

Algebra & Number Theory

Volume 5

2011

No. 2

**Correction to a proof in the article
Patching and admissibility over two-dimensional
complete local domains**

Danny Neftin and Elad Paran



mathematical sciences publishers

Correction to a proof in the article Patching and admissibility over two-dimensional complete local domains

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Volume 4:6 (2010), 743–762

The proof of Lemma 1.8 of [the article in the title](#) is incorrect. We supply an alternate argument for Proposition 1.10, whose proof invoked that lemma.

We are grateful to Yong Hu for pointing out to us a gap in the proof of Lemma 1.8 of our article “[Patching and admissibility over two-dimensional complete local domains](#)”, namely, the isomorphism $R/\mathfrak{p} \cong S/\mathfrak{q}$ implies only that $S = R + \mathfrak{q}$ and not $S = R + \mathfrak{p}S$, as required for this argument.

The lemma is applied for the rings

$$R_0 = D_{J \cup J'}, \quad R_1 = D_J, \quad R_2 = D_{J'}, \quad R = D_\emptyset,$$

where $J \cap J' = \emptyset$, to show that $S := R_1 \cap R_2 = R$. Let us show this assertion directly. In particular, this will trivially imply that for these rings $\mathfrak{q} = \mathfrak{p}S$.

All references are to [the article in question](#).

Recall that I is a finite set and that v is the extension of the order function of the ideal $\mathfrak{p} := (x, y) \triangleleft K[x, y]$ to $K(x, y)$. For $i \in I$, let $z_i = y/(x - c_i y)$ and for a subset $J \subset I$, D_J is defined as the completion of $K[z_j \mid j \in J][x, y]$ with respect to v .

Lemma. *Let $i, j \in I$ be two distinct indices. Then $D_{\{i\}} \cap D_{\{j\}} = D_\emptyset$.*

Proof. By Proposition 1.5, $D_{\{i\}} = K[z_i][[x - c_i y]]$ and hence an element $f \in D_{\{i\}}$ can be written as $\sum_{k=0}^{\infty} f_k(z_i)(x - c_i y)^k$ for some polynomials $f_k, k \geq 0$. Assume $f \in D_{\{i, j\}}$ can also be written as $\sum_{k=0}^{\infty} g_k(z_j)(x - c_j y)^k \in D_{\{j\}} = K[z_j][[x - c_j y]]$, where g_k are polynomials for $k \geq 0$.

In particular,

$$f_k(z_i)(x - c_i y)^k = g_k(z_j)(x - c_j y)^k \pmod{\mathfrak{p}^{k+1} D_{\{i, j\}}} \quad (1)$$

MSC2010: primary 12F12; secondary 12E30, 12E15, 12F10.

Keywords: patching, crossed product, admissible groups, division algebras, complete local domains.

for all $k \geq 0$. We claim that equality (1) in fact holds in $D_{\{i,j\}}$. Indeed, since $x - c_i y = (1 + (c_j - c_i)z_j)(x - c_j y)$, the difference between the sides of (1) is

$$(f_k(z_i)(1 + (c_j - c_i)z_j)^k - g_k(z_j))(x - c_j y)^k \in K[z_i, z_j](x - c_j y)^k. \quad (2)$$

Since \mathfrak{p} is contained in the center of the valuation v , Proposition 1.5 implies that the difference (2) is in $\mathfrak{p}^{k+1} D_{\{i,j\}}$ only if it is zero, proving the claim.

By finding a common denominator, one can write an element $f_k(z_i)(x - c_i y)^k$ as $p_k(x, y)/(x - c_i y)^m$ where $m \geq 0$ and p_k is a homogenous polynomial of degree $k + m$ that is prime to $(x - c_i y)^m$. Writing $g_k(z_j)(x - c_j y)^k = q_k(x, y)/(x - c_j y)^l$ for $l \geq 0$ and q_k a homogenous polynomial of degree $k + l$ that is prime to $(x - c_i y)^l$, the equality

$$\frac{p_k(x, y)}{(x - c_i y)^m} = \frac{q_k(x, y)}{(x - c_j y)^l}$$

implies that $m = l = 0$ and hence that $f_k(z_i)(x - c_i y)^k \in K[x, y]$ for all $k \geq 0$. It follows that $f = \sum_{k=0}^{\infty} f_k(z_i)(x - c_i y)^k \in K[[x, y]]$, as required. \square

Let us complete the proof of Proposition 1.10:

Proposition. *Suppose $J, J' \subseteq I$. Then $D_J \cap D_{J'} = D_{J \cap J'}$.*

Proof. Clearly $D_{J \cap J'} \subseteq D_J \cap D_{J'}$. For the converse inclusion, we distinguish between two cases. First suppose that $J \cap J' \neq \emptyset$ and fix $j \in J \cap J'$. Then $D_J = K[z_k \mid k \in J][[x - c_j y]]$, $D_{J'} = K[z_k \mid k \in J'][[x - c_j y]]$ and hence

$$D_J \cap D_{J'} = (K[z_k \mid k \in J] \cap K[z_k \mid k \in J'])[[x - c_j y]].$$

By Lemma 1.9, $K[z_k \mid k \in J] \cap K[z_k \mid k \in J'] = K[z_k \mid k \in J \cap J']$.

Now suppose that $J \cap J' = \emptyset$. If $|J| = |J'| = 1$, then the claim follows from the Lemma. Assume without loss of generality $|J| \geq 2$. For distinct $j_1, j_2 \in J$, we have by the previous case $D_J \cap D_{J' \cup \{j_i\}} = D_{\{j_i\}}$, for $i = 1, 2$. In particular, $D_J \cap D_{J'} \subseteq D_{\{j_1\}} \cap D_{\{j_2\}}$. By the Lemma, $D_{\{j_1\}} \cap D_{\{j_2\}} = D_{\emptyset}$ implying that $D_J \cap D_{J'} = D_{\emptyset}$ as required. \square

Communicated by Jean-Louis Colliot-Thélène

Received 2011-07-06

Revised 2011-07-13

Accepted 2011-08-10

neftin@umich.edu

Department of Mathematics, University of Michigan, Ann Arbor, 530 Church St., Ann Arbor 48109, United States

paranela@post.tau.ac.il

School of Mathematical Sciences, Tel Aviv University, Ramat Aviv, 69978 Tel Aviv, Israel
<http://www.tau.ac.il/~paranela/>

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Algebra & Number Theory (ISSN 1937-0652) at Mathematical Sciences Publishers, Department of Mathematics, University of California, Berkeley, CA 94720-3840 is published continuously online. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices.

ANT peer review and production are managed by EditFLOW™ from Mathematical Sciences Publishers.

PUBLISHED BY

 **mathematical sciences publishers**

<http://msp.org/>

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Typeset in L^AT_EX

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Algebra & Number Theory

Volume 5 No. 2 2011

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