Algebraic & Geometric Topology 17 (2017) 1–16

Three-manifold mutations detected by Heegaard Floer homology

CORRIN CLARKSON

Given an orientation-preserving self-diffeomorphism φ of a closed, orientable surface S with genus at least two and an embedding f of S into a three-manifold M, we construct a mutant manifold by cutting M along $f(S)$ and regluing by $f\varphi f^{-1}$. We will consider whether there exist nontrivial gluings such that for any embedding, the manifold M and its mutant have isomorphic Heegaard Floer homology. In particular, we will demonstrate that if φ is not isotopic to the identity map, then there exists an embedding of S into a three-manifold M such that the rank of the nontorsion summands of $\widehat{\mathrm{HF}}$ of M differs from that of its mutant. We will also show that if the gluing map is isotopic to neither the identity nor the genus-two hyperelliptic involution, then there exists an embedding of S into a three-manifold M such that the total rank of $\widehat{\mathrm{HF}}$ of M differs from that of its mutant.

57M27, 57M60

1 Introduction

In 2001, Ozsváth and Szabó introduced Heegaard Floer homology, a topological invariant that assigns a collection of abelian groups to each closed, oriented three-manifold equipped with a Spin^c–structure [25]. Given a topological invariant, it is natural to ask which topological operations it detects. In this paper, we will consider whether or not Heegaard Floer homology detects *mutation*, the operation of cutting a three-manifold along an embedded surface and regluing by a surface diffeomorphism. In particular, we will show that the version of Heegaard Floer homology denoted by $\widehat{\mathrm{HF}}$ can detect mutation by any nontrivial diffeomorphisms of a closed, orientable surface of genus greater than one.

In order to make this statement more precise, we introduce the following terminology and notation. Let $g \geq 2$ be a natural number and let S_g be a genus-g, closed, connected, orientable, smooth surface. By a *manifold–surface pair*, we will mean a pair (M, f) where M is a closed, connected, oriented, smooth three-manifold and $f\colon S_g \to M$ is a smooth embedding of S_g into M such that $f(S_g)$ separates M. To an orientation-preserving diffeomorphism $\varphi\colon S_g \to S_g$ and a manifold–surface pair (M, f), we

Published: 26 January 2017 DOI: 10.2140/agt.2017.17.1

associate the *mutant manifold* M_f^φ that results from cutting M along $f(S_g)$ and regluing by $f\varphi f^{-1}$. The mutant manifold M_f^φ inherits an orientation from M. Finally, we will denote the nontorsion summands of $\widehat{\mathrm{HF}}$ in the following way:

$$\widehat{\mathrm{HF}}_{\mathrm{NT}}(M,\mathfrak{s}) := \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{\mathrm{HF}}(M,\mathfrak{s}).$$

Here, $c_1(\mathfrak{s})$ is the first Chern class of the Spin^c–structure $\mathfrak{s}$.

Theorem 1.1 *Let φ be an orientation-preserving self-diffeomorphism of S_g that is not isotopic to the identity map. Then there exists a manifold–surface pair (M, f) such that*

$$\mathrm{rk}\, \widehat{\mathrm{HF}}_{\mathrm{NT}}(M,\mathfrak{s}) \neq \mathrm{rk}\, \widehat{\mathrm{HF}}_{\mathrm{NT}}(M_f^\varphi,\mathfrak{s}).$$

Our proof of this result begins with a reformulation of the theorem statement. In Section 2, we use Ivanov and Long's results about subgroups of mapping class groups to show that Theorem 1.1 is equivalent to the statement that a particular subgroup of the mapping class group $\mathrm{Mod}(S_g)$ contains neither the genus-2 hyperelliptic involution nor any pseudo-Anosov elements. In Section 3, we show that the genus-2 hyperelliptic involution is not an element of this subgroup by giving an example of a mutation by this map that changes the rank of $\widehat{\mathrm{HF}}_{\mathrm{NT}}$. In Section 4, we use the fact that $\widehat{\mathrm{HF}}$ detects the Thurston seminorm on homology to establish the existence of mutations by pseudo-Anosov maps that change the rank of $\widehat{\mathrm{HF}}_{\mathrm{NT}}$. This step uses work of Ozsváth and Szabó [23], Ni [22] and Hedden and Ni [12]. We conclude the proof of Theorem 1.1 in Section 5.

Then in Section 6, we use similar techniques to show that the total rank of $\widehat{\mathrm{HF}}$ can detect mutations by noncentral mapping classes:

Theorem 1.2 *Let $[\varphi] \in \mathrm{Mod}(S_g)$ be a mapping class that is isomorphic to neither the identity nor the genus-2 hyperelliptic involution. Then there exists a manifold–surface pair (M, f) such that*

$$\mathrm{rk}\, \widehat{\mathrm{HF}}(M) \neq \mathrm{rk}\, \widehat{\mathrm{HF}}(M_f^\varphi).$$

The question of whether the total rank of $\widehat{\mathrm{HF}}$ is preserved by mutation along a separating surface by the genus-2 hyperelliptic involution remains open.

The effect of mutating by the genus-2 hyperelliptic involution has been considered for invariants related to $\widehat{\mathrm{HF}}$. In particular, Ozsváth and Szabó showed that the Heegaard Floer knot invariant $\widehat{\mathrm{HFK}}$ can detect knot genus, which can be changed by mutations of this form [23, Theorem 1.2]. Conversely, Moore and Starkston produced computational evidence that the total rank of $\widehat{\mathrm{HFK}}$ in each δ–grading is preserved by mutation by the genus-2 hyperelliptic involution [21]. Finally, Ruberman showed that the instanton Floer homology with $\mathbb{Z}/2\mathbb{Z}$ coefficients of an oriented homology 3–sphere is preserved by mutations of this form [26, Theorem 1].[1]

The results of this paper also fit into the growing body of work on group actions on triangulated categories. See Section 7 for a more detailed discussion.

Acknowledgements I am grateful to Robert Lipshitz for suggesting this problem and for many useful discussions and ideas. I am also grateful to Jason Behrstock, Ian Biringer, Nathan Dunfield, Julia Elisenda Grigsby, Adam Levine, Dan Margalit, Walter Neumann and Dylan Thurston for helpful conversations. Finally, I would like to thank the referee for helpful comments and suggestions.

While working on this project, I was partially supported by NSF grant number DMS-0739392.

2 Theorem reformulation

Let $\mathrm{Mod}(S_g)$ be the mapping class group of S_g. In this section, we will reformulate Theorem 1.1 as a statement about the triviality of a normal subgroup of $\mathrm{Mod}(S_g)$.

Definition 2.1 A mapping class $[\varphi] \in \mathrm{Mod}(S_g)$ is $\widehat{\mathrm{HF}}$–*invisible* if for all manifold–surface pairs (M, f) we have that

$$\mathrm{rk}\,\widehat{\mathrm{HF}}_{\mathrm{NT}}(M, \mathfrak{s}) = \mathrm{rk}\,\widehat{\mathrm{HF}}_{\mathrm{NT}}(M_f^{\varphi}, \mathfrak{s}).$$

The $\widehat{\mathrm{HF}}$–invisible mapping classes are well defined, because mutating by isotopic diffeomorphisms results in diffeomorphic mutant manifolds. Moreover, they form a normal subgroup.

Proposition 2.2 *The* $\widehat{\mathrm{HF}}$*–invisible mapping classes form a normal subgroup of the mapping class group* $\mathrm{Mod}(S_g)$.

Proof The mapping class of the identity map is $\widehat{\mathrm{HF}}$–invisible, because mutating by any of its representatives preserves the diffeomorphism class of the manifold. We will

[1] In private communication, Ruberman indicated that there is an issue with the signs in [26] due to a particular moduli space not being orientable. However, this is not relevant when one considers $\mathbb{Z}/2\mathbb{Z}$ coefficients.

show that mutations by products, inverses and conjugates of $\widehat{\mathrm{HF}}$–invisible mapping classes preserve the rank of $\widehat{\mathrm{HF}}_{\mathrm{NT}}$.

Let (M, f) be a manifold–surface pair and let M_1 and M_2 be the closures of the two connected components of $M \setminus f(S_g)$. Finally, let α and β be arbitrary orientation-preserving self-diffeomorphisms of S_g. The mutant manifold M_f^{α} can be made into a manifold–surface pair by composing the embedding $f\colon S_g \mapsto M_1$ with the inclusion of M_1 into M_f^{α}. Let (N, h) denote this pair. Mutating (N, h) by β gives the mutant N_h^{β} which is constructed by using $(f\alpha)\beta f^{-1}$ to glue M_1 to M_2. Thus, N_h^{β} is diffeomorphic to $M_f^{\alpha\beta}$ by construction, and we can view mutation by a composite map as a sequence of mutations.

Let $[\varphi]$ and $[\tau]$ be $\widehat{\mathrm{HF}}$–invisible mapping classes. It follows that mutating by either φ or τ preserves the rank of $\widehat{\mathrm{HF}}_{\mathrm{NT}}$. Thus, if we view mutating (M, f) by the composition $\varphi\tau$ as a mutation by φ followed by a mutation by τ, we find that

$$\operatorname{rk} \widehat{\mathrm{HF}}_{\mathrm{NT}}(M, \mathfrak{s}) = \operatorname{rk} \widehat{\mathrm{HF}}_{\mathrm{NT}}(M_f^{\varphi}, \mathfrak{s}) = \operatorname{rk} \widehat{\mathrm{HF}}_{\mathrm{NT}}(M_f^{\varphi\tau}, \mathfrak{s}).$$

Therefore, the product $[\varphi][\tau] = [\varphi\tau]$ is $\widehat{\mathrm{HF}}$–invisible.

Mutating (M, f) by the composite map $\varphi^{-1}\varphi$ does not change its diffeomorphism class. Furthermore, if we view this mutation sequentially, the second mutation preserves the rank of $\widehat{\mathrm{HF}}_{\mathrm{NT}}$. Thus, we have that

$$\operatorname{rk} \widehat{\mathrm{HF}}_{\mathrm{NT}}(M, \mathfrak{s}) = \operatorname{rk} \widehat{\mathrm{HF}}_{\mathrm{NT}}(M_f^{\varphi^{-1}\varphi}, \mathfrak{s}) = \operatorname{rk} \widehat{\mathrm{HF}}_{\mathrm{NT}}(M_f^{\varphi^{-1}}, \mathfrak{s}).$$

Therefore, the inverse mapping class $[\varphi]^{-1} = [\varphi^{-1}]$ is $\widehat{\mathrm{HF}}$–invisible.

Let $[\psi] \in \operatorname{Mod}(S_g)$ be an arbitrary mapping class. Composing f with ψ gives a new embedding $f\psi\colon S_g \to M$. Mutating the manifold–surface pair $(M, f\psi)$ by φ gives the mutant manifold $M_{f\psi}^{\varphi}$. This mutant is constructed by using $(f\psi)\varphi(f\psi)^{-1}$ to glue M_1 to M_2. In a similar manner, the mutant $M_f^{\psi\varphi\psi^{-1}}$ is constructed by using $f(\psi\varphi\psi^{-1})f^{-1}$ to glue M_1 to M_2 and is thus diffeomorphic to $M_{f\psi}^{\varphi}$. Moreover, the rank of $\widehat{\mathrm{HF}}_{\mathrm{NT}}(M_{f\psi}^{\varphi})$ is the same as that of M, because $[\varphi]$ is $\widehat{\mathrm{HF}}$–invisible. Thus, we have that

$$\operatorname{rk} \widehat{\mathrm{HF}}_{\mathrm{NT}}(M, \mathfrak{s}) = \operatorname{rk} \widehat{\mathrm{HF}}_{\mathrm{NT}}(M_{f\psi}^{\varphi}, \mathfrak{s}) = \operatorname{rk} \widehat{\mathrm{HF}}_{\mathrm{NT}}(M_f^{\psi\varphi\psi^{-1}}, \mathfrak{s}).$$

Therefore, the conjugate $[\psi][\varphi][\psi]^{-1} = [\psi\varphi\psi^{-1}]$ is $\widehat{\mathrm{HF}}$–invisible. It follows that the $\widehat{\mathrm{HF}}$–invisible mapping classes form a normal subgroup of $\operatorname{Mod}(S_g)$. □

Theorem 1.1 is equivalent to the statement that the normal subgroup of $\widehat{\mathrm{HF}}$–invisible mapping classes is trivial. Reformulating the theorem statement in this way allows us to leverage the group structure of $\operatorname{Mod}(S_g)$. We begin by recalling a few definitions from the theory of mapping class groups.

The *Torelli group* is the normal subgroup consisting of those mapping classes whose representatives induce the identity map on homology and is denoted by $\mathcal{I}(S_g)$. If $g=2$, then $\mathrm{Mod}(S_g)$ has a unique order two element that acts by $-\mathrm{id}$ on $H_1(S_2;\mathbb{Z})$. See Farb and Margalit [5, Section 7.4]. This element is called the *genus*-2 *hyperelliptic involution*. A subgroup $G \le \mathrm{Mod}(S_g)$ is called *irreducible* if for any simple closed curve C on S_g there exists an element $[\varphi] \in G$ such that $\varphi(C)$ is not isotopic to C.

We are now ready to state and prove the following proposition:

Proposition 2.3 *If a normal subgroup $G \lhd \mathrm{Mod}(S_g)$ contains no pseudo-Anosov elements of the Torelli group, then it is either the trivial subgroup or the order two subgroup generated by the genus-2 hyperelliptic involution.*

Proof Let $G \lhd \mathrm{Mod}(S_g)$ be a normal subgroup of the mapping class group that contains no pseudo-Anosov elements of the Torelli group. Also let $H = G \cap \mathcal{I}(S_G)$ be the intersection of G with the Torelli group. Thus, H is a normal subgroup that contains no pseudo-Anosov elements.

It follows from a theorem of Ivanov that H is either finite or reducible [14, Theorem 1]. Furthermore, the Torelli group is torsion free and thus H must be either trivial or infinite and reducible [14, Corollary 1.5]. However, Ivanov also showed [14, Corollary 7.13] that there are no infinite, reducible, normal subgroups of $\mathrm{Mod}(S_g)$. Therefore, H must be trivial.

Long showed that if the intersection of two normal subgroups of $\mathrm{Mod}(S_g)$ is trivial, then one of those groups must either be central or trivial [19, Lemma 2.1]. The Torelli group is neither central nor trivial, so we must conclude that G is either central or trivial. If $g \ge 3$, then the center of $\mathrm{Mod}(S_g)$ is trivial [5, Theorem 3.10] and thus G must also be trivial. In the genus-2 case, things are only slightly more complicated. The center of $\mathrm{Mod}(S_2)$ is the order two subgroup generated by the hyperelliptic involution [5, Section 3.4]. Therefore, G is either trivial or the order two subgroup generated by the genus-2 hyperelliptic involution. □

By combining Proposition 2.2 and Proposition 2.3, we see that Theorem 1.1 is equivalent to the statement that neither the genus-2 hyperelliptic involution nor any pseudo-Anosov elements of the Torelli group are $\widehat{\mathrm{HF}}$–invisible. In the next two sections, we will consider mutations by these two types of mapping classes.

3 Genus-two hyperelliptic involution

In this section, we will show that mutating by the genus-2 hyperelliptic involution can change the rank of the nontorsion summands of $\widehat{\mathrm{HF}}$. To accomplish this, we will consider the seminorm on $H_2(M;\mathbb{R})$ defined by Thurston in [29]. This is a useful invariant to consider, because it is detected by $\widehat{\mathrm{HF}}$ and is much easier to compute. See Ozsváth and Szabó [23], Ni [22] and Hedden and Ni [12].

Proposition 3.1 *The genus-2 hyperelliptic involution is not $\widehat{\mathrm{HF}}$–invisible.*

Proof We consider the pair of mutant knots that form the basis of Moore and Starkston's examples of mutations by the genus-2 hyperelliptic involution [21]. Let K and K^τ be the knots denoted respectively by 14^n_{22185} and 14^n_{22589} in Knotscape notation [21, Figure 2]. These two knots are related by a mutation of S^3 by the genus-2 hyperelliptic involution [21, Figure 3]. From the computations of $\widehat{\mathrm{HFK}}$ in Table 1 of [21], we see that K has genus two and K^τ has genus one.

Now, let M and M^τ be the results of 0–surgery on K and K^τ respectively. Because the mutation of S^3 that transforms K into K^τ involves a surface that is disjoint from the knot, there is a corresponding surface in M. Moreover, mutating M along that corresponding surface by the genus-2 hyperelliptic involution will result in a manifold diffeomorphic to M^τ.

A Mayer–Vietoris argument shows that both $H_2(M;\mathbb{R})$ and $H_2(M^\tau;\mathbb{R})$ are isomorphic to $\mathbb{R}$. Furthermore, it follows from the work of Gabai that the genera of the knots K and K^τ determine the Thurston seminorm on these homology groups [7, Corollary 8.3]. In particular, the seminorm is constantly zero on $H_2(M^\tau;\mathbb{R})$ and nonzero on $H_2(M;\mathbb{R})$. This implies that $\widehat{\mathrm{HF}}(M^\tau)$ is supported entirely in the Spin^c–structure whose first Chern class is zero and $\widehat{\mathrm{HF}}(M)$ is nontrivial in at least one Spin^c–structure with nonzero first Chern class by Hedden and Ni [12, Theorem 2.2]. □

4 Pseudo-Anosov gluings

In this section, we examine mutations by pseudo-Anosov elements of the Torelli group. In particular, we will show that mutating by any such element will change the Thurston seminorm of some three-manifold.

Proposition 4.1 *Let $[\varphi] \in \mathcal{I}(S_g)$ be a pseudo-Anosov element of the Torelli group. Then there exists a natural number N and a manifold–surface pair (M, f) such that $M = S^1 \times S^2$ and the mutant manifold $M_f^{\varphi^N}$ has a homology class with nonzero Thurston seminorm.*

In order to determine the effect of mutation on the Thurston seminorm, we must first establish a relationship between the homology of a three-manifold and that of its mutants. In the case of mutation by elements of the Torelli group, this is achieved by the following lemma.

Lemma 4.2 *If $[\psi] \in \mathcal{I}(S_g)$ is an element of the Torelli group and (M, f) is a manifold–surface pair, then M and its mutant M_f^ψ have isomorphic homology groups*

$$H_i(M) \cong H_i(M_f^\psi) \quad \text{for all } i.$$

Proof Because M and its mutant M_f^ψ are closed three-manifolds, it suffices to show that the first homology groups are isomorphic. In order to do this, we decompose M into two open sets that overlap in a tubular neighborhood of the separating surface $f(S_g)$. A comparison of the Mayer–Vietoris sequence coming from this decomposition to that coming from a similar decomposition of the mutant M_f^ψ shows that the first homology groups are indeed isomorphic. □

Our inquiry will focus on mutating $S^1 \times S^2$ along Heegaard surfaces. We proceed by considering the relationship between the complexity of the Heegaard splittings of a three-manifold and the minimal genera of its homology classes.

4.1 Homology and Hempel distance

A genus-g *Heegaard splitting* is a decomposition of a three-manifold into two genus-g handlebodies glued together along their boundaries. Such a splitting is determined by two handlebodies with parametrized boundaries. A handlebody with parametrized boundary is in turn determined by the curves on the boundary that bound disks in the handlebody.

Definition 4.3 For a genus-g handlebody X with boundary parametrized by a map to S_g, let $\mathcal{V}_X$ be the set of isotopy classes of essential simple closed curves in S_g whose preimages bound disks in X. We will refer to the elements of $\mathcal{V}_X$ as *compression curves* of X.

Given two genus-g handlebodies X and Y with boundaries parametrized respectively by maps a and b to S_g, we can construct a three-manifold M by using $b^{-1}a\colon \partial X \to \partial Y$ to glue X to Y. We will write $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$ for the corresponding Heegaard splitting of M.

The compression curves of a genus-g handlebody can be viewed as points in the curve complex, $C(S_g)$. See Harvey [11]. The *curve complex* is a simplicial complex with 0–simplices corresponding to isotopy classes of essential closed curves and n–simplices corresponding to $(n+1)$–tuples of isotopy classes that can be realized disjointly. There is a natural distance function d on the 0–simplices of the curve complex given by viewing the 1–skeleton as a graph with edge length one. Applying this distance function to the sets of compression curves in a Heegaard splitting can provide information about the minimal genera of homology classes.

Lemma 4.4 *If $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$ is a Heegaard splitting of a manifold M and the distance $d(\mathcal{V}_X, \mathcal{V}_Y)$ is greater than two, then M is irreducible and has no essential tori.*

Proof Haken showed that if M were reducible, then $\mathcal{V}_X$ and $\mathcal{V}_Y$ would have a point in common and thus $d(\mathcal{V}_X, \mathcal{V}_Y)$ would be zero [8, page 84]. Furthermore, Hempel demonstrated [13, Corollary 3.7] that if M had an essential torus, then $d(\mathcal{V}_X, \mathcal{V}_Y)$ would be ≤ 2. Thus, $d(\mathcal{V}_X, \mathcal{V}_Y) > 2$ implies that M is irreducible and has no essential tori. □

The distance between the sets of compression curves in a Heegaard splitting is called the *Hempel distance* of that splitting. Combining this language with the definition of Thurston's seminorm gives the following corollary to Lemma 4.4.

Corollary 4.5 *If a three-manifold M has a Heegaard splitting with Hempel distance greater than two, then the Thurston seminorm is in fact a norm on $H_2(M;\mathbb{R})$.*

Now that we have established a relationship between Hempel distance and the Thurston norm, we turn our attention to the effect of mutating by a pseudo-Anosov map on the Hempel distance of a Heegaard splitting.

4.2 Effects of pseudo-Anosov mutations

Each pseudo-Anosov map $\varphi\colon S_g \to S_g$ has two associated projective measured laminations on S_g called its *stable* and *unstable laminations*. See Casson and Bleiler [4, Theorem 5.5]. Furthermore, a set of compression curves can be viewed as a subset of $\mathrm{PML}(S_g)$, the space of projective measured laminations on S_g, by simply applying the counting measure to each curve. See Hamenstädt [10, Section 2]. We will use $\overline{\mathcal{V}_H}$ to denote the closure of $\mathcal{V}_H$ in $\mathrm{PML}(S_g)$. Hempel showed that repeatedly twisting by a pseudo-Anosov map can increase the Hempel distance of a Heegaard splitting:

Theorem 4.6 (Hempel [13, page 640]; see also Abrams and Schleimer [1, Section 2]) *Let X and Y be genus-g handlebodies with their boundaries parametrized by maps to S_g and let $\varphi\colon S_g \to S_g$ be a pseudo-Anosov map with stable lamination s and unstable lamination u. If s and u are not in $\overline{\mathcal{V}_X} \cup \overline{\mathcal{V}_Y}$, then the distance between $\mathcal{V}_X$ and $\varphi^n(\mathcal{V}_Y)$ tends to infinity:*

$$\lim_{n\to\infty} d(\mathcal{V}_X, \varphi^n(\mathcal{V}_Y)) = \infty.$$

It is worth noting that $(S_g, \mathcal{V}_X, \varphi^n(\mathcal{V}_Y))$ is the Heegaard splitting of the mutant manifold that results from mutating $X \cup Y$ by φ^n along the Heegaard surface ∂X. We would like to use Hempel's theorem to make statements about mutations of $S^1 \times S^2$ by pseudo-Anosov maps. However, we must first verify that $S^1 \times S^2$ admits Heegaard splittings of the appropriate form.

Lemma 4.7 *Let $\varphi\colon S_g \to S_g$ be a pseudo-Anosov map with stable lamination s and unstable lamination u. Then there exists a genus-g Heegaard splitting $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$ of $S^1 \times S^2$ such that s and u are not in $\overline{\mathcal{V}_X} \cup \overline{\mathcal{V}_Y}$.*

Proof For an arbitrary handlebody X, the stable lamination s is in $\overline{\mathcal{V}_X}$ if and only if the unstable lamination u is also in $\overline{\mathcal{V}_X}$ by Biringer, Johnson and Minsky [3, Theorem 1.1]. Thus, it is enough to find a Heegaard splitting of $S^1 \times S^2$ such that s is not in the closure of either set of compression curves.

Let $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$ be a genus-g Heegaard splitting of $S^1 \times S^2$. The union $\overline{\mathcal{V}_X} \cup \overline{\mathcal{V}_Y}$ is nowhere dense in $\mathrm{PML}(S_g)$ by Masur [20, Theorem 1.2]. Furthermore, the stable laminations of pseudo-Anosov elements of $\mathrm{Mod}(S_g)$ form a dense subset of $\mathrm{PML}(S_g)$. See Farb and Margalit [6, Theorem 6.19]. Thus, there exists a pseudo-Anosov map $\psi\colon S_g \to S_g$ with stable lamination t such that t is not in $\overline{\mathcal{V}_X} \cup \overline{\mathcal{V}_Y}$ and t is not equal to s or u.

We will now show that translating the set $\mathcal{V}_X$ by a high power of ψ will move it away from s. In particular, we show that the set of natural numbers n for which $s \in \overline{\psi^n(\mathcal{V}_X)}$ is either empty or bounded above. Suppose there exists a $k \in \mathbb{N}$ such that $s \in \overline{\psi^k(\mathcal{V}_X)}$. By Theorem 4.6, the distance $d(\psi^k(\mathcal{V}_X), \psi^{k+\ell}(\mathcal{V}_X))$ goes to infinity as ℓ grows. Therefore, it is enough to show that if s is an element of $\overline{\psi^{k+\ell}(\mathcal{V}_X)}$, then the sets $\psi^k(\mathcal{V}_X)$ and $\psi^{k+\ell}(\mathcal{V}_X)$ must be close together in the curve complex.

Suppose s is an element of $\overline{\psi^{k+\ell}(\mathcal{V}_X)}$. Let (a_i) and (b_i) be sequences of points in $\psi^k(\mathcal{V}_X)$ and $\psi^{k+\ell}(\mathcal{V}_X)$ respectively that converge to s in $\mathrm{PML}(S_g)$. It follows from work of Klarreich that the sequences (a_i) and (b_i) converge to the same point in the Gromov boundary of the curve complex $C(S_g)$ [15]. See also Abrams and Schleimer [1, Theorem 8.4] and Hamenstädt [9, Theorem 1]. This in turn implies that the Hempel distance between $\psi^k(\mathcal{V}_X)$ and $\psi^{k+\ell}(\mathcal{V}_X)$ is bounded above by a constant which depends only on the genus g [1, Lemma 9.2].

Therefore, the set of n for which $s \in \overline{\psi^n(\mathcal{V}_X)}$ is either empty or bounded above. By a similar argument, the corresponding results holds for $\mathcal{V}_Y$. Thus, there exists an N such that s is not in $\overline{\psi^N(\mathcal{V}_X)} \cup \overline{\psi^N(\mathcal{V}_Y)}$. By construction, $(S_g, \psi^N(\mathcal{V}_X), \psi^N(\mathcal{V}_Y))$ is a Heegaard splitting for $S^1 \times S^2$. □

4.3 Proof of Proposition 4.1

Proposition 4.1 *Let $[\varphi] \in \mathcal{I}(S_g)$ be a pseudo-Anosov element of the Torelli group. Then there exists a natural number N and a manifold–surface pair (M, f) such that $M = S^1 \times S^2$ and the mutant manifold $M_f^{\varphi^N}$ has a homology class with nonzero Thurston seminorm.*

Proof Let $s, u \in \mathrm{PML}(S_g)$ be respectively the stable and unstable laminations of φ. Also, let $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$ be a genus-g Heegaard splitting of $S^1 \times S^2$ such that s and u are not in $\overline{\mathcal{V}_X} \cup \overline{\mathcal{V}_Y}$. The existence of such a splitting is guaranteed by Lemma 4.7. Finally, let (M, f) be the manifold–surface pair where $M = S^1 \times S^2$ and f is the embedding of S_g as the Heegaard surface ∂X from the splitting $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$.

By Theorem 4.6, we have that

$$\lim_{n\to\infty} d(\mathcal{V}_X, \varphi^n(\mathcal{V}_Y)) = \infty.$$

Thus, there is a natural number N such that $d(\mathcal{V}_X, \varphi^N(\mathcal{V}_Y)) > 2$. Furthermore, $(S_g, \mathcal{V}_X, \varphi^N(\mathcal{V}_Y))$ is a Heegaard splitting for the mutant $M_f^{\varphi^N}$. This implies that $M_f^{\varphi^N}$ is irreducible and has no essential tori (Lemma 4.4).

A simple calculation shows that the $H_2(M;\mathbb{Z}) = H_2(S^1 \times S^2;\mathbb{Z}) \cong \mathbb{Z}$. It follows that $H_2(M_f^{\varphi^N};\mathbb{Z}) \cong \mathbb{Z}$, because $[\varphi]$ is in the Torelli group (Lemma 4.2). Let ω be a nonzero element of $H_2(M_f^{\varphi^N};\mathbb{Z}) \cong \mathbb{Z}$ and let $F \subseteq M_f^{\varphi^N}$ be a surface that represents ω. Because $M_f^{\varphi^N}$ is irreducible and has no essential tori, the genus of F must be at least 2. It follows that the Thurston seminorm of $\omega = [F] \in H_2(M_f^{\varphi^N};\mathbb{R})$ is nonzero. □

5 Proof of Theorem 1.1

Theorem 1.1 *Let φ be an orientation-preserving self-diffeomorphism of S_g that is not isotopic to the identity map. Then there exists a manifold–surface pair (M, f) such that*

$$\mathrm{rk}\,\widehat{\mathrm{HF}}_{\mathrm{NT}}(M,\mathfrak{s}) \neq \mathrm{rk}\,\widehat{\mathrm{HF}}_{\mathrm{NT}}(M_f^{\varphi},\mathfrak{s}).$$

Proof Let $G \lhd \mathrm{Mod}(S_g)$ be the set of $\widehat{\mathrm{HF}}$–invisible mapping classes. We begin by showing that G contains no pseudo-Anosov element of the Torelli group. Let $[\varphi] \in \mathcal{I}(S_g)$ be a pseudo-Anosov element of the Torelli group. Also let (M, f) be a manifold–surface pair such that $M = S^1 \times S^2$ and for some $N \in \mathbb{N}$ the mutant manifold $M_f^{\varphi^N}$ has a homology class with nonzero Thurston seminorm. The existence of such a pair is guaranteed by Proposition 4.1.

A simple computation shows that the Heegaard Floer homology of $M = S^1 \times S^2$ is isomorphic to $\mathbb{Z} \oplus \mathbb{Z}$ and is supported entirely in the Spin^c–structure whose first Chern class is zero. See Ozsváth and Szabó [24, Section 3]. Thus, the rank of the nontorsion summands of $\widehat{\mathrm{HF}}(M)$ is zero:

$$\mathrm{rk}\,\widehat{\mathrm{HF}}_{\mathrm{NT}}(M,\mathfrak{s}) = 0.$$

By construction $M_f^{\varphi^N}$ has a homology class with nonzero Thurston seminorm. It follows from work of Hedden and Ni that $\widehat{\mathrm{HF}}(M_f^{\varphi^N})$ is nontrivial in at least one Spin^c–structure with nonzero first Chern class [12, Theorem 2.2]. In particular, the rank of the nontorsion summands is positive:

$$\mathrm{rk}\,\widehat{\mathrm{HF}}_{\mathrm{NT}}(M_f^{\varphi^N}, \mathfrak{s}) > 0.$$

Therefore

$$\mathrm{rk}\,\widehat{\mathrm{HF}}_{\mathrm{NT}}(M, \mathfrak{s}) \neq \mathrm{rk}\,\widehat{\mathrm{HF}}_{\mathrm{NT}}(M_f^{\varphi^N}, \mathfrak{s}).$$

Thus, the mapping class $[\varphi^N] = [\varphi]^N$ is not $\widehat{\mathrm{HF}}$–invisible. Because the $\widehat{\mathrm{HF}}$–invisible mapping classes form a subgroup of $\mathrm{Mod}(S_g)$, we concluded that $[\varphi]$ is also not $\widehat{\mathrm{HF}}$–invisible (Proposition 2.2). Therefore, no pseudo-Anosov element of the Torelli group is an element of G.

Furthermore, we showed in Proposition 2.2 and Proposition 3.1 respectively that G is normal and does not contain the genus-2 hyperelliptic involution. Hence, G is trivial by Proposition 2.3. □

6 Total rank detects mutation

Theorem 1.2 *Let $[\varphi] \in \mathrm{Mod}(S_g)$ be a mapping class that is isomorphic to neither the identity nor the genus-2 hyperelliptic involution. Then there exists a manifold–surface pair (M, f) such that*

$$\mathrm{rk}\,\widehat{\mathrm{HF}}(M) \neq \mathrm{rk}\,\widehat{\mathrm{HF}}(M_f^{\varphi}).$$

Proof Let G be the set of mapping classes such that $[\varphi] \in G$ if and only if $\mathrm{rk}\,\widehat{\mathrm{HF}}(M)$ is equal to $\mathrm{rk}\,\widehat{\mathrm{HF}}(M_f^{\varphi})$ for all manifold–surface pairs (M, f). The set G, like the set of $\widehat{\mathrm{HF}}$–invisible mapping classes, is a normal subgroup of $\mathrm{Mod}(S_g)$. This follows from the proof of Proposition 2.2 with the appropriate notation changes. Thus, it suffices to show that G contains no pseudo-Anosov elements of the Torelli group (Proposition 2.3).

Let $[\varphi] \in \mathcal{I}(S_g)$ be a pseudo-Anosov element of the Torelli group. Also let (M, f) be a manifold–surface pair such that $M = S^1 \times S^2$ and for some $N \in \mathbb{N}$ the mutant manifold $M_f^{\varphi^N}$ has a homology class with nonzero Thurston seminorm. The existence of such a pair is guaranteed by Proposition 4.1.

Let T be the result of 0–surgery on the trefoil. Hedden and Ni showed that T and M are the only closed, orientable, irreducible three-manifolds with nonzero first Betti number and $\mathrm{rk}\,\widehat{\mathrm{HF}} = 2$ [12, Theorem 1.1]. In the proof of Proposition 4.1, we showed that the mutant $M_f^{\varphi^N}$ is closed, orientable and irreducible, and its first Betti number is nonzero. Thus, it is enough to show that $M_f^{\varphi^N}$ is not diffeomorphic to either T or M.

A Mayer–Vietoris argument shows that $H_2(T;\mathbb{R}) \cong \mathbb{R}$. The Thurston seminorm is constantly zero on $H_2(T;\mathbb{R})$, because the trefoil is a genus-1 knot by Gabai [7, Corollary 8.3]. The homology group $H_2(M;\mathbb{Z}) = H_2(S^1 \times S^2;\mathbb{Z})$ is isomorphic to $\mathbb{Z}$ and is generated by the homology class of a sphere. Thus, the Thurston seminorm of any homology class in $H_2(S^1 \times S^2;\mathbb{R})$ is zero. Therefore, the Thurston seminorm differentiates $M_f^{\varphi^N}$ from both T and $S^1 \times S^2$. □

7 Implications

There are two ways to interpret Theorem 1.1 and Theorem 1.2 as statements about actions of mapping class groups of surfaces on categories. The first uses bordered Heegaard Floer homology and results in a statement about an action on a category of $\mathcal{A}_\infty$–modules. The second uses the definition of $\widehat{\mathrm{HF}}$ and results in a statement about an action on a Fukaya category.

7.1 Bordered Heegaard Floer homology

In [16] and [18], Lipshitz, Ozsváth and Thurston developed a variant of Heegaard Floer homology for three-manifolds with parametrized boundary called *bordered Heegaard Floer homology*. These bordered invariants are related to $\widehat{\mathrm{HF}}$ by pairing theorems [16, Theorem 1.3] and [18, Theorem 11]. The pairing theorems provide a method for computing $\widehat{\mathrm{HF}}(M)$ by cutting M along separating surfaces and computing the bordered Heegaard Floer homology of the resulting components. By applying this method to manifold–surface pairs and their mutants, we can use Theorem 1.1 to infer information about the bordered Heegaard Floer homology of mapping cylinders of surface diffeomorphisms.

Let $\mathrm{Mod}_0(S_g)$ denote the *strongly based mapping class group* of S_g that is the isotopy classes of diffeomorphisms that fix a given disk in S_g. There is a canonical projection

$$p\colon \mathrm{Mod}_0(S_g) \to \mathrm{Mod}(S_g)$$

given by quotienting out by the copy of $\pi_1(S_g)$ that corresponds to pushing the disk around closed curves in S_g as well as by the Dehn twist around the boundary of the disk. Following [18, Section 8], we assign to each strongly based mapping class $[\varphi] \in \mathrm{Mod}_0(S_g)$ the bimodule $\widehat{\mathrm{CFDA}}(\varphi, 0)$ associated to its mapping cylinder. By considering Theorem 1.1 from the perspective of bordered Heegaard Floer homology, we get the following result about these bimodules.

Corollary 7.1 *If $[\varphi] \in \mathrm{Mod}_0(S_g)$ is a strongly based mapping class such that $[\varphi]$ is not in the kernel of p, then the action of $[\varphi]$ on the category of $\mathcal{G}(Z)$–graded $\mathcal{A}(Z)$–modules given by tensoring with $\widehat{\mathrm{CFDA}}(\varphi, 0)$ is not the trivial action.*

Proof Let $[\varphi] \in \mathrm{Mod}_0(S_g)$ such that $[\varphi]$ is not in the kernel of p. Also, let (M, f) be a manifold–surface pair such that the rank of $\widehat{\mathrm{HF}}_{\mathrm{NT}}(M)$ differs from that of $\widehat{\mathrm{HF}}_{\mathrm{NT}}(M_f^\varphi)$. The existence of such a pair is guaranteed by Theorem 1.1. Finally, let M_1 and M_2 be the connected components of $M \setminus f(S_g)$.

The Heegaard Floer homology of M can be computed from the bordered invariants of M_1 and M_2 as follows:

$$\widehat{\mathrm{HF}}(M) \cong H_*(\widehat{\mathrm{CFA}}(M_1) \mathbin{\widetilde{\otimes}} \widehat{\mathrm{CFD}}(M_2)),$$

where $\widetilde{\otimes}$ is the $\mathcal{A}_\infty$–tensor product.

Similarly, decomposing the mutant manifold M_f^φ as the union $M_1 \cup C_\varphi \cup M_2$, where C_φ is the mapping cylinder of φ, corresponds to the following module decomposition of $\widehat{\mathrm{HF}}(M_f^\varphi)$:

$$\widehat{\mathrm{HF}}(M_f^\varphi) \cong H_*(\widehat{\mathrm{CFA}}(M_1) \widetilde{\otimes} \widehat{\mathrm{CFDA}}(\varphi, 0) \widetilde{\otimes} \widehat{\mathrm{CFD}}(M_2)).$$

Thus, the difference between $\widehat{\mathrm{HF}}(M)$ and $\widehat{\mathrm{HF}}(M_f^\varphi)$ must result from the effect of tensoring with $\widehat{\mathrm{CFDA}}(\varphi, 0)$. Therefore, the action of $[\varphi]$ on $\mathcal{A}(Z)$–modules given by tensoring with $\widehat{\mathrm{CFDA}}(\varphi, 0)$ must not be the trivial action. □

A similar reformulation of Theorem 1.2 gives the following result about the action of $\mathrm{Mod}_0(S_g)$ on the category of ungraded $\mathcal{A}(Z)$–modules.

Corollary 7.2 *If $[\varphi] \in \mathrm{Mod}_0(S_g)$ is a strongly based mapping class such that $p([\varphi])$ is neither the identity nor the genus-2 hyperelliptic involution, then the action of $[\varphi]$ on the category of ungraded $\mathcal{A}(Z)$–modules given by tensoring with $\widehat{\mathrm{CFDA}}(\varphi, 0)$ is not the trivial action.*

These results are closely related to work of Lipshitz, Ozsváth and Thurston. In [17], they showed that the action of a nontrivial strongly based mapping class $[\varphi]$ on the ungraded $\mathcal{A}(Z)$–modules given by tensoring with $\widehat{\mathrm{CFDA}}(\varphi, \pm(g-1))$ is not the trivial action.

7.2 Fukaya categories

When viewed from another perspective, the work of Lipshitz, Ozsváth and Thurston shows that the strongly based mapping class group $\mathrm{Mod}_0(S_g)$ acts freely on a version of the Fukaya category of S_g with a disk removed as well as on a version of the Fukaya category of the $(2g-1)$–fold symmetric product $\mathrm{Sym}^{2g-1}(S_g - D)$. See Auroux [2].

Theorem 1.2 is also related to mapping class group actions on Fukaya categories. In particular, the chain complex that underlies $\widehat{\mathrm{HF}}$ of a three-manifold with a genus-g Heegaard splitting corresponds to a morphism group in the Fukaya category of the g–fold symmetric product of S_g with a point removed. Furthermore, the action of the based mapping class group of S_g on the symmetric product $\mathrm{Sym}^g(S_g - z)$ induces a strict action on the Fukaya category $\mathrm{Fuk}(\mathrm{Sym}^g(S_g - z))$. See Seidel [27, Section 10b].

Corollary 7.3 *If $[\varphi] \in \mathrm{Mod}(S_g - z)$ is a based mapping class such that the corresponding element of $\mathrm{Mod}(S_g)$ is neither the identity nor the genus-2 hyperelliptic involution, then the action of $[\varphi]$ on the Fukaya category $\mathrm{Fuk}(\mathrm{Sym}^g(S_g - z))$ is not the trivial action. In particular, the map induced by φ on $\mathrm{Sym}^g(S_g - z)$ is not Hamiltonian isotopic to the identity.*

Proof Let $[\varphi] \in \mathrm{Mod}(S_g - z)$ be a based mapping class such that the corresponding element of $\mathrm{Mod}(S_g)$ is neither the identity nor the genus-2 hyperelliptic involution. Also, let (M, f) be a manifold–surface pair such that $f(S_g)$ is a Heegaard surface and

$$\mathrm{rk}\,\widehat{\mathrm{HF}}(M) \neq \mathrm{rk}\,\widehat{\mathrm{HF}}(M_f^\varphi).$$

The existence of such a manifold is guaranteed by the fact that the proof of Theorem 1.2 only uses manifold–surface pairs where the embedded surface is a Heegaard surface. Finally, let T_α and T_β be the corresponding Heegaard tori in $\mathrm{Sym}^g(S_g - z)$.

The action of $[\varphi]$ on $\mathrm{Fuk}(\mathrm{Sym}^g(S_g - z))$ sends T_β to $T_{\varphi(\beta)}$, the Heegaard torus that results from translating the curves of β by φ. Furthermore, T_α and $T_{\varphi(\beta)}$ are the Heegaard tori of a splitting of the mutant manifold M_f^φ. It then follows from the definitions that

$$\widehat{\mathrm{CF}}(M) = \mathrm{Mor}(T_\alpha, T_\beta) \quad \text{and} \quad \widehat{\mathrm{CF}}(M_f^\varphi) = \mathrm{Mor}(T_\alpha, T_{\varphi(\beta)}).$$

Since $\widehat{\mathrm{HF}}(M)$ and $\widehat{\mathrm{HF}}(M_f^\varphi)$ do not have the same rank, we concluded that their underlying chain complexes $\widehat{\mathrm{CF}}(M)$ and $\widehat{\mathrm{CF}}(M_f^\varphi)$ are not quasi-isomorphic. Thus, the morphism groups $\mathrm{Mor}(T_\alpha, T_\beta)$ and $\mathrm{Mor}(T_\alpha, T_{\varphi(\beta)})$ are not quasi-isomorphic. Therefore, T_β is not isomorphic to $T_{\varphi(\beta)}$. □

It should also be possible to reformulate Theorem 1.1 as a statement about an action of the based mapping class group of S_g on a version of the Fukaya category of $\mathrm{Sym}^g(S_g - z)$. Such a reformulation would likely require working with grading data like that described by Sheridan [28]. We will return to this in a future paper.

References

[1] **A Abrams**, **S Schleimer**, *Distances of Heegaard splittings*, Geom. Topol. 9 (2005) 95–119 MR

[2] **D Auroux**, *Fukaya categories of symmetric products and bordered Heegaard–Floer homology*, J. Gökova Geom. Topol. 4 (2010) 1–54 MR

[3] **I Biringer**, **J Johnson**, **Y Minsky**, *Extending pseudo-Anosov maps into compression bodies*, J. Topol. 6 (2013) 1019–1042 MR

[4] **A J Casson**, **S A Bleiler**, *Automorphisms of surfaces after Nielsen and Thurston*, London Mathematical Society Student Texts 9, Cambridge University Press (1988) MR

[5] **B Farb**, **D Margalit**, *A primer on mapping class groups*, Princeton Mathematical Series 49, Princeton University Press (2012) MR

[6] **A Fathi**, **F Laudenbach**, **V Poénaru**, *Thurston's work on surfaces*, Mathematical Notes 48, Princeton University Press (2012) MR

[7] **D Gabai**, *Foliations and the topology of 3–manifolds, III*, J. Differential Geom. 26 (1987) 479–536 MR

[8] **W Haken**, *Some results on surfaces in 3–manifolds*, from "Studies in modern topology" (P J Hilton, editor), Math. Assoc. Amer., Englewood Cliffs, NJ (1968) 39–98 MR

[9] **U Hamenstädt**, *Train tracks and the Gromov boundary of the complex of curves*, from "Spaces of Kleinian groups" (Y N Minsky, M Sakuma, C Series, editors), London Math. Soc. Lecture Note Ser. 329, Cambridge University Press (2006) 187–207 MR

[10] **U Hamenstädt**, *Geometry of the complex of curves and of Teichmüller space*, from "Handbook of Teichmüller theory, I" (A Papadopoulos, editor), IRMA Lect. Math. Theor. Phys. 11, Eur. Math. Soc., Zürich (2007) 447–467 MR

[11] **W J Harvey**, *Boundary structure of the modular group*, from "Riemann surfaces and related topics" (I Kra, B Maskit, editors), Ann. of Math. Stud. 97, Princeton University Press (1981) 245–251 MR

[12] **M Hedden**, **Y Ni**, *Manifolds with small Heegaard Floer ranks*, Geom. Topol. 14 (2010) 1479–1501 MR

[13] **J Hempel**, *3–manifolds as viewed from the curve complex*, Topology 40 (2001) 631–657 MR

[14] **N V Ivanov**, *Subgroups of Teichmüller modular groups*, Translations of Mathematical Monographs 115, Amer. Math. Soc., Providence, RI (1992) MR

[15] **E Klarreich**, *The boundary at infinity of the curve complex and the relative Teichmüller space*, preprint (1999) Available at http://tinyurl.com/Klarreich-cc

[16] **R Lipshitz**, **P Ozsvath**, **D Thurston**, *Bordered Heegaard Floer homology: invariance and pairing*, preprint (2011) arXiv

[17] **R Lipshitz**, **P S Ozsváth**, **D P Thurston**, *A faithful linear-categorical action of the mapping class group of a surface with boundary*, J. Eur. Math. Soc. (JEMS) 15 (2013) 1279–1307 MR

[18] **R Lipshitz**, **P S Ozsváth**, **D P Thurston**, *Bimodules in bordered Heegaard Floer homology*, Geom. Topol. 19 (2015) 525–724 MR

[19] **D D Long**, *A note on the normal subgroups of mapping class groups*, Math. Proc. Cambridge Philos. Soc. 99 (1986) 79–87 MR

[20] **H Masur**, *Measured foliations and handlebodies*, Ergodic Theory Dynam. Systems 6 (1986) 99–116 MR

[21] **A H Moore**, **L Starkston**, *Genus-two mutant knots with the same dimension in knot Floer and Khovanov homologies*, Algebr. Geom. Topol. 15 (2015) 43–63 MR

[22] **Y Ni**, *Nonseparating spheres and twisted Heegaard Floer homology*, Algebr. Geom. Topol. 13 (2013) 1143–1159 MR

[23] **P Ozsváth**, **Z Szabó**, *Holomorphic disks and genus bounds*, Geom. Topol. 8 (2004) 311–334 MR

[24] **P Ozsváth**, **Z Szabó**, *Holomorphic disks and three-manifold invariants: properties and applications*, Ann. of Math. 159 (2004) 1159–1245 MR

[25] **P Ozsváth**, **Z Szabó**, *Holomorphic disks and topological invariants for closed three-manifolds*, Ann. of Math. 159 (2004) 1027–1158 MR

[26] **D Ruberman**, *Mutation and gauge theory, I: Yang–Mills invariants*, Comment. Math. Helv. 74 (1999) 615–641 MR

[27] **P Seidel**, *Fukaya categories and Picard–Lefschetz theory*, Eur. Math. Soc., Zürich (2008) MR

[28] **N Sheridan**, *Homological mirror symmetry for Calabi–Yau hypersurfaces in projective space*, Invent. Math. 199 (2015) 1–186 MR

[29] **W P Thurston**, *A norm for the homology of 3–manifolds*, Mem. Amer. Math. Soc. 339, Amer. Math. Soc., Providence, RI (1986) 99–130 MR

Department of Mathematics, Indiana University
831 E Third St, Bloomington, IN 47405, United States

cjclarks@indiana.edu

http://pages.iu.edu/~cjclarks/

Received: 16 October 2013 Revised: 8 June 2016