & \\ -2 & -2 & -4 & i = 0, \\ 0 & -2 & -2 & i = \pm 1, \end{array}$$

$$= \begin{cases} \begin{array}{cccc} \Delta_1 & \Delta_2 & \Delta_{2'} & \\ -\frac{3}{2} & -\frac{3}{2} & -\frac{7}{2} & i = 0, \\ -\frac{1}{6} & -\frac{13}{6} & -\frac{13}{6} & i = \pm 1. \end{array} \end{cases}$$

On the other hand, $d(Y,\mathfrak{t})$ may be calculated as in Example 15; $-Y=(-2;\frac{1}{2},\frac{2}{3},\frac{2}{3})$ has $e(-Y)<0$, and Γ is



where $\mathfrak{v}_1$ is the degree-3 vertex on the left, $\mathfrak{v}_2$ is on the top arm, $\mathfrak{v}_3,\mathfrak{v}_4$ are on the middle arm, and $\mathfrak{v}_5,\mathfrak{v}_6$ are on the bottom arm.

There are three nice full paths, starting with the vectors (written in the form $v^T Q$)

$$(0,0,0,0,0,0),$$
$$(0,0,0,2,0,0),$$
$$(0,0,0,0,0,2),$$

with squares

$$0,\quad \tfrac{16}{3},\quad \tfrac{16}{3},$$

so the correction terms $d(Y(\Gamma),\mathfrak{t})$ are, in some order,

$$-\tfrac{3}{2},\quad -\tfrac{1}{6},\quad -\tfrac{1}{6}.$$

These terms do not match the correction terms coming from surgery on a knot with Alexander polynomial $\Delta_2(T)$ or $\Delta_{2'}(T)$, so the Alexander polynomial can only be $\Delta_1(T)$, and K is a trefoil. Note $S^3_3(T_{3,2})=(-1;\frac{1}{2},\frac{1}{3},\frac{1}{3})$.

$\boldsymbol{|H_1(Y)|=4}$ Then $Y=\pm(-1;\frac{1}{2},\frac{1}{2},\frac{1}{n})$ with n odd. Either $p/q=\frac{4}{3}$ with $g(K)=1$ and K the trefoil (but this Y is not elliptic),[4] or $p/q=4$ with $g(K)=1$ or 2 and Alexander polynomial $\Delta_1(T)$, $\Delta_2(T)$ or $\Delta_{2'}(T)$. In the latter case,

$$d(S^3_4(K),i)=\begin{array}{c} \\ \\ \\ \end{array}\begin{cases} \begin{array}{ccc|l} \Delta_1 & \Delta_2 & \Delta_{2'} & \\ -\frac{5}{4} & -\frac{5}{4} & -\frac{13}{4} & i=0,\\ 0 & -2 & -2 & i=\pm1,\\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & i=2. \end{array} \end{cases}$$

These terms match the $d(Y,\mathfrak{t})$ calculated in Example 15 only for $\Delta_1(T)$ and $n=3$, which corresponds to $S^3_4(T_{3,2})=(-1;\frac{1}{2},\frac{1}{2},\frac{1}{3})$.

[4] It was previously known that there is no finite $\frac{4}{3}$–surgery: finite surgery on a hyperbolic knot must be integral or half-integral [4, Theorems 1.1 and 1.2], and no $(\pm\frac{4}{3})$–surgery on a torus or satellite knot gives this Y [2, Theorem 7].

$|H_1(Y)| = 5$ or 6 All elliptic Y with these first homologies are lens spaces.

$|H_1(Y)| = 7$ Then $Y = \pm(-1; \frac{1}{2}, \frac{1}{3}, \frac{2}{5})$ with $d(Y, \mathfrak{t})$ as in Table 2. By Boyer and Zhang [4, Theorem 1.2], elliptic hyperbolic surgeries must be integral or half-integral, so it may be $p/q = \frac{7}{2}$ with $g(K) \leq 2$ and Alexander polynomial $\Delta_1(T)$, $\Delta_2(T)$ or $\Delta_{2'}(T)$, or $p/q = 7$ with $g(K) \leq 4$ and appropriate Alexander polynomial. Of the 18 possible sets of $d(S^3_{7/q}(K), \mathfrak{t})$, the only ones that match $d(Y, \mathfrak{t})$ are $\frac{7}{2}$–surgery and $\Delta_1(T)$ (which must be the trefoil) and $\frac{7}{1}$–surgery and $\Delta_2(T)$ (which could be $T_{5,2}$). Finally,

$$S^3_7(T_{5,2}) = S^3_{7/2}(T_{3,2}) = (-1; \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{2}{5}).$$

p	1	2	3	7	9
d	-2	$-\frac{7}{4}$	$-\frac{1}{6}$	$\frac{1}{14}$	0
		$-\frac{1}{4}$	$-\frac{3}{2}$	$-\frac{3}{14}$	$-\frac{10}{9}$
			$-\frac{1}{6}$	$-\frac{19}{14}$	$-\frac{4}{9}$
				$-\frac{1}{2}$	$\frac{2}{9}$
				$-\frac{19}{14}$	0
				$-\frac{3}{14}$	$\frac{2}{9}$
				$\frac{1}{14}$	$-\frac{4}{9}$
					$-\frac{10}{9}$
					0

Table 2: The correction terms for Y for $p = |H_1(Y)| < 10$. (See Table 3 for $p = 4$ and 8.) Y is given the canonical orientation as in Theorem 6.

$|H_1(Y)| = 8$ Then $Y = \pm(-1; \frac{1}{2}, \frac{1}{2}, \frac{2}{n})$ with n odd. The $d(Y, \mathfrak{t})$ are listed in Table 3. If $p/q = \pm 8$, then $g(K) \leq 4$. There are only two possible choices of $\Delta_K(T)$ and n that give the same correction terms; for these cases, $d(S^3_8(K), \mathfrak{t})$, $\Delta_i(T)$ and n are listed in Table 4.

$|H_1(Y)| = 9$ Then $Y = (-1; \frac{1}{2}, \frac{1}{3}, \frac{2}{3})$ and the correction terms are given in Table 2. If $p/q = 9$, then $g(K) \leq 5$; if $p/q = \frac{9}{2}$, then $g(K) \leq 2$. Comparing the correction terms shows that the Alexander polynomial must be $\Delta_1(T)$ when $q = 1$, so K is the trefoil. □

Proof of Theorem 4 The calculations were performed by computer and are similar to the calculations for $p = 8$. We summarize the results:

p	4	8	12	16	20	20	24	28	28	28	32	32
k	1	1	1	1	1	2	1	1	2	3	1	3
d											$-\frac{7}{4}$	$\frac{1}{4}$
								$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
							$\frac{5}{4}$	$-\frac{3}{14}$	$-\frac{3}{14}$	$\frac{3}{14}$	$-\frac{1}{4}$	$-\frac{1}{4}$
					1	0	$-\frac{1}{4}$	$-\frac{3}{14}$	$-\frac{3}{14}$	$\frac{3}{14}$	$-\frac{1}{4}$	$-\frac{1}{4}$
				0	$-\frac{1}{5}$	$-\frac{2}{5}$	$-\frac{1}{12}$	$\frac{1}{14}$	$\frac{1}{14}$	$-\frac{1}{14}$	$\frac{7}{8}$	$\frac{5}{8}$
			$\frac{1}{2}$	0	$-\frac{1}{5}$	$-\frac{2}{5}$	$-\frac{1}{12}$	$\frac{1}{14}$	$\frac{1}{14}$	$-\frac{1}{14}$	$\frac{7}{8}$	$\frac{5}{8}$
		$\frac{1}{4}$	$-\frac{1}{6}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{5}{12}$	$\frac{9}{14}$	$\frac{9}{14}$	$-\frac{9}{14}$	$-\frac{1}{8}$	$-\frac{3}{8}$
	0	$-\frac{1}{4}$	$-\frac{1}{6}$	$-\frac{1}{4}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{5}{12}$	$\frac{9}{14}$	$\frac{9}{14}$	$-\frac{9}{14}$	$-\frac{1}{8}$	$-\frac{3}{8}$
	$\frac{2-n}{4}^*$	$\frac{4-n}{8}$	$\frac{8-n}{12}$	$\frac{14-n}{16}$	$\frac{2-n}{20}$	$\frac{10-n}{20}$	$\frac{16-n}{24}$	$\frac{44-n}{28}$	$\frac{4-n}{28}$	$\frac{8-n}{28}$	$\frac{26-n}{32}$	$\frac{22-n}{32}$
	$\frac{-2-n}{4}^*$	$\frac{-4-n}{8}$	$\frac{-4-n}{12}$	$\frac{-2-n}{16}$	$\frac{22-n}{20}$	$\frac{-10-n}{20}$	$\frac{-16-n}{24}$	$\frac{24-n}{28}$	$\frac{-4-n}{28}$	$\frac{-8-n}{28}$	$\frac{58-n}{32}$	$\frac{-10-n}{32}$
			$\frac{4-n}{12}^*$	$\frac{6-n}{16}$	$\frac{10-n}{20}$	$\frac{14-n}{20}$	$\frac{8-n}{24}$	$\frac{16-n}{28}$	$\frac{20-n}{28}$	$\frac{20-n}{28}$	$\frac{2-n}{32}$	$\frac{30-n}{32}$
			$\frac{-8-n}{12}^*$	$\frac{-10-n}{16}$	$\frac{-10-n}{20}$	$\frac{-6-n}{20}$	$\frac{-8-n}{24}$	$\frac{-16-n}{28}$	$\frac{-8-n}{28}$	$\frac{-20-n}{28}$	$\frac{34-n}{32}$	$\frac{-2-n}{32}$
					$\frac{6-n}{20}^*$	$\frac{2-n}{20}^*$	$\frac{8-n}{24}$	$\frac{12-n}{28}$	$\frac{24-n}{28}$	$\frac{16-n}{28}$	$\frac{18-n}{32}$	$\frac{6-n}{32}$
					$\frac{-14-n}{20}^*$	$\frac{-18-n}{20}^*$	$\frac{32-n}{24}$	$\frac{-4-n}{28}$	$\frac{-24-n}{28}$	$\frac{-12-n}{28}$	$\frac{-14-n}{32}$	$\frac{-26-n}{32}$
								$\frac{8-n}{28}^*$	$\frac{-12-n}{28}^*$	$\frac{-4-n}{28}^*$	$\frac{-22-n}{32}$	$\frac{-18-n}{32}$
								$\frac{-20-n}{28}^*$	$\frac{16-n}{28}^*$	$\frac{-32-n}{28}^*$	$\frac{10-n}{32}$	$\frac{14-n}{32}$

Table 3: The correction terms of $Y = (-1; \frac{1}{2}, \frac{1}{2}, \frac{m}{n})$ with $n > 0$ and $|H_1(Y)| = p \leq 32$ where $p = 4m$ and $k = n \bmod m$. For $n < 0$, use $|n|$ and reverse the sign on each formula. Note that $d(Y_{m,-n}, \mathfrak{t}) = -d(Y_{m,n}, -\mathfrak{t}) = -d(Y_{m,n}, \mathfrak{t})$ since $(-1; \frac{1}{2}, \frac{1}{2}, -\frac{m}{n}) = -(-1; \frac{1}{2}, \frac{1}{2}, \frac{m}{n})$. The correction terms come in two flavors, those constant for the entire family, which are of the form $(\cdot)/4m$ (not necessarily reduced), and those dependent on n, which are of the form $(n + \cdot)/4m$. The correction terms marked by $*$ correspond to $\mathfrak{t}$ which occur once each; in fact, they are in $\mathrm{Spin}(Y)$. The other values occur for two $\mathfrak{t} \in \mathrm{Spin}^c(Y)$ each.

For each choice of $|H_1(Y)| = 4m$, Theorem 6 gives a description like $(-1; \frac{1}{2}, \frac{1}{2}, \frac{m}{n})$ for all possible Y. The correction terms for each such manifold are listed in Table 3. On the other hand, assuming Y is a knot surgery, Theorem 11 restricts the surgery coefficients

p	4	8	8	12	12	16	16	20	20	24	24	28	28	28	28	32	32
n	–3	3	–5	5	–7	7	–9	9	–11	11	–13	5	–11	13	–15	15	–17
Δ_i	Δ_1	Δ_1	Δ_2	Δ_2	Δ_3	Δ_3	Δ_4	Δ_4	Δ_5	Δ_5	Δ_6	$\Delta_{8'}$	$\Delta_{9'}$	Δ_6	Δ_7	Δ_7	Δ_8
d	$-\frac{5^*}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{3^*}{4}$	$-\frac{5^*}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{3^*}{4}$	$-\frac{5^*}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{3^*}{4}$	$\frac{3^*}{4}$	$\frac{3^*}{4}$	$-\frac{5^*}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$
	0	$\frac{7}{8}$	$-\frac{9}{8}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{13}{16}$	$-\frac{19}{16}$	$-\frac{1}{5}$	$-\frac{1}{5}$	$\frac{19}{24}$	$-\frac{29}{24}$	$-\frac{3}{14}$	$-\frac{3}{14}$	$-\frac{3}{14}$	$-\frac{3}{14}$	$\frac{25}{32}$	$-\frac{39}{32}$
	$-\frac{1^*}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{13}{12}$	$-\frac{11}{12}$	0	0	$\frac{19}{20}$	$-\frac{21}{20}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$\frac{25}{28}$	$-\frac{31}{28}$	$\frac{25}{28}$	$-\frac{31}{28}$	$-\frac{1}{8}$	$-\frac{1}{8}$
		$-\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{21}{16}$	$-\frac{11}{16}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{9}{8}$	$-\frac{7}{8}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{33}{32}$	$-\frac{31}{32}$
				$\frac{1}{12}$	$\frac{1}{12}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{31}{20}$	$-\frac{9}{20}$	$\frac{5}{12}$	$\frac{5}{12}$	$-\frac{19}{28}$	$-\frac{19}{28}$	$\frac{37}{28}$	$-\frac{19}{28}$	$\frac{1}{4}$	$\frac{1}{4}$
				$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{5}{16}$	$\frac{5}{16}$	1	1	$\frac{43}{24}$	$-\frac{5}{24}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{49}{32}$	$-\frac{15}{32}$
				$-\frac{1^*}{4}$	$-\frac{1^*}{4}$	0	0	$\frac{11}{20}$	$\frac{11}{20}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{1}{28}$	$\frac{1}{28}$	$\frac{57}{28}$	$\frac{1}{28}$	$\frac{7}{8}$	$\frac{7}{8}$
						$-\frac{3}{16}$	$-\frac{3}{16}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{19}{24}$	$\frac{19}{24}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{73}{32}$	$\frac{9}{32}$
								$-\frac{1}{20}$	$-\frac{1}{20}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{29}{28}$	$-\frac{27}{28}$	$\frac{29}{28}$	$\frac{29}{28}$	$\frac{7}{4}$	$\frac{7}{4}$
								$-\frac{1}{5}$	$-\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{9}{14}$	$\frac{41}{32}$	$\frac{41}{32}$
								$-\frac{1^*}{4}$	$-\frac{1^*}{4}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$\frac{9}{28}$	$\frac{9}{28}$	$\frac{9}{28}$	$\frac{9}{28}$	$\frac{7}{8}$	$\frac{7}{8}$
										$-\frac{5}{24}$	$-\frac{5}{24}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{17}{32}$	$\frac{17}{32}$
												$-\frac{3}{28}$	$-\frac{3}{28}$	$-\frac{3}{28}$	$-\frac{3}{28}$	$\frac{1}{4}$	$\frac{1}{4}$
												$-\frac{3}{14}$	$-\frac{3}{14}$	$-\frac{3}{14}$	$-\frac{3}{14}$	$\frac{1}{32}$	$\frac{1}{32}$
												$-\frac{1^*}{4}$	$-\frac{1^*}{4}$	$-\frac{1^*}{4}$	$-\frac{1^*}{4}$	$-\frac{1}{8}$	$-\frac{1}{8}$
																$-\frac{7}{32}$	$-\frac{7}{32}$

Table 4: All possible cases for $p \leq 32$ where $Y = (-1; \frac{1}{2}, \frac{1}{2}, \frac{m}{n})$ has the same correction terms as some $S^3_p(K)$, if the latter exists. Also listed are n and the Alexander polynomial $\Delta_i(T)$ from Table 1. Note that $p = 4m = |H_1(Y)|$. The correction terms marked by $*$ correspond to $\mathfrak{t}$ which occur once each; in fact, they are in $\mathrm{Spin}(Y)$. The other values occur for two $\mathfrak{t} \in \mathrm{Spin}^c(Y)$ each.

that can give Y, and Theorems 9 and 10 restrict the possible Alexander polynomials of the knot, of which there are slightly fewer than 2^x for $x = p/(2q)$. Using Theorem 13, it is possible to calculate the correction terms of the resulting surgeries for each knot (assuming they are indeed L–spaces). Table 4 lists the cases where the correction terms for surgeries on the knots with the given Alexander polynomials from Table 1 match the correction terms for the appropriate elliptic manifolds.

For uniqueness of the first set of cases in Theorem 4, note that these manifolds do not appear in the list obtainable by $p/1$ surgery and so are not surgery on hyperbolic K. They are also not on the list of finite satellite surgeries from [2] and are not obtainable from surgery on any other torus knots by Theorem 7. □

5 Conjectures

We have applied the correction terms to obstruct a manifold being surgery on a knot, and it was a sufficient obstruction in all but one of the cases studied, where it was inconclusive; that manifold is realized by a nonintegral surgery on a torus knot. On the basis of this evidence, we feel compelled to state the following conjecture, although we do not have any deeper intuition about why it would be true.

Conjecture 16 *The Heegaard Floer correction terms $d(Y, \mathfrak{t})$ are sufficient to distinguish which finite manifolds are surgeries on knots in S^3.*

A careful examination of Theorem 4 also suggests a more specific conjecture: all known examples of $(-1; \frac{1}{2}, \frac{1}{2}, \frac{m}{n})$ which are knot surgeries obey $n \leq 2m+1$ (and the cases $n = 2m \pm 1$ are realized by torus knots). Since this paper appeared on the arXiv, the author has proven that each family of dihedral manifolds with a fixed $|H_1(Y)| = 4m$ includes finitely many knot surgeries [9], but the bounds given there appear to be capable of improvement:

Conjecture 17 *If $n > 2m+1$, then $(-1; \frac{1}{2}, \frac{1}{2}, \frac{m}{n})$ is never a knot surgery.*

Finally, work of Ni and Zhang [28] indicates that 7 and 8 may be characterizing slopes for $T_{5,2}$ (p/q is a *characterizing slope* for $T_{5,2}$ if $S^3_{p/q}(K) \cong S^3_{p/q}(T_{5,2})$ means $K = T_{5,2}$).

Conjecture 18 *The phrase "With the possible exception of $S^3_7(T_{5,2})$ and $S^3_8(T_{5,2})$" in Theorem 2 may be removed. The bound in Corollary 3 may be increased from 7 to 10.*

References

[1] **J Berge**, *Obtaining lens spaces by surgeries on knots*, unpublished manuscript

[2] **S A Bleiler**, **C D Hodgson**, *Spherical space forms and Dehn filling*, Topology 35 (1996) 809–833 MR1396779

[3] **S Boyer**, **X Zhang**, *Finite Dehn surgery on knots*, J. Amer. Math. Soc. 9 (1996) 1005–1050 MR1333293

[4] **S Boyer**, **X Zhang**, *A proof of the finite filling conjecture*, J. Differential Geom. 59 (2001) 87–176 MR1909249

[5] **G Burde**, **H Zieschang**, *Knots*, de Gruyter Studies in Mathematics 5, de Gruyter, Berlin (1985) MR808776

[6] **M Culler**, **C M Gordon**, **J Luecke**, **P B Shalen**, *Dehn surgery on knots*, Ann. of Math. 125 (1987) 237–300 MR881270

[7] **J C Dean**, *Small Seifert-fibered Dehn surgery on hyperbolic knots*, Algebr. Geom. Topol. 3 (2003) 435–472 MR1997325

[8] **A Deruelle**, **K Miyazaki**, **K Motegi**, *Networking Seifert surgeries on knots*, Mem. Amer. Math. Soc. 1021 (2012) MR2951665

[9] **M Doig**, *Obstructing finite surgery* `arXiv:1302.6130`

[10] **M Doig**, *Spherical Seifert fibered spaces, knot surgeries, and Heegaard Floer homology*, PhD thesis, Princeton (2010) Available at `http://search.proquest.com/docview/755500657`

[11] **R Fintushel**, **R J Stern**, *Constructing lens spaces by surgery on knots*, Math. Z. 175 (1980) 33–51 MR595630

[12] **D Futer**, **M Ishikawa**, **Y Kabaya**, **T W Mattman**, **K Shimokawa**, *Finite surgeries on three-tangle pretzel knots*, Algebr. Geom. Topol. 9 (2009) 743–771 MR2496889

[13] **D Gabai**, *Foliations and the topology of* 3*–manifolds, III*, J. Differential Geom. 26 (1987) 479–536 MR910018

[14] **P Ghiggini**, *Knot Floer homology detects genus-one fibred knots*, Amer. J. Math. 130 (2008) 1151–1169 MR2450204

[15] **R E Gompf**, **A I Stipsicz**, 4*–manifolds and Kirby calculus*, Graduate Studies in Mathematics 20, Amer. Math. Soc. (1999) MR1707327

[16] **C M Gordon**, *Dehn surgery and satellite knots*, Trans. Amer. Math. Soc. 275 (1983) 687–708 MR682725

[17] **C M Gordon**, **J Luecke**, *Knots are determined by their complements*, Bull. Amer. Math. Soc. 20 (1989) 83–87 MR972070

[18] **J E Greene**, *The lens space realization problem*, Ann. of Math. 177 (2013) 449–511 MR3010805

[19] **A Juhász**, *Holomorphic discs and sutured manifolds*, Algebr. Geom. Topol. 6 (2006) 1429–1457 MR2253454

[20] **R Kirby**, *A calculus for framed links in* S^3, Invent. Math. 45 (1978) 35–56 MR0467753

[21] **R Kirby**, *Problems in low dimensional manifold theory*, from: "Algebraic and geometric topology, part 2", (R J Milgram, editor), Proc. Sympos. Pure Math. 32, Amer. Math. Soc. (1978) 273–312 MR520548

[22] **W B R Lickorish**, *A representation of orientable combinatorial 3–manifolds*, Ann. of Math. 76 (1962) 531–540 MR0151948

[23] **T W Mattman**, *Cyclic and finite surgeries on pretzel knots*, J. Knot Theory Ramifications 11 (2002) 891–902 MR1936241

[24] **T W Mattman**, **K Miyazaki**, **K Motegi**, *Seifert-fibered surgeries which do not arise from primitive/Seifert-fibered constructions*, Trans. Amer. Math. Soc. 358 (2006) 4045–4055 MR2219009

[25] **L Moser**, *Elementary surgery along a torus knot*, Pacific J. Math. 38 (1971) 737–745 MR0383406

[26] **W D Neumann**, *A calculus for plumbing applied to the topology of complex surface singularities and degenerating complex curves*, Trans. Amer. Math. Soc. 268 (1981) 299–344 MR632532

[27] **Y Ni**, *Knot Floer homology detects fibred knots*, Invent. Math. 170 (2007) 577–608 MR2357503

[28] **Y Ni**, **X Zhang**, *Characterizing slopes for torus knots*, Algebr. Geom. Topol. 14 (2014) 1249–1274

[29] **P S Ozsváth**, **Z Szabó**, *Absolutely graded Floer homologies and intersection forms for four-manifolds with boundary*, Adv. Math. 173 (2003) 179–261 MR1957829

[30] **P S Ozsváth**, **Z Szabó**, *On the Floer homology of plumbed three-manifolds*, Geom. Topol. 7 (2003) 185–224 MR1988284

[31] **P S Ozsváth**, **Z Szabó**, *Holomorphic disks and genus bounds*, Geom. Topol. 8 (2004) 311–334 MR2023281

[32] **P S Ozsváth**, **Z Szabó**, *Holomorphic disks and topological invariants for closed three-manifolds*, Ann. of Math. 159 (2004) 1027–1158 MR2113019

[33] **P S Ozsváth**, **Z Szabó**, *Holomorphic disks and three-manifold invariants: properties and applications*, Ann. of Math. 159 (2004) 1159–1245 MR2113020

[34] **P S Ozsváth**, **Z Szabó**, *On knot Floer homology and lens space surgeries*, Topology 44 (2005) 1281–1300 MR2168576

[35] **P S Ozsváth**, **Z Szabó**, *Holomorphic triangles and invariants for smooth four-manifolds*, Adv. Math. 202 (2006) 326–400 MR2222356

[36] **P S Ozsváth**, **Z Szabó**, *Knot Floer homology and rational surgeries*, Algebr. Geom. Topol. 11 (2011) 1–68 MR2764036

[37] **G Perelman**, *The entropy formula for the Ricci flow and its geometric applications* `arXiv:math/0211159`

[38] **G Perelman**, *Ricci flow with surgery on three-manifolds* `arXiv:math/0303109`

[39] **G Perelman**, *Finite extinction time for the solutions to the Ricci flow on certain three-manifolds* `arXiv:math/0307245`

[40] **T Perutz**, *Hamiltonian handleslides for Heegaard Floer homology*, from: "Proceedings of Gökova Geometry–Topology Conference 2007", (S Akbulut, T Önder, R J Stern, editors) (2008) 15–35 MR2509747

[41] **P Scott**, *The geometries of 3–manifolds*, Bull. London Math. Soc. 15 (1983) 401–487 MR705527

[42] **H Seifert**, *Topologie dreidimensionaler gefaserter Räume*, Acta Math. 60 (1933) 147–238 MR1555366

[43] **H Seifert**, **W Threlfall**, *Seifert and Threlfall: A textbook of topology*, Pure and Applied Mathematics 89, Academic, New York (1980) MR575168

[44] **A H Wallace**, *Modifications and cobounding manifolds, IV*, J. Math. Mech. 12 (1963) 445–484 MR0154297

[45] **X Zhang**, *On property I for knots in S^3*, Trans. Amer. Math. Soc. 339 (1993) 643–657 MR1154545

Department of Mathematics, Syracuse University
215 Carnegie Building, Syracuse, NY 13244-1150, USA

midoig@syr.edu

Received: 9 July 2012 Revised: 7 March 2014