Algebra & Number Theory

Volume 8
2014
No. 8

Relative cohomology of cuspidal forms on PEL-type Shimura varieties

Kai-Wen Lan and Benoît Stroh

msp

ALGEBRA AND NUMBER THEORY 8:8 (2014)
dx.doi.org/10.2140/ant.2014.8.1787

Relative cohomology of cuspidal forms on PEL-type Shimura varieties

Kai-Wen Lan and Benoît Stroh

We present a short proof that, for PEL-type Shimura varieties, subcanonical extensions of automorphic bundles, whose global sections over toroidal compactifications of Shimura varieties are represented by cuspidal automorphic forms, have no higher direct images under the canonical morphism to the minimal compactification, in characteristic zero or in positive characteristics greater than an explicitly computable bound.

1. Introduction 1787
2. Proof of the theorem 1789
3. Elementary computations 1792
4. Simpler proof for the trivial weight case 1796
Acknowledgements 1797
References 1797

1. Introduction

The main goal of this article is to present a short proof of Theorem 1.1 below, as an application of a certain vanishing theorem of automorphic bundles in mixed characteristics. (We refer to [Lan 2013; Lan and Suh 2012; 2013] for the precise definitions and descriptions of smooth integral models of PEL-type Shimura varieties and their various compactifications, and of the automorphic bundles and their canonical and subcanonical extensions.)

Let $\pi : \mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma} \to \mathsf{M}^{\mathrm{min}}_{\mathcal{H}}$ denote the canonical proper morphism from any projective smooth toroidal compactification to the minimal compactification of a p-integral model $\mathsf{M}_{\mathcal{H}}$ of a PEL-type Shimura variety at a neat level $\mathcal{H} \subset \mathrm{G}(\widehat{\mathbb{Z}}^p)$, where p is *good* for the integral PEL datum $(\mathcal{O}, \star, L, \langle \cdot, \cdot \rangle, h_0)$ defining $\mathsf{M}_{\mathcal{H}}$, as in [Lan

Lan is partially supported by the National Science Foundation under agreements Nos. DMS-1258962 and DMS-1352216, and by an Alfred P. Sloan Research Fellowship. Stroh is partially supported by the A.N.R. (Agence Nationale de la Recherche) under the project ANR-10-BLAN-0114 ArShiFo.
MSC2010: primary 14G35; secondary 14K10, 14G17, 32L20.
Keywords: Shimura varieties, vanishing theorem, toroidal compactification, minimal compactification, cuspidal forms.

and Suh 2013, §4.1] (and the references there). Let $W_{\nu_0,R} := W_{\nu_0,\mathbb{Z}} \otimes_{\mathbb{Z}} R$ be a representation of M_1 of weight $\nu_0 \in \mathrm{X}^+_{\mathrm{M}_1}$ over a coefficient ring R, where $W_{\nu_0,\mathbb{Z}}$ denotes a Weyl module of weight ν_0 of a split model $\mathrm{M}_{\mathrm{split}}$ of M_1 over $\mathbb{Z}$, as in [Lan and Suh 2012, §2.6]. Let $\underline{W}_{\nu_0,R} := \mathcal{E}_{\mathrm{M}_1,R}(W_{\nu_0,R})$ be the corresponding automorphic bundle over $\mathsf{M}_{\mathcal{H}}$, as in [Lan and Suh 2012, Definition 1.16 and §6.3], and let $\underline{W}^{\mathrm{sub}}_{\nu_0,R} := \mathcal{E}^{\mathrm{sub}}_{\mathrm{M}_1,R}(W_{\nu_0,R})$ be its subcanonical extension over $\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},R}$, as in [Lan and Suh 2013, Definition 4.12 and §7]. (We similarly define $W_{\nu,R}$, $\underline{W}_{\nu,R}$, and $\underline{W}^{\mathrm{sub}}_{\nu,R}$ for all $\nu \in \mathrm{X}^+_{\mathrm{M}_1}$.)

Theorem 1.1. *With the setting as above, there exists a bound $C(\nu_0)$ depending only on the integral PEL datum $(\mathcal{O}, \star, L, \langle\cdot,\cdot\rangle, h_0)$ and the weight ν_0, such that*

$$R^i\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0,R} = 0 \tag{1.2}$$

for all $i > 0$ when the residue characteristics of R are zero or p greater than $C(\nu_0)$. (See Lemma 3.3 below for an explicit choice of $C(\nu_0)$.)

To help the reader understand the restriction imposed by $C(\nu_0)$, let us spell out the bound in some simple special cases. If $\nu_0 = 0$, then we can take $C(\nu_0)$ to be the relative dimension d of $\mathsf{M}_{\mathcal{H}}$ over the base scheme S_0 (see Example 3.9 below). If $\mathsf{M}_{\mathcal{H}}$ is a p-integral model of the Siegel modular variety of genus three, then the weight ν_0 is of the form $(k_1, k_2, k_3; k_0)$ for some integers k_0 and $k_1 \geq k_2 \geq k_3$, and we can take $C(\nu_0)$ to be $6 + (k_1 - k_3) + (k_2 - k_3)$ (see Example 3.10 below with $r = 3$ there). If $\mathsf{M}_{\mathcal{H}}$ is a p-integral model of a Picard modular surface, then the weight ν_0 is of the form $(k_1, k_2, k_3; k_0)$ for some integers k_0, k_1, and $k_2 \geq k_3$, and we can take $C(\nu_0)$ to be $2 + (k_2 - k_3)$ (see Example 3.12 below with $(r-q, q) = (2, 1)$ there). (In all cases, $C(\nu_0)$ is insensitive to shifting the weight ν_0 by a "parallel weight". See Section 3C below for more examples.)

We note that, when $R = \mathbb{C}$, global sections of $\underline{W}^{\mathrm{sub}}_{\nu_0,R}$ over $\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma}$ can be represented by holomorphic cuspidal automorphic forms. (See, e.g., [Harris 1990b, Proposition 5.4.2]; see also [Harris 1990a] for a survey on how the higher cohomology of $\underline{W}^{\mathrm{sub}}_{\nu_0,R}$ can be represented by nonholomorphic automorphic forms. See [Lan 2012] for the comparison between algebraic and analytic constructions hidden behind this.) Combined with the Leray spectral sequence, Theorem 1.1 allows one to identify the cohomology of $\underline{W}^{\mathrm{sub}}_{\nu_0,R}$ over $\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma}$ with the cohomology of $\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0,R}$ over $\mathsf{M}^{\mathrm{min}}_{\mathcal{H}}$. Although the coherent sheaf $\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0,R}$ is not locally free in general, there are reasons for $\mathsf{M}^{\mathrm{min}}_{\mathcal{H}}$ to be useful for the construction of p-adic modular forms and p-adic Galois representations.

Special cases of Theorem 1.1 have been independently proved in [Andreatta et al. 2013a; 2013b] (in the Siegel and Hilbert cases, for trivial weight ν_0) and in [Harris et al. 2013] (in the unitary case, for all weights ν_0), without any assumption on the residue characteristic p. The idea in [Harris et al. 2013] has also been carried out for

all PEL-type cases in [Lan 2014]. Such results have played crucial roles in positive characteristics in [Andreatta et al. 2013a; 2013b; Emerton et al. 2013; Pilloni and Stroh 2013], and in characteristic zero in [Harris et al. 2013; Tian and Xiao 2013]. The proofs in [Andreatta et al. 2013a; 2013b] and [Harris et al. 2013; Lan 2014] directly used the toroidal and minimal boundary structures, and hence can be considered more elementary, which is why they work for all residue characteristics p; but they are lengthier and arguably more complicated. It is not easy to see from their proofs why Theorem 1.1 should be true. (It is not even clear how the two strategies in [Andreatta et al. 2013a; 2013b] and [Harris et al. 2013; Lan 2014] are related to each other.) Thus it is desirable to find a proof more closely related to other vanishing statements, at least when the residue characteristics are zero or sufficiently large.

It was first observed by the second author that this is indeed possible — in characteristic zero, the trivial weight case can be deduced from Grauert and Riemenschneider's vanishing theorem [1970]; in positive characteristics, under suitable assumptions (involving choices of projective but generally nonsmooth cone decompositions Σ for the toroidal compactification $\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma}$, whose existence is not very clearly documented in the literature), it is also possible to deduce the statement from Deligne and Illusie's [1987] and Kato's [1989] vanishing theorems. Then the first author made the observations that the assumption on cone decompositions can be relaxed by using Esnault and Viehweg's [1992] vanishing theorem as in [Lan and Suh 2011], and that (along similar lines) cases of nontrivial weights can be treated using stronger vanishing theorems in [Lan and Suh 2013]. (In the Siegel case, one can also use [Stroh 2010; 2013].)

In Section 2, we will present the proof of Theorem 1.1 and highlight the main inputs. In Section 3, we will carry out some elementary computations needed in the proof of Theorem 1.1, and find an explicit choice of $C(\nu_0)$. In Section 4, we sketch a logically simpler proof for the trivial weight case.

2. Proof of the theorem

Let $\pi : \mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma} \to \mathsf{M}^{\mathrm{min}}_{\mathcal{H}}$, $\nu_0 \in \mathrm{X}^+_{\mathrm{M}_1}$, and $\underline{W}^{\mathrm{sub}}_{\nu_0,R}$ be as in Section 1. Since $\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,1}$ and $\mathsf{M}^{\mathrm{min}}_{\mathcal{H},1}$ are proper over $\mathsf{S}_1 = \mathrm{Spec}(R_1)$ (see [Lan and Suh 2013, §4.1] and the references there for the notation), which are in particular separated and of finite type, for the purpose of proving Theorem 1.1 we may write R as an inductive limit over its sub-R_1-algebras and assume that R is of finite type over R_1, which is in particular noetherian. Then we may base change to R and abusively denote $\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,R} \to \mathsf{M}^{\mathrm{min}}_{\mathcal{H},R}$ by the same notation π. Our goal is to show that $R^i\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0,R} = 0$ for all $i > 0$.

As in [Lan and Suh 2012, §2.6], we shall denote by $\mathrm{X}^{+,<p}_{\mathrm{M}_1}$ the subset of $\mathrm{X}^+_{\mathrm{M}_1}$ consisting of p-small weights, namely the weights $\nu \in \mathrm{X}^+_{\mathrm{M}_1}$ such that $(\nu + \rho_{\mathrm{M}_1}, \alpha) \le p$ for all roots $\alpha \in \Phi_{\mathrm{M}_1}$, where ρ_{M_1} is the usual half sum of positive roots.

2A. ***Application of Serre's fundamental theorem.*** By [Lan and Suh 2013, Proposition 7.13], there exists some weight $\nu_1 \in \mathrm{X}^{+,<p}_{\mathrm{M}_1}$ such that $W_{\nu_1,R}$ is free of rank one as an R-module, and such that there exists an *ample line bundle* ω_{ν_1} over $\mathsf{M}^{\min}_{\mathcal{H},R}$ such that

$$\pi^*\omega_{\nu_1} \cong \underline{W}^{\mathrm{can}}_{\nu_1,R}, \tag{2.1}$$

the canonical extension $\underline{W}^{\mathrm{can}}_{\nu_1,R}$ of $\underline{W}_{\nu_1,R}$. Since (by definition)

$$\underline{W}^{\mathrm{sub}}_{\nu_0+N\nu_1,R} \cong \underline{W}^{\mathrm{sub}}_{\nu_0,R} \otimes_{\mathcal{O}_{\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,R}}} (\underline{W}^{\mathrm{can}}_{\nu_1,R})^{\otimes N} \tag{2.2}$$

for all integers N, by the projection formula [EGA 1960, 0_{I}, (5.4.10.1), p. 52] we have

$$R^i\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0+N\nu_1,R} \cong (R^i\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0,R}) \otimes_{\mathcal{O}_{\mathsf{M}^{\min}_{\mathcal{H},R}}} \omega^{\otimes N}_{\nu_1}. \tag{2.3}$$

Then we have the following:

Lemma 2.4. *There exists some integer $N_1 \geq 0$ such that, for all integers $N \geq N_1$ and all $i \geq 0$, the sheaves $R^i\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0+N\nu_1,R}$ over $\mathsf{M}^{\min}_{\mathcal{H},R}$ are generated by their global sections and satisfy $H^j(\mathsf{M}^{\min}_{\mathcal{H},R}, R^i\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0+N\nu_1,R}) = 0$ for all $j > 0$.*

Proof. Since π is proper and $\mathsf{M}^{\min}_{\mathcal{H},R}$ is noetherian, by the theorem of finiteness [EGA 1961, III, Théorème (3.2.1), p. 116], the sheaves $R^i\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0,R}$ are coherent over $\mathsf{M}^{\min}_{\mathcal{H},R}$ for all $i \geq 0$, and are nonzero only for finitely many i. Since ω_{ν_1} is ample over $\mathsf{M}^{\min}_{\mathcal{H},R}$, the lemma follows from (2.3) and Serre's fundamental theorem for projective schemes [EGA 1961, III, Théorème (2.2.1), p. 100]. □

2B. ***Shifting weights into the holomorphic chamber.*** Let w_0 (resp. w_1) be the longest Weyl element in $\mathrm{W}_{\mathrm{M}_1}$ (resp. $\mathrm{W}^{\mathrm{M}_1}$) (see [Lan and Suh 2012, §2.4]), so that $(-w_0)\Phi^+_{\mathrm{M}_1} = \Phi^+_{\mathrm{M}_1}$ and $W_\nu \cong W^\vee_{-w_0(\nu)}$ for all $\nu \in \mathrm{X}^{+,<p}_{\mathrm{M}_1}$ and $l(w_1) = d = \dim_{\mathsf{S}_1}(\mathsf{M}_{\mathcal{H},1})$.

Remark 2.5. When $R = \mathbb{C}$, for any $\mu \in \mathrm{X}^+_{\mathrm{G}_1}$, sections in $H^0(\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,R}, (\underline{W}^\vee_{w_1\cdot\mu,R})^{\mathrm{sub}})$ are represented by holomorphic cusp forms of weight $(-w_0)(w_1\cdot\mu) \in \mathrm{X}^+_{\mathrm{M}_1}$, which contribute via the dual BGG spectral sequence to

$$H^d_{\log-\mathrm{dR}}(\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},R}, (\underline{V}^\vee_{[\mu],R})^{\mathrm{sub}}) \cong H^d_{\mathrm{dR,c}}(\mathsf{M}_{\mathcal{H},R}, \underline{V}^\vee_{[\mu],R})$$

(compactly supported of middle degree), compatible with their contribution to the better-understood L^2 cohomology of $\mathsf{M}_{\mathcal{H},R}$. (For more explanations see [Faltings 1983, Theorem 9; Harris 1990a, §2; 1990b, Proposition 5.4.2]; see also the comparisons with transcendental results in [Lan and Suh 2012; 2013] and the references there.) Thus we consider weights of the form

$$(-w_0)(w_1\cdot\mu) = (-w_0w_1)(\mu) + (-w_0)(w_1\cdot 0)$$

holomorphic; these holomorphic weights form a translation of the dominant chamber $\mathrm{X}^+_{\mathrm{G}_1}$ because $(-w_0w_1)$ preserves $\mathrm{X}^+_{\mathrm{G}_1}$.

Proposition 2.6. *There exists an integer N_2, a positive parallel weight $\nu_2 \in \mathrm{X}^+_{\mathrm{M}_1}$, and a weight $\mu_0 \in \mathrm{X}^+_{\mathrm{G}_1}$, all of which can be explicitly determined, such that*

$$\nu_0 + N_2\nu_1 - \nu_2 = -w_0(w_1 \cdot \mu_0) \tag{2.7}$$

This proposition is elementary in nature. One can prove Proposition 2.6 using general principles that also work for all reductive groups defining Shimura varieties. However, we shall spell out a (less elegant) case-by-case argument, which has the advantage of giving explicit choices of N_2, ν_2, and μ_0 of small sizes.

We will assume Proposition 2.6 in the remainder of this section, and postpone its proof until Section 3A. In Lemma 3.3, we will give an explicit choice of $C(\mu_0)$, depending only on $(\mathcal{O}, \star, L, \langle\cdot,\cdot\rangle, h_0)$ and the weight ν_0, such that $C(\nu_0) \geq |\mu_0|_{\mathrm{re}}$ (see [Lan and Suh 2012, Definition 3.9]) for some triple (N_2, ν_2, μ_0) as in Proposition 2.6.

2C. *Application of automorphic vanishing.*

Corollary 2.8. *Let (N_2, ν_2, μ_0) be any triple as in Proposition 2.6. Suppose that $p > |\mu_0|_{\mathrm{re}}$ and that N is any integer satisfying $N \geq N_2$. Then we have*

$$H^i(\mathrm{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,R}, \underline{W}^{\mathrm{sub}}_{\nu_0+N\nu_1,R}) = 0 \quad \textit{for every } i > 0.$$

Proof. By definition, the subset $\mathrm{X}^{+,<p}_{\mathrm{M}_1}$ of $\mathrm{X}^+_{\mathrm{M}_1}$ is preserved by translations by parallel weights. Moreover, by [Lan and Suh 2012, Remark 2.30], and by the same argument as in the proof of [Lan and Suh 2012, Lemma 7.20], we have $\nu_0 \in \mathrm{X}^{+,<p}_{\mathrm{M}_1}$ under the assumption that $p > |\mu_0|_{\mathrm{re}}$. Then the assertion $H^i(\mathrm{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,R}, \underline{W}^{\mathrm{sub}}_{\nu_0+N\nu_1,R}) = 0$ follows from [Lan and Suh 2013, Theorem 8.13(2)], because $\nu := \nu_0 + N\nu_1$ and $\nu_+ := (N-N_2)\nu_1 + \nu_2$ satisfy the condition there, with $\mu(\nu - \nu_+) = \mu_0 \in \mathrm{X}^{+,<_{\mathrm{re}}p}_{\mathrm{G}_1}$ and $w(\nu) = w_1$ (so that $d - l(w(\nu)) = d - l(w_1) = 0$). □

Remark 2.9 (erratum). There are typos in [Lan and Suh 2013, Theorem 8.13]: both instances of $\mathrm{X}^{+,<_{\mathrm{w}}p}_{\mathrm{G}_1}$ there should be $\mathrm{X}^{+,<_{\mathrm{re}}p}_{\mathrm{G}_1}$, which is what was used in [Lan and Suh 2013, Corollary 7.24], on which the theorem depends.

2D. *End of the proof of Theorem 1.1.*

Let N_1 be as in Lemma 2.4, and let (N_2, ν_2, μ_0) be any triple as in Proposition 2.6 satisfying $C(\nu_0) \geq |\mu_0|_{\mathrm{re}}$ for some $C(\nu_0)$ (which will be given in Lemma 3.3 below). Suppose that $p > C(\nu_0)$ and that N is any integer satisfying $N \geq N_1$ and $N \geq N_2$. By Lemma 2.4 and by the Leray spectral sequence, and by Corollary 2.8, we have

$$H^0(\mathrm{M}^{\mathrm{min}}_{\mathcal{H},R}, R^i\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0+N\nu_1,R}) \cong H^i(\mathrm{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,R}, \underline{W}^{\mathrm{sub}}_{\nu_0+N\nu_1,R}) = 0 \tag{2.10}$$

for all $i > 0$. Since $R^i\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0+N\nu_1,R}$ is generated by its global sections (by Lemma 2.4) it follows that

$$R^i\pi_*\underline{W}^{\mathrm{sub}}_{\nu_0+N\nu_1,R} = 0 \tag{2.11}$$

for all $i > 0$. By combining (2.3) and (2.11), we obtain the desired vanishing (1.2) for all $i > 0$ (under the assumption that $p > C(\nu_0) \geq |\mu_0|_{\mathrm{re}}$).

Suppose that the residue characteristics of R are all zero. By shrinking R and enlarging R by flat descent, we may replace the setup with a different one in which $p > C(\nu_0) \geq |\mu_0|_{\mathrm{re}}$, and obtain the desired vanishing from the above.

Thus, Theorem 1.1 follows. □

3. Elementary computations

We shall freely use the notation in [Lan and Suh 2012, §2 and §7]. The material in this section can be read without any knowledge of algebraic geometry or Shimura varieties.

3A. ***Proof of Proposition 2.6.*** We can rewrite (2.7) as

$$\nu_0 + N_2\nu_1 - \nu_2 = -w_0(w_1\mu_0 + w_1\rho - \rho) = \mu_0' + (-w_0)(w_1 \cdot 0),$$

where $\mu_0' = -(w_0w_1)(\mu_0) \in \mathrm{X}^+_{\mathrm{G}_1}$ satisfies $V_{[\mu_0']} \cong V^\vee_{[\mu_0]}$, because w_0w_1 is the longest Weyl element in $\mathrm{W}_{\mathrm{G}_1}$. Hence it suffices to find N_2 and ν_2 such that

$$\mu_0' = \nu_0 + N_2\nu_1 - \nu_2 - (-w_0)(w_1 \cdot 0) \in \mathrm{X}^+_{\mathrm{G}_1}. \tag{3.1}$$

Let us write $\nu_j = ((\nu_{j,\tau})_{\tau\in\Upsilon/c};\, \nu_{j,0}) = \big(((\nu_{j,\tau,i_\tau})_{1\leq i_\tau\leq r_\tau})_{\tau\in\Upsilon/c};\, \nu_{j,0}\big) \in \mathrm{X}^+_{\mathrm{M}_1}$ for $j = 0, 1, 2$. We shall also denote by ρ_τ (resp. $w_{0,\tau}$, $w_{1,\tau}$) the corresponding factors of ρ (resp. w_0, w_1). Then we need

$$\mu_{0,\tau}' = \nu_{0,\tau} + N_2\nu_{1,\tau} - \nu_{2,\tau} - (-w_{0,\tau})(w_{1,\tau} \cdot 0) \in \mathrm{X}^+_{\mathrm{G}_\tau} \tag{3.2}$$

for each factor G_τ of G_1. There are two cases:

(1) If $\tau = \tau \circ c$, then $\mathrm{G}_\tau \cong \mathrm{Sp}_{2r_\tau} \otimes_{\mathbb{Z}} R_1$ or $\mathrm{G}_\tau \cong \mathrm{O}_{2r_\tau} \otimes_{\mathbb{Z}} R_1$, and $\mathrm{M}_\tau \cong \mathrm{GL}_{r_\tau} \otimes_{\mathbb{Z}} R_1$. If $\mathrm{G}_\tau \cong \mathrm{Sp}_{2r_\tau} \otimes_{\mathbb{Z}} R_1$, set $d_\tau = \frac{1}{2}r_\tau(r_\tau + 1)$ and $r_\tau' = r_\tau + 1$. If $\mathrm{G}_\tau \cong \mathrm{O}_{2r_\tau} \otimes_{\mathbb{Z}} R_1$, set $d_\tau = \frac{1}{2}r_\tau(r_\tau - 1)$ and $r_\tau' = r_\tau$. Set $e_\tau = (1, 1, \ldots, 1)$. If $d_{[\tau]_{\mathbb{Q}}} = \sum_{\tau'\in[\tau]_{\mathbb{Q}}} d_{\tau'} = 0$, then we must have $\mathrm{G}_\tau \cong \mathrm{O}_{2r_\tau} \otimes_{\mathbb{Z}} R_1$ and $r_\tau \leq 1$, in which case (3.2) is trivially true if we take $\mu_{0,\tau}' = \nu_{0,\tau}$, any $N_2 \in \mathbb{Z}$, and $\nu_{2,\tau} = N_2\nu_{1,\tau} - (-w_{0,\tau})(w_{1,\tau} \cdot 0)$. Hence we may assume that $d_{[\tau]_{\mathbb{Q}}} > 0$. By assumption, we know that $\nu_{0,\tau,1} \geq \nu_{0,\tau,2} \geq \cdots \geq \nu_{0,\tau,r_\tau}$, and that $\nu_{1,\tau} = k_{1,\tau}e_\tau$, where $k_{1,\tau} > 0$ depends only on the equivalence class $[\tau]_{\mathbb{Q}}$ of τ (see [Lan and Suh 2012, Definition 7.12]). Also, we have $\rho_\tau = (r_\tau', r_\tau' - 1, \ldots, r_\tau' - r_\tau)$ and $(-w_{0,\tau})(w_{1,\tau} \cdot 0) = r_\tau' e_\tau$. Thus, in order for (3.2) to hold, we need

$$\nu_{0,r_\tau} + Nk_{1,\tau} - k_{2,\tau} \geq r_\tau + 1 = r_\tau' \quad \text{if } \mathrm{G}_\tau \cong \mathrm{Sp}_{2r_\tau} \otimes_{\mathbb{Z}} R_1,$$

or

$$\nu_{0,r_\tau - 1} + Nk_{1,\tau} - k_{2,\tau} - r_\tau \geq |\nu_{0,r_\tau} + Nk_{1,\tau} - k_{2,\tau} - r_\tau| \quad \text{if } \mathrm{G}_\tau \cong \mathrm{O}_{2r_\tau} \otimes_{\mathbb{Z}} R_1.$$

We may take:

(a) $\mu'_{0,\tau} := \nu_{0,\tau} - \nu_{0,[\tau]_{\mathbb{Q}}} e_\tau$, where $\nu_{0,[\tau]_{\mathbb{Q}}} := \min_{\tau' \in [\tau]_{\mathbb{Q}}} (\nu_{0,\tau',r_\tau})$;

(b) $\mu_{0,\tau} := -(w_{0,\tau} w_{1,\tau})(\mu'_{0,\tau}) = \mu'_{0,\tau}$; and

(c) N_τ to be any integer satisfying $\nu_{0,[\tau]_{\mathbb{Q}}} + N_\tau k_{1,\tau} > r'_\tau$, so that

$$\nu_{0,\tau} + N\nu_{1,\tau} - \mu'_{0,\tau} - (-w_{0,\tau})(w_{1,\tau} \cdot 0) = (\nu_{0,[\tau]_{\mathbb{Q}}} + Nk_{1,\tau} - r'_\tau)\, e_\tau,$$

with a positive coefficient $\nu_{0,[\tau]_{\mathbb{Q}}} + Nk_{1,\tau} - r'_\tau > 0$ for every $N \geq N_\tau$.

(2) If $\tau \neq \tau \circ c$, then $\mathrm{G}_\tau \cong \mathrm{GL}_{r_\tau} \otimes_{\mathbb{Z}} R_1$ and $\mathrm{M}_\tau \cong (\mathrm{GL}_{q_\tau} \times \mathrm{GL}_{p_\tau}) \otimes_{\mathbb{Z}} R_1$. Set $d_\tau = p_\tau q_\tau$,

$$e_\tau = (\underbrace{1, 1, \ldots, 1}_{q_\tau}, 0, 0, \ldots, 0), \quad \text{and} \quad e'_\tau = (0, 0, \ldots, 0, \underbrace{-1, -1, \ldots, -1}_{p_\tau}).$$

If $d_{[\tau]_{\mathbb{Q}}} = \sum_{\tau' \in [\tau]_{\mathbb{Q}}/c} d_{\tau'} = 0$, then we must have $p_\tau q_\tau = 0$ for all $\tau \in [\tau]_{\mathbb{Q}}$, in which case (3.2) is trivially true if we take $\mu'_{0,\tau} = \nu_{0,\tau}$, any $N_2 \in \mathbb{Z}$, and $\nu_{2,\tau} = N_2 \nu_{1,\tau} - (-w_{0,\tau})(w_{1,\tau} \cdot 0)$. Hence we may assume that $d_{[\tau]_{\mathbb{Q}}} > 0$. By assumption, we know that

$$\nu_{0,\tau,1} \geq \nu_{0,\tau,2} \geq \cdots \geq \nu_{0,\tau,q_\tau} \quad \text{and} \quad \nu_{0,\tau,q_\tau+1} \geq \nu_{0,\tau,q_\tau+2} \geq \cdots \geq \nu_{0,\tau,r_\tau},$$

and that $\nu_{1,\tau} = k_{1,\tau} e_\tau + k_{1,\tau\circ c} e'_\tau$, where $[k_1]_\tau = k_{1,\tau} + k_{1,\tau\circ c} > 0$ depends only on the equivalence class $[\tau]_{\mathbb{Q}}$ of τ (see [Lan and Suh 2012, Proposition 7.15]). Also, we have $\rho_\tau = \frac{1}{2}(r_\tau - 1, r_\tau - 3, \ldots, -r_\tau + 1)$ and $(-w_{0,\tau})(w_{1,\tau} \cdot 0) = p_\tau e_\tau + q_\tau e'_\tau$. Thus, in order for (3.2) to hold, we need

$$\nu_{0,q_\tau} + Nk_{1,\tau} - k_{2,\tau} - p_\tau \geq \nu_{0,q_\tau+1} - Nk_{1,\tau\circ c} + k_{2,\tau\circ c} + q_\tau,$$

or equivalently

$$(\nu_{0,q_\tau} - \nu_{0,q_\tau+1}) + N[k_1]_\tau - [k_2]_\tau \geq p_\tau + q_\tau = r_\tau.$$

We may take:

(a) $\mu'_{0,\tau} := \nu_{0,\tau} - \nu_{0,[\tau]_{\mathbb{Q}}} e_\tau - (\nu'_{0,\tau,1} - \nu_{0,[\tau]_{\mathbb{Q}}})(e_\tau - e'_\tau)$, where

$$\nu_{0,[\tau]_{\mathbb{Q}}} := \min_{\tau' \in [\tau]_{\mathbb{Q}}, d_{\tau'} \neq 0} (\nu_{0,\tau',q_{\tau'}} - \nu_{0,\tau',q_{\tau'}+1}),$$

$$\nu'_{0,\tau,1} := \begin{cases} \nu_{0,\tau,1} & \text{if } q_\tau > 0, \\ \nu_{0,\tau,1} + \nu_{0,[\tau]_{\mathbb{Q}}} & \text{if } q_\tau = 0. \end{cases}$$

(b) $\mu_{0,\tau} := -(w_{0,\tau} w_{1,\tau})(\mu'_{0,\tau})$, which ends with $\mu_{0,\tau,r_\tau} = 0$ because $\mu'_{0,\tau}$ starts with $\mu'_{0,\tau,1} = 0$; and

(c) N_τ to be any integer satisfying $\nu_{0,[\tau]_{\mathbb{Q}}} + N_\tau [k_1]_\tau > r_\tau$, so that

$$\begin{aligned} &\nu_{0,\tau} + N\nu_{1,\tau} - \mu'_{0,\tau} - (-w_{0,\tau})(w_{1,\tau} \cdot 0) \\ &\qquad = (\nu_{0,\tau,1} + Nk_{1,\tau} - p_\tau)\, e_\tau + (\nu_{0,[\tau]_{\mathbb{Q}}} - \nu_{0,\tau,1} + Nk_{1,\tau\circ c} - q_\tau)\, e_\tau \end{aligned}$$

with sum of coefficients, for every $N \geq N_\tau$,

$$(\nu_{0,\tau,1}+Nk_{1,\tau}-p_\tau)+(\nu_{0,[\tau]_\mathbb{Q}}-\nu_{0,\tau,1}+Nk_{1,\tau\circ c}-q_\tau)=\nu_{0,[\tau]_\mathbb{Q}}+N[k_1]_\tau-r_\tau>0.$$

Now set:

$$\begin{aligned}
N_2 &:= \max_{\tau\in\Upsilon/c}(N_\tau);\\
\mu_0 &:= ((\mu_{0,\tau})_{\tau\in\Upsilon/c};\, \mu_{0,0}) \quad \text{with any value of } \mu_{0,0};\\
\mu_0' &:= (-w_0w_1)(\mu_0);\\
\nu_2 &:= \nu_0 + N_2\nu_1 - \mu_0' - (-w_0)(w_1\cdot 0).
\end{aligned}$$

Then the triple (N_2, ν_2, μ_0) satisfies (3.1) and hence also (2.7), as desired, because each of its factors $(N_2, \nu_{2,\tau}, \mu_{0,\tau})$ satisfies (3.2) by the above. □

3B. *Explicit choice of $C(\nu_0)$.*

Lemma 3.3. *The minimal size $|\mu_0|_{\mathrm{re}}$ (see [Lan and Suh 2012, Definition 3.9]) among all μ_0 appearing in some (N_2, ν_2, μ_0) satisfying (2.7) in Proposition 2.6 is smaller than or equal to*

$$C(\nu_0) := \sum_{\tau\in\Upsilon/c} C_\tau(\nu_{0,\tau}), \tag{3.4}$$

where each $C_\tau(\nu_{0,\tau})$ is defined as follows:

(1) *If $\tau = \tau\circ c$, then we set $d_\tau := \frac{1}{2}r_\tau(r_\tau+1)$ (resp. $d_\tau := \frac{1}{2}r_\tau(r_\tau-1)$) if $\mathrm{G}_\tau \cong \mathrm{Sp}_{2r_\tau}\otimes_\mathbb{Z} R_1$ (resp. $\mathrm{G}_\tau \cong \mathrm{O}_{2r_\tau}\otimes_\mathbb{Z} R_1$), $\nu_{0,[\tau]_\mathbb{Q}} := \min_{\tau'\in[\tau]_\mathbb{Q}}(\nu_{0,\tau',r_\tau})$, and*

$$C_\tau(\nu_{0,\tau}) := d_\tau + \sum_{1\leq i_\tau\leq r_\tau}(\nu_{0,\tau,i_\tau}-\nu_{0,[\tau]_\mathbb{Q}}). \tag{3.5}$$

(2) *If $\tau \neq \tau\circ c$, then we set $d_\tau := p_\tau q_\tau$,*

$$\nu_{0,[\tau]_\mathbb{Q}} := \min_{\tau'\in[\tau]_\mathbb{Q}, d_{\tau'}\neq 0}(\nu_{0,\tau',q_{\tau'}}-\nu_{0,\tau',q_{\tau'}+1}),$$

$$\nu'_{0,\tau,1} := \begin{cases} \nu_{0,\tau,1} & \text{if } q_\tau > 0,\\ \nu_{0,\tau,1}+\nu_{0,[\tau]_\mathbb{Q}} & \text{if } q_\tau = 0,\end{cases}$$

and

$$C_\tau(\nu_{0,\tau}) := d_\tau + \sum_{1\leq i_\tau\leq q_\tau}(\nu'_{0,\tau,1}-\nu_{0,\tau,i_\tau}) + \sum_{q_\tau<i_\tau\leq r_\tau}(\nu'_{0,\tau,1}-\nu_{0,[\tau]_\mathbb{Q}}-\nu_{0,\tau,i_\tau}). \tag{3.6}$$

Proof. These follow from the definition of $|\mu_0|_{\mathrm{re}} = d + \sum_{\tau\in\Upsilon/c}\big(\sum_{1\leq i_\tau\leq r_\tau}\mu_{0,\tau,i_\tau}\big)$ and the explicit choices of $\mu_{0,\tau}$ in the proof of Proposition 2.6. □

Remark 3.7. By using [Lan and Suh 2013, (7.9) and (7.11)], it is possible to reduce the proof of Theorem 1.1 to the case where the integral PEL datum is $\mathbb{Q}$-simple,

and replace (3.4) with

$$C'(\nu_0) := \max_{[\tau]_{\mathbb{Q}}} \big(C_{[\tau]_{\mathbb{Q}}}(\nu_{0,[\tau]_{\mathbb{Q}}}) \big), \tag{3.8}$$

where:

(1) $C_{[\tau]_{\mathbb{Q}}}(\nu_{0,[\tau]_{\mathbb{Q}}}) = 0$ if $d_{[\tau]_{\mathbb{Q}}} = \sum_{\tau' \in [\tau]_{\mathbb{Q}}/c} d_\tau \le 1$;

(2) $C_{[\tau]_{\mathbb{Q}}}(\nu_{0,[\tau]_{\mathbb{Q}}}) = \sum_{\tau' \in [\tau]_{\mathbb{Q}}/c} C_\tau(\nu_{0,\tau})$, where $C_\tau(\nu_{0,\tau})$ are as in (3.5) and (3.6), otherwise.

We leave the details to the interested readers.

3C. ***Some examples.*** To help the reader understand the notation and formulas, we include some examples of familiar special cases.

Example 3.9 (trivial weight). If $\nu_0 = 0$, then (2.7) holds for $\mu_0 = 0$ and any sufficiently large N_2, and we have $C(\nu_0) = \sum_{\tau \in \Upsilon/c} C_\tau(\nu_{0,\tau}) = \sum_{\tau \in \Upsilon/c} d_\tau = d$ in (3.4).

Example 3.10 (Siegel case). Suppose $(\mathcal{O}, \star, L, \langle \cdot, \cdot \rangle, h_0)$ is given with $\mathcal{O} = \mathbb{Z}$ with trivial $\star$, with $(L, \langle \cdot, \cdot \rangle)$ given by $\mathbb{Z}^{\oplus 2r}$ with some standard self-dual symplectic pairing, and with any conventional choice of h_0. Then we are in the so-called *Siegel case*. There is a unique $\tau \in \Upsilon$ with $\tau = \tau \circ c$, which we can suppress in our notation, and each $\nu_0 \in \mathrm{X}^+_{\mathrm{M}_1}$ can be represented by a tuple $((\nu_{0,1}, \nu_{0,2}, \ldots, \nu_{0,r}); \nu_{0,0})$, where $\nu_{0,1} \ge \nu_{0,2} \ge \cdots \ge \nu_{0,r}$ are integers. Then μ_0 can be chosen to be

$$\nu_0 - \nu_{0,r}((1, 1, \ldots, 1, 1); 0) = ((\nu_{0,1} - \nu_{0,r}, \ldots, \nu_{0,r-1} - \nu_{0,r}, 0); \nu_{0,0})$$

(where the last entry is irrelevant), and then $C(\nu_0) = \frac{1}{2} r(r+1) + \sum_{1 \le i < r} (\nu_{0,i} - \nu_{0,r})$ (see (3.5)).

Example 3.11 ("$\mathbb{Q}$-similitude Hilbert case"). Suppose $(\mathcal{O}, \star, L, \langle \cdot, \cdot \rangle, h_0)$ is given with $\mathcal{O} = \mathcal{O}_F$ with trivial $\star$, where F is a totally real number field, with $(L, \langle \cdot, \cdot \rangle)$ given by $\mathcal{O}_F^{\oplus 2}$ with some standard symplectic pairing defined by trace, and with any conventional choice of h_0; and suppose p is any prime number unramified in $\mathcal{O}_F$. Then we are essentially in the so-called *Hilbert case*, although we only consider elements in $\mathrm{Res}_{F/\mathbb{Q}} \mathrm{GL}_2$ with similitudes in $\mathbb{G}_{\mathrm{m}}$ (rather than $\mathrm{Res}_{F/\mathbb{Q}} \mathbb{G}_{\mathrm{m}}$). There are d elements $\tau \in \Upsilon$ corresponding to the $d = [F : \mathbb{Q}]$ homomorphisms from $\mathcal{O}_F$ to an algebraic closure of $\mathbb{Q}_p$, which all satisfy $\tau = \tau \circ c$ and determine a unique equivalence class $[\tau]_{\mathbb{Q}}$ (of Galois orbits of τ), and our coefficient ring R is chosen to contain the images of all these homomorphisms, over which all linear algebraic data are split. Each $\nu_0 \in \mathrm{X}^+_{\mathrm{M}_1}$ can be represented by a tuple $((\nu_{0,\tau})_{\tau \in \Upsilon}; \nu_{0,0})$, where each $\nu_{0,\tau} = (\nu_{0,\tau,1})$ consists of just one integer $\nu_{0,\tau,1}$. Then $\nu_{0,[\tau]_{\mathbb{Q}}} = \min_{\tau \in \Upsilon}(\nu_{0,\tau,1})$, and μ_0 can be chosen to be $\nu_0 - \nu_{0,[\tau]_{\mathbb{Q}}}((1)_{\tau \in \Upsilon}; 0) = ((\nu_{0,\tau,1} - \nu_{0,[\tau]_{\mathbb{Q}}})_{\tau \in \Upsilon}; \nu_{0,0})$, and we have $C(\nu_0) = d + \sum_{\tau \in \Upsilon}(\nu_{0,\tau,1} - \nu_{0,[\tau]_{\mathbb{Q}}})$ (see (3.5)).

Example 3.12 (simplest unitary case). Suppose $(\mathcal{O}, \star, L, \langle\cdot,\cdot\rangle, h_0)$ is given with $\mathcal{O} = \mathcal{O}_F$, where F is an imaginary quadratic extension of $\mathbb{Q}$ with an embedding $F \hookrightarrow \mathbb{C}$, with $\star$ given by complex conjugation, with $(L, \langle\cdot,\cdot\rangle)$ given by a Hermitian module over $\mathcal{O}_F^{\oplus r}$ with signature $(r-q, q)$ at ∞ (using the given $F \hookrightarrow \mathbb{C}$), and with any conventional choice of h_0 (respecting the signature); and suppose p is any prime number unramified in $\mathcal{O}_F$. Then we obtain the simplest (nontrivial) *unitary case*. There is a unique representative τ of orbits in Υ/c such that $\tau \neq \tau \circ c$ and $(p_\tau, q_\tau) = (r-q, q)$, matching the signatures at ∞ and at p; hence we shall always choose this τ and suppress τ from the notation. Each $\nu_0 \in \mathrm{X}^+_{\mathrm{M}_1}$ can be represented by a tuple $((\nu_{0,1}, \nu_{0,2}, \ldots, \nu_{0,q}, \nu_{0,q+1}, \ldots, \nu_{0,r}); \nu_{0,0})$, where $\nu_{0,1} \geq \nu_{0,2} \geq \ldots \geq \nu_{0,q}$ and $\nu_{0,q+1} \geq \ldots \geq \nu_{0,r}$ are integers. If $q > 0$, then μ_0 can be chosen to be $(\nu_{0,1} - \nu_{0,q} + \nu_{0,q+1} - \nu_{0,r}, \ldots, \nu_{0,1} - \nu_{0,q}, \nu_{0,1} - \nu_{0,q}, \ldots, \nu_{0,1} - \nu_{0,2}, 0; \nu_{0,0})$ (note the reversed order and the repeated term $\nu_{0,1} - \nu_{0,q}$), and we have

$$C(\nu_0) = (r-q)q + \sum_{1 \leq i \leq q} (\nu_{0,1} - \nu_{0,i}) + \sum_{q < i \leq r} (\nu_{0,1} - \nu_{0,q} + \nu_{0,q+1} - \nu_{0,i}).$$

If $q = 0$, then μ_0 can be chosen to be $(\nu_{0,1} - \nu_{0,r}, \ldots, \nu_{0,1} - \nu_{0,2}, 0; \nu_{0,0})$ and we have $C(\nu_0) = \sum_{1 \leq i \leq r} (\nu_{0,\tau,1} - \nu_{0,i})$; but $d = 0$ and the map π is trivial — $C(\nu_0) = 0$ suffices. (See (3.6) and Remark 3.7.)

4. Simpler proof for the trivial weight case

In this final section, we sketch a logically simpler proof for the trivial weight case $\nu_0 = 0$, which does not require the various advanced technical inputs in [Lan and Suh 2013, §§1–3] (such as the theory of F-spans in [Ogus 1994]). The key is to give a simpler proof of the vanishing statement in Corollary 2.8 when $\nu_0 = 0$ (with a suitable choice of (N_2, ν_2, μ_0)). By standard arguments, as in the proof of [Lan and Suh 2013, Theorem 8.2], we may and we shall assume that R is a perfect field extension of the residue field of R_1.

Using the extended Kodaira–Spencer isomorphism — see [Lan 2013, Theorem 6.4.1.1(4)] — and the very construction of canonical extensions of automorphic bundles using the relative Lie algebra of the universal abelian scheme, one can show that

$$\underline{W}^{\mathrm{can}}_{(-w_0)(w_1\cdot 0)} \cong (\underline{W}^{\vee}_{w_1\cdot 0})^{\mathrm{can}} \cong \Omega^d_{\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,1}/\mathsf{S}_1}(\log\infty) := \bigwedge^d (\Omega^1_{\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,1}/\mathsf{S}_1}(\log\infty))$$

as line bundles over $\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,1}$ (ignoring Tate twists). (The proof is left to the interested readers.) Moreover, the proof of Proposition 2.6 in Section 3A shows that we can take $\mu_0 = 0$ in Proposition 2.6, with some integer N_2 such that the weight $\nu_2 = N_2\nu_1 - (-w_0)(w_1 \cdot 0)$ is positive and parallel. Then we have

$$\underline{W}^{\mathrm{sub}}_{N\nu_1} \cong \underline{W}^{\mathrm{sub}}_{\nu_2} \otimes_{\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,1}} \underline{W}^{\mathrm{can}}_{(-w_0)(w_1\cdot 0)} \cong \underline{W}^{\mathrm{sub}}_{\nu_2} \otimes_{\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,1}} \Omega^d_{\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,1}/\mathsf{S}_1}(\log \mathsf{D}),$$

where D is the boundary divisor $\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,1} - \mathsf{M}_{\mathcal{H},1}$ (with reduced subscheme structure).

By [Lan and Suh 2013, Proposition 4.2(5) and Corollary 7.14], there exists a (usually nonreduced) divisor D' with $\mathsf{D}'_{\mathrm{red}} = \mathsf{D}$, and some $r_0 > 0$, such that the line bundle $(\underline{W}^{\mathrm{can}}_{\nu_2})^{\otimes r}(-\mathsf{D}')$ is ample for all integers $r \geq r_0$. (This follows from [Lan 2013, Theorem 7.3.3.4], which implies that there exists some D' as above such that $\mathbb{O}_{\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,1}}(-\mathsf{D}')$ is relatively ample over $\mathsf{M}^{\mathrm{min}}_{\mathcal{H},1}$.) By base change from R_1 to R, this is exactly the condition $(*)$ needed in [Esnault and Viehweg 1992, Theorem 11.5]. Then, by [Esnault and Viehweg 1992, Theorem 11.5] and by Serre duality, we obtain

$$H^i(\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,R}, \underline{W}^{\mathrm{sub}}_{N\nu_1,R}) = H^i(\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,R}, \underline{W}^{\mathrm{sub}}_{\nu_2,R} \otimes_{\mathbb{O}_{\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,1}}} \Omega^d_{\mathsf{M}^{\mathrm{tor}}_{\mathcal{H},\Sigma,1}/\mathsf{S}_1}(\log \mathsf{D})) = 0$$

for all $i > 0$. (This is the same approach taken in [Lan and Suh 2011].) This gives the desired vanishing statement in Corollary 2.8 when $\nu_0 = 0$, and we can conclude as in Section 2D. This argument does not depend on [Lan and Suh 2013, Theorem 8.13(2)], and hence not on the various advanced technical inputs in [Lan and Suh 2013, §§1–3].

Acknowledgements

The authors would like to thank the anonymous referee for valuable suggestions. Stroh would like to thank Esnault for very helpful discussions, and to thank Scholze for pointing out an error in an earlier approach.

References

[Andreatta et al. 2013a] F. Andreatta, A. Iovita, and V. Pilloni, "p-adic families of Siegel modular cuspforms", preprint, 2013, Available at http://perso.ens-lyon.fr/vincent.pilloni/AIP.pdf. To appear in *Ann. Math.* arXiv 1212.3812

[Andreatta et al. 2013b] F. Andreatta, A. Iovita, and V. Pilloni, "p-adic families of Hilbert modular forms", preprint, 2013, Available at http://perso.ens-lyon.fr/vincent.pilloni/AIP2.pdf.

[Deligne and Illusie 1987] P. Deligne and L. Illusie, "Relèvements modulo p^2 et décomposition du complexe de de Rham", *Invent. Math.* **89**:2 (1987), 247–270. MR 88j:14029 Zbl 0632.14017

[EGA 1960] A. Grothendieck, *Eléments de géométrie algébrique, I: Le langage des schémas*, Publications Mathématiques de l'I.H.E.S. **4**, Inst. Hautes Études Sci., Paris, 1960. MR 29 #1207 Zbl 0118.36206

[EGA 1961] A. Grothendieck, *Eléments de géométrie algébrique, III: Étude cohomologique des faisceaux cohérents, I*, Publications Mathématiques de l'I.H.E.S. **11**, Inst. Hautes Études Sci., Paris, 1961. MR 0163910 Zbl 0118.36206

[Emerton et al. 2013] M. Emerton, D. A. Reduzzi, and L. Xiao, "Galois representations and torsion in the coherent cohomology of Hilbert modular varieties", preprint, 2013. arXiv 1307.8003

[Esnault and Viehweg 1992] H. Esnault and E. Viehweg, *Lectures on vanishing theorems*, DMV Seminar **20**, Birkhäuser, Basel, 1992. MR 94a:14017 Zbl 0779.14003

[Faltings 1983] G. Faltings, "On the cohomology of locally symmetric Hermitian spaces", pp. 55–98 in *Séminaire d'algèbre Paul Dubreil et Marie-Paule Malliavin, 35ème année* (Paris, 1982), edited by M.-P. Malliavin, Lecture Notes in Math. **1029**, Springer, Berlin, 1983. MR 85k:22028 Zbl 0539.22008

[Grauert and Riemenschneider 1970] H. Grauert and O. Riemenschneider, "Verschwindungssätze für analytische Kohomologiegruppen auf komplexen Räumen", *Invent. Math.* **11** (1970), 263–292. MR 46 #2081 Zbl 0202.07602

[Harris 1990a] M. Harris, "Automorphic forms and the cohomology of vector bundles on Shimura varieties", pp. 41–91 in *Automorphic forms, Shimura varieties, and L-functions* (Ann Arbor, MI, 1988), vol. II, edited by L. Clozel and J. S. Milne, Perspect. Math. **11**, Academic Press, Boston, 1990. MR 91g:11063 Zbl 0716.14011

[Harris 1990b] M. Harris, "Automorphic forms of $\overline{\partial}$-cohomology type as coherent cohomology classes", *J. Differential Geom.* **32**:1 (1990), 1–63. MR 91g:11064 Zbl 0711.14012

[Harris et al. 2013] M. Harris, K.-W. Lan, R. Taylor, and J. Thorne, "On the rigid cohomology of certain Shimura varieties", preprint, Institute for Advanced Study, Princeton, NJ, 2013, Available at http://www.math.ias.edu/~rtaylor/rigcoh.pdf.

[Kato 1989] K. Kato, "Logarithmic structures of Fontaine–Illusie", pp. 191–224 in *Algebraic analysis, geometry, and number theory: proceedings of the JAMI Inaugural Conference* (Baltimore, MD, 1988), edited by J. I. Igusa, Johns Hopkins University Press, Baltimore, MD, 1989. MR 99b:14020 Zbl 0776.14004

[Lan 2012] K.-W. Lan, "Comparison between analytic and algebraic constructions of toroidal compactifications of PEL-type Shimura varieties", *J. Reine Angew. Math.* **664** (2012), 163–228. MR 2980135 Zbl 1242.14022

[Lan 2013] K.-W. Lan, *Arithmetic compactifications of PEL-type Shimura varieties*, London Mathematical Society Monographs Series **36**, Princeton University Press, 2013. MR 3186092 Zbl 1284.14004

[Lan 2014] K.-W. Lan, "Compactifications of PEL-type Shimura varieties and Kuga families with ordinary loci", preprint, 2014, Available at http://math.umn.edu/~kwlan/articles/cpt-ram-ord.pdf.

[Lan and Suh 2011] K.-W. Lan and J. Suh, "Liftability of mod p cusp forms of parallel weights", *Int. Math. Res. Not.* **2011**:8 (2011), 1870–1879. MR 2012e:11085 Zbl 1233.11042

[Lan and Suh 2012] K.-W. Lan and J. Suh, "Vanishing theorems for torsion automorphic sheaves on compact PEL-type Shimura varieties", *Duke Math. J.* **161**:6 (2012), 1113–1170. MR 2913102 Zbl 06029039

[Lan and Suh 2013] K.-W. Lan and J. Suh, "Vanishing theorems for torsion automorphic sheaves on general PEL-type Shimura varieties", *Adv. Math.* **242** (2013), 228–286. MR 3055995 Zbl 1276.11103

[Ogus 1994] A. Ogus, *F-crystals, Griffiths transversality, and the Hodge decomposition*, Astérisque **221**, Société Mathématique de France, Paris, 1994. MR 95g:14025 Zbl 0801.14004

[Pilloni and Stroh 2013] V. Pilloni and B. Stroh, "Surconvergence, ramification et modularité", preprint, 2013, Available at http://www.math.univ-paris13.fr/~stroh/Artin.pdf.

[Stroh 2010] B. Stroh, "Relèvement de formes modulaires de Siegel", *Proc. Amer. Math. Soc.* **138**:9 (2010), 3089–3094. MR 2011g:11093 Zbl 1257.11046

[Stroh 2013] B. Stroh, "Classicité en théorie de Hida", *Amer. J. Math.* **135**:4 (2013), 861–889. MR 3086063 Zbl 06203651

[Tian and Xiao 2013] Y. Tian and L. Xiao, "p-adic cohomology and classicality of overconvergent Hilbert modular forms", preprint, 2013. arXiv 1308.0779

Communicated by Richard Taylor
Received 2013-08-23 Revised 2014-08-03 Accepted 2014-10-08

kwlan@math.umn.edu *School of Mathematics, University of Minnesota, 127 Vincent Hall, 206 Church Street SE, Minneapolis, MN 55455, United States*

stroh@math.univ-paris13.fr *C.N.R.S, Université Paris 13, LAGA, 99 avenue J.B. Clément, 93430 Villetaneuse, France*

Algebra & Number Theory

msp.org/ant

EDITORS

MANAGING EDITOR

Bjorn Poonen
Massachusetts Institute of Technology
Cambridge, USA

EDITORIAL BOARD CHAIR

David Eisenbud
University of California
Berkeley, USA

BOARD OF EDITORS

Georgia Benkart	University of Wisconsin, Madison, USA	Shigefumi Mori	RIMS, Kyoto University, Japan
Dave Benson	University of Aberdeen, Scotland	Raman Parimala	Emory University, USA
Richard E. Borcherds	University of California, Berkeley, USA	Jonathan Pila	University of Oxford, UK
John H. Coates	University of Cambridge, UK	Anand Pillay	University of Notre Dame, USA
J-L. Colliot-Thélène	CNRS, Université Paris-Sud, France	Victor Reiner	University of Minnesota, USA
Brian D. Conrad	University of Michigan, USA	Peter Sarnak	Princeton University, USA
Hélène Esnault	Freie Universität Berlin, Germany	Joseph H. Silverman	Brown University, USA
Hubert Flenner	Ruhr-Universität, Germany	Michael Singer	North Carolina State University, USA
Edward Frenkel	University of California, Berkeley, USA	Vasudevan Srinivas	Tata Inst. of Fund. Research, India
Andrew Granville	Université de Montréal, Canada	J. Toby Stafford	University of Michigan, USA
Joseph Gubeladze	San Francisco State University, USA	Bernd Sturmfels	University of California, Berkeley, USA
Roger Heath-Brown	Oxford University, UK	Richard Taylor	Harvard University, USA
Craig Huneke	University of Virginia, USA	Ravi Vakil	Stanford University, USA
János Kollár	Princeton University, USA	Michel van den Bergh	Hasselt University, Belgium
Yuri Manin	Northwestern University, USA	Marie-France Vignéras	Université Paris VII, France
Barry Mazur	Harvard University, USA	Kei-Ichi Watanabe	Nihon University, Japan
Philippe Michel	École Polytechnique Fédérale de Lausanne	Efim Zelmanov	University of California, San Diego, USA
Susan Montgomery	University of Southern California, USA	Shou-Wu Zhang	Princeton University, USA

PRODUCTION

production@msp.org
Silvio Levy, Scientific Editor

See inside back cover or msp.org/ant for submission instructions.

The subscription price for 2014 is US $225/year for the electronic version, and $400/year (+$55, if shipping outside the US) for print and electronic. Subscriptions, requests for back issues and changes of subscribers address should be sent to MSP.

Algebra & Number Theory (ISSN 1944-7833 electronic, 1937-0652 printed) at Mathematical Sciences Publishers, 798 Evans Hall #3840, c/o University of California, Berkeley, CA 94720-3840 is published continuously online. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices.

ANT peer review and production are managed by EditFLOW® from MSP.

PUBLISHED BY

mathematical sciences publishers

nonprofit scientific publishing

http://msp.org/

© 2014 Mathematical Sciences Publishers

Algebra & Number Theory

Volume 8 No. 8 2014

Relative cohomology of cuspidal forms on PEL-type Shimura varieties 1787
KAI-WEN LAN and BENOÎT STROH

ℓ-modular representations of unramified p-adic U(2,1) 1801
ROBERT JAMES KURINCZUK

McKay natural correspondences on characters 1839
GABRIEL NAVARRO, PHAM HUU TIEP and CAROLINA VALLEJO

Quantum matrices by paths 1857
KAREL CASTEELS

Twisted Bhargava cubes 1913
WEE TECK GAN and GORDAN SAVIN

Proper triangular $\mathbb{G}_a$-actions on $\mathbb{A}^4$ are translations 1959
ADRIEN DUBOULOZ, DAVID R. FINSTON and IMAD JARADAT

Multivariate Apéry numbers and supercongruences of rational functions 1985
ARMIN STRAUB

The image of Carmichael's λ-function 2009
KEVIN FORD, FLORIAN LUCA and CARL POMERANCE

1937-0652(2014)8:8;1-3