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**Correction to the article
Finite generation of the cohomology of
some skew group algebras**

Van C. Nguyen and Sarah Witherspoon



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For the class of examples in Section 5 of the article in question, the proof of finite generation of cohomology is incomplete. We give here a proof of existence of a polynomial subalgebra needed there. The rest of the proof of finite generation given by the authors then applies.

Let k be a field of characteristic $p > 2$. Let A be the augmented k -algebra generated by a and b , with relations

$$a^p = 0, \quad b^p = 0, \quad ba = ab + \frac{1}{2}a^2,$$

and augmentation $\varepsilon : A \rightarrow k$ given by $\varepsilon(a) = \varepsilon(b) = 0$. Let G be a cyclic group of order p with generator g , acting on A by

$$g(a) = a, \quad g(b) = a + b.$$

The corresponding skew group algebra $A\#kG$ is a pointed Hopf algebra described in [Cibils et al. 2009, Corollary 3.14]. We remark that in Section 4 of the article we are correcting, referred to as [NW 2014], we used the left G -module structure with $g(a) = a$ and $g(b) = b - a$, whereas the authors in [Cibils et al. 2009; Nguyen et al. 2017] used the right G -module structure given as above. We will apply the results in [Nguyen et al. 2017] to prove that the cohomology $H^*(A\#kG, k) := \text{Ext}_{A\#kG}^*(k, k)$ is finitely generated, and this will fill a gap in the proof in [NW 2014, Section 5]. Thus we will now also adopt the choices of group actions in [Cibils et al. 2009; Nguyen et al. 2017] instead of that in [NW 2014]. This change does not affect the results discussed in [NW 2014, Section 4].

Let k be an $A\#kG$ -module via the augmentation map ε . To prove finite generation of $H^*(A\#kG, k)$, we wish to apply [NW 2014, Theorem 3.1]. We use results in [Nguyen et al. 2017], where the notation is slightly different, with x in place of a and y in place of b . There it is shown that there are 2-cocycles ξ_a, ξ_b in $H^*(A, k)$ generating a polynomial subring $k[\xi_a, \xi_b]$. These 2-cocycles are not both G -invariant, as was claimed in [NW 2014]; specifically, in [Nguyen et al. 2017] it is shown that ξ_a is G -invariant while ξ_b is not. The claimed G -invariance was used in [NW 2014, Section 5] to show that ξ_a and ξ_b are

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in the image $\text{Im}(\text{res}_{A\#kG,A})$ of the restriction map from $H^*(A\#kG, k)$ to $H^*(A, k)$. However, results in [Nguyen et al. 2017, Section 5.1] imply directly that ξ_a, ξ_b are in $\text{Im}(\text{res}_{A\#kG,A})$; the needed elements in $H^*(A\#kG, k)$ are constructed explicitly using a twisted tensor product resolution in [Nguyen et al. 2017, Section 3.3]. Now the rest of the finite generation proof in [NW 2014, Section 5] can proceed as before, since it is shown there that the rest of the hypotheses of [NW 2014, Theorem 3.1] are satisfied. An alternative proof is given in [Nguyen et al. 2017, Section 5.1].

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