Classification of connected edge-transitive graphs on 20 vertices or less

supplement to "Edge-transitive graphs and combinatorial designs", *Involve*, v. 12 (2019)

Notation

- PM stands for "Perfect Matching"
- $\overline{G}$ stands for the complement graph of G
- $K_{s,s,\ldots,s}$ is the complete multipartite graph with partite sets of size s
- Graphs labelled NoncayleyTransitive(n, k) come from Gordon Royle's list of non-Cayley vertex-transitive graphs at: http://staffhome.ecm.uwa.edu.au/$\tilde{0}$0013890/trans/

For graphs with fewer than six vertices, all of the graphs are either cycles, complete graphs, or star graphs.

Edge-Transitive Graphs on 6 Vertices

1. $K_{1,5}$
2. C_6
3. $K_{2,4}$
4. $K_{3,3}$
5. C_6^2
6. K_6

Edge-Transitive Graphs on 7 Vertices

1. $K_{1,6}$
2. C_7
3. $K_{2,5}$
4. $K_{3,4}$
5. K_7

Edge-Transitive Graphs on 8 Vertices

1. K_8
2. $K_{1,7}$
3. $K_{2,6}$
4. $K_{3,5}$
5. $K_{4,4}$
6. C_8
7. C_8^3
8. Cube (with 6 faces and 12 edges)

Edge-Transitive Graphs on 9 Vertices

1. K_9
2. $K_{1,8}$
3. C_9
4. $K_{2,7}$

5. $K_{3,6}$
6. $K_{4,5}$
7. $C_3 \times C_3$
8. $K_{3,3,3}$
9. $(4,2)$ bi-regular subgraph of $K_{3,6}$

Edge-Transitive Graphs on 10 Vertices

1. $K_{1,9}$
2. C_{10}
3. $(3,2)$ bi-regular subgraph of $K_{4,6}$
4. $Petersen$
5. $K_{2,8}$
6. $K_{5,5} - PM$
7. $Wreath(5,2)$
8. $K_{3,7}$
9. $K_{4,6}$
10. $K_{5,5}$
11. $\overline{Petersen}$

12 .$K_{10} - PM$

13. K_{12}

Edge-Transitive Graphs on 11 Vertices

1. $K_{1,10}$
2. C_{11}
3. $K_{2,9}$
4. $K_{3,8}$
5. $K_{4,7}$
6. $K_{5,6}$
7. K_{11}

Edge-Transitive Graphs on 12 Vertices

There are 19 edge-transitive graphs on 12 vertices:

1 graph : $n = 12$; $e = 11$; $K_{1,11}$
1 graph : $n = 12$; $e = 12$; C_{12}
1 graph : $n = 12$; $e = 16$; $(4,2)$ subgraph of $K_{4,8}$
1 graph : $n = 12$; $e = 18$; $(6,2)$ subgraph of $K_{3,9}$
1 graph : $n = 12$; $e = 20$; $K_{2,10}$
3 graphs : $n = 12$; $e = 24$; $(3,6)$ subgraph of $K_{8,4}$; Wreath(6,2); Cuboctahedral
1 graph : $n = 12$; $e = 27$; $K_{3,9}$
2 graphs : $n = 12$; $e = 30$; $K_{6,6} - PM$; Icosahedral
1 graph : $n = 12$; $e = 32$; $K_{4,8}$
1 graph : $n = 12$; $e = 35$; $K_{5,7}$
2 graphs : $n = 12$; $e = 36$; $K_{6,6}$; $\overline{K_3 \times K_4}$
1 graph : $n = 12$; $e = 48$; $K_{4,4,4}$
1 graph : $n = 12$; $e = 54$; $K_{3,3,3,3}$
1 graph : $n = 12$; $e = 60$; $K_{12} - PM$
1 graph : $n = 12$; $e = 66$; K_{12}

Edge-Transitive Graphs on 13 Vertices
There are 10 edge-transitive graphs on 13 vertices:

1 graph : $n = 13$; $e = 12$; $K_{1,12}$
1 graph : $n = 13$; $e = 13$; C_{13}
1 graph : $n = 13$; $e = 22$; $K_{2,11}$
1 graph : $n = 13$; $e = 26$; $C_{13}(1, 5)$
1 graph : $n = 13$; $e = 30$; $K_{3,10}$
1 graph : $n = 13$; $e = 36$; $K_{4,9}$
1 graph : $n = 13$; $e = 39$; Paley(13)
1 graph : $n = 13$; $e = 40$; $K_{5,8}$
1 graph : $n = 13$; $e = 42$; $K_{6,7}$
1 graph : $n = 13$; $e = 78$; K_{13}

Edge-Transitive Graphs on 14 Vertices
There are 16 edge-transitive graphs on 14 vertices:

1 graph : $n = 14$; $e = 13$; $K_{1,13}$
1 graph : $n = 14$; $e = 14$; C_{14}
1 graph : $n = 14$; $e = 21$; Heawood = $(3, 3)$ subgraph of $K_{7,7}$
3 graphs : $n = 14$; $e = 24$; $K_{2,12}$; 2 $(4, 3)$ bi-regular subgraphs of $K_{6,8}$
2 graphs : $n = 14$; $e = 28$; Wreath(7,2); $(4, 4)$ subgraph of $K_{7,7}$
1 graph : $n = 14$; $e = 33$; $K_{3,11}$
1 graph : $n = 14$; $e = 40$; $K_{4,10}$
1 graph : $n = 14$; $e = 42$; $K_{7,7} - PM$
1 graph : $n = 14$; $e = 45$; $K_{5,9}$
1 graph : $n = 14$; $e = 48$; $K_{6,8}$
1 graph : $n = 14$; $e = 49$; $K_{7,7}$
1 graph : $n = 14$; $e = 84$; $K_{14} - PM$
1 graph : $n = 14$; $e = 91$; K_{14}

Edge-Transitive Graphs on 15 Vertices
There are 25 edge-transitive (connected) graphs on 15 vertices.

1 graph : $n = 15$; $e = 14$; $K_{1,14}$
1 graph : $n = 15$; $e = 15$; C_{15}
1 graph : $n = 15$; $e = 18$; $(3, 2)$ subgraph of $K_{6,9}$
2 graphs : $n = 15$; $e = 20$; 2 $(4, 2)$ subgraphs of $K_{5,10}$
1 graph : $n = 15$; $e = 24$; $(8, 2)$ subgraph of $K_{3,12}$
1 graph : $n = 15$; $e = 26$; $K_{2,13}$
3 graphs : $n = 15$; $e = 30$; $(6, 3)$ subgraph of $K_{5,10}$; L(Petersen); $C_{15}(1, 4)$
3 graphs : $n = 15$; $e = 36$; $K_{3,12}$; 2 $(6, 4)$ subgraphs of $K_{6,9}$
1 graph : $n = 15$; $e = 40$; $(8, 4)$ subgraph of $K_{5,10}$
1 graph : $n = 15$; $e = 44$; $K_{4,11}$
2 graphs : $n = 15$; $e = 45$; $(6, 2)$ Kneser graph; Trpl(C_5)
1 graph : $n = 15$; $e = 50$; $K_{5,10}$
1 graph : $n = 15$; $e = 54$; $K_{6,9}$
1 graph : $n = 15$; $e = 56$; $K_{7,8}$
2 graphs : $n = 15$; $e = 60$; $\overline{K_3 \times K_5}$; (6,2) Johnson graph

1 graph : $n = 15$; $e = 75$; $K_{5,5,5}$
1 graph : $n = 15$; $e = 90$; $K_{3,3,3,3,3}$
1 graph : $n = 15$; $e = 105$; K_{15}

Edge-Transitive Graphs on 16 Vertices
There are 26 edge-transitive (connected) graphs on 16 vertices:

1 graph : $n = 16$; $e = 15$; $K_{1,15}$
1 graph : $n = 16$; $e = 16$; C_{16}
3 graphs : $n = 16$; $e = 24$; Möbius-Kantor graph; 2 $(6, 2)$ subgraphs of $K_{4,12}$
1 graph : $n = 16$; $e = 28$; $K_{2,14}$
1 graph : $n = 16$; $e = 30$; $(5, 3)$ subgraph of $K_{6,10}$
2 graphs : $n = 16$; $e = 32$; Q_4; Wreath(8,2)
1 graph : $n = 16$; $e = 36$; $(9, 3)$ subgraph of $K_{4,12}$
1 graph : $n = 16$; $e = 39$; $K_{3,13}$
1 graph : $n = 16$; $e = 40$; Clebsch graph
4 graphs : $n = 16$; $e = 48$; $K_{4,12}$; Shrikhande graph; $K_4 \times K_4$; $Haar(187)$
1 graph : $n = 16$; $e = 55$; $K_{5,11}$
1 graph : $n = 16$; $e = 56$; $K_{8,8} - PM \cong \overline{K_8 \times K_2}$
1 graph : $n = 16$; $e = 60$; $K_{6,10}$
1 graph : $n = 16$; $e = 63$; $K_{7,9}$
1 graph : $n = 16$; $e = 64$; $K_{8,8}$
1 graph : $n = 16$; $e = 72$; $\overline{K_4 \times K_4}$
1 graph : $n = 16$; $e = 80$; Complement of Clebsch
1 graph : $n = 16$; $e = 96$; $K_{4,4,4,4}$
1 graph : $n = 16$; $e = 112$; $K_{16} - PM$
1 graph : $n = 16$; $e = 120$; K_{16}

Edge-Transitive Graphs on 17 Vertices
There are 12 edge-transitive (connected) graphs on 17 vertices:

1 graph : $n = 17$; $e = 16$; $K_{1,16}$
1 graph : $n = 17$; $e = 17$; C_{17}
1 graph : $n = 17$; $e = 30$; $K_{2,15}$
1 graph : $n = 17$; $e = 34$; $C_{17}(1, 4)$
1 graph : $n = 17$; $e = 42$; $K_{3,14}$
1 graph : $n = 17$; $e = 52$; $K_{4,13}$
1 graph : $n = 17$; $e = 60$; $K_{5,12}$
1 graph : $n = 17$; $e = 66$; $K_{6,11}$
1 graph : $n = 17$; $e = 68$; Paley(17)
1 graph : $n = 17$; $e = 70$; $K_{7,10}$
1 graph : $n = 17$; $e = 72$; $K_{8,9}$
1 graph : $n = 17$; $e = 136$; K_{17}

Edge-Transitive Graphs on 18 Vertices
There are 28 edge-transitive (connected) graphs on 18 vertices:

1 graph : $n = 18$; $e = 17$; $K_{1,17}$
1 graph : $n = 18$; $e = 18$; C_{18}
2 graphs : $n = 18$; $e = 24$; 2 $(4, 2)$ subgraphs of $K_{6,12}$
1 graph : $n = 18$; $e = 27$; Pappus graph

1 graph : $n = 18$; $e = 30$; $(10, 2)$ subgraph of $K_{3,15}$
1 graph : $n = 18$; $e = 32$; $K_{2,16}$
3 graphs : $n = 18$; $e = 36$; $(6, 3)$ subgraph of $K_{6,12}$; $(4, 4)$ subgraph of $K_{9,9}$, Wreath$(9, 2)$
1 graph : $n = 18$; $e = 45$; $K_{3,15}$
1 graph : $n = 18$; $e = 48$; $(8, 4)$ subgraph of $K_{6,12}$
2 graphs : $n = 18$; $e = 54$; 2 $(6, 6)$ subgraphs of $K_{9,9}$
1 graph : $n = 18$; $e = 56$; $K_{4,14}$
1 graph : $n = 18$; $e = 60$; $(10, 5)$ subgraph of $K_{6,12}$
1 graph : $n = 18$; $e = 65$; $K_{5,13}$
3 graphs : $n = 18$; $e = 72$; $K_{9,9} - PM \cong \overline{K_9 \times K_2}$; $K_{6,12}$; H_1
1 graph : $n = 18$; $e = 77$; $K_{7,11}$
1 graph : $n = 18$; $e = 80$; $K_{8,10}$
1 graph : $n = 18$; $e = 81$; $K_{9,9}$
1 graph : $n = 18$; $e = 90$; $\overline{K_6 \times K_3}$
1 graph : $n = 18$; $e = 108$; $K_{6,6,6}$
1 graph : $n = 18$; $e = 135$; $K_{3,3,3,3,3,3}$
1 graph : $n = 18$; $e = 144$; K_{18} - PM
1 graph : $n = 18$; $e = 153$; K_{18}

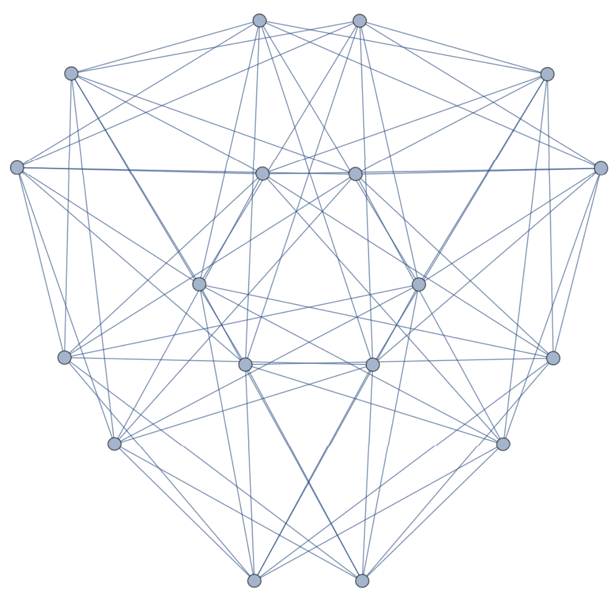

Figure 4: The graph H_1 on 18 vertices.

Mathematica Code: Graph[{0<->1, 0<->2, 0<->3, 0<->4, 0<->5, 0<->6, 0<->7, 0<->8, 1<->2, 1<->3, 1<->9, 1<->10, 1<->11, 1<->12, 1<->13, 2<->4, 2<->9, 2<->14, 2<->15, 2<->16, 2<->17,3<->4, 3<->9, 3<->14, 3<->15, 3<->16, 3<->17, 4<->9, 4<->10, 4<->11, 4<->12, 4<->13, 5<->6, 5<->7, 5<->9, 5<->10, 5<->11, 5<->14, 5<->15, 6<->8, 6<->9, 6<->12,6<->13, 6<->16, 6<->17, 7<->8, 7<->9, 7<->12, 7<->13, 7<->16, 7<->17, 8<->9, 8<->10, 8<->11, 8<->14, 8<->15, 10<->12, 10<->13, 10<->14, 10<->15, 11<->12,11<->13,11<->14, 11<->15, 12<->16, 12<->17, 13<->16, 13<->17, 14<->16, 14<->17, 15<->16, 15<->17}]

Edge-Transitive Graphs on 19 Vertices
There are 12 edge-transitive (connected) graphs on 19 vertices:

1 graph : $n = 19$; $e = 18$; $K_{1,18}$
1 graph : $n = 19$; $e = 19$; C_{19}
1 graph : $n = 19$; $e = 34$; $K_{2,17}$
1 graph : $n = 19$; $e = 48$; $K_{3,16}$
1 graph : $n = 19$; $e = 57$; $C_{19}(1, 7, 8)$
1 graph : $n = 19$; $e = 60$; $K_{4,15}$
1 graph : $n = 19$; $e = 70$; $K_{5,14}$
1 graph : $n = 19$; $e = 78$; $K_{6,13}$
1 graph : $n = 19$; $e = 84$; $K_{7,12}$
1 graph : $n = 19$; $e = 88$; $K_{8,11}$
1 graph : $n = 19$; $e = 90$; $K_{9,10}$
1 graph : $n = 19$; $e = 171$; K_{19}

Edge-Transitive Graphs on 20 Vertices
There are 43 edge-transitive (connected) graphs on 20 vertices.

1 graph : $n = 20$; $e = 19$; $K_{1,19}$
1 graph : $n = 20$; $e = 20$; C_{20}
1 graph : $n = 20$; $e = 24$; $(3, 2)$ subgraph of $K_{8,12}$ =1-Menger sponge graph
3 graphs : $n = 20$; $e = 30$; $(6, 2)$ subgraph of $K_{5,15}$; Desargues Graph; Dodecahedral graph
1 graph : $n = 20$; $e = 32$; $(8, 2)$ subgraph of $K_{4,16}$
1 graph : $n = 20$; $e = 36$; $K_{2,18}$
4 graphs : $n = 20$; $e = 40$; Folkman; $Wreath(10, 2)$; $Haar(525)$; NoncayleyTransitive(20,4)
5 graphs : $n = 20$; $e = 48$; 4 $(6, 4)$ subgraphs of $K_{8,12}$; 1 $(12, 3)$ subgraph of $K_{4,16}$
1 graph : $n = 20$; $e = 51$; $K_{3,17}$
7 graphs : $n = 20$; $e = 60$; 1 $(12, 4)$ subgraph of $K_{5,15}$; 2 $(6, 6)$ subgraphs of $K_{10,10}$ (one is NoncayleyTransitive(20,12); $C_{20}(1, 6, 9)$; G_1; G_2; (5,2)-arrangement graph
1 graph : $n = 20$; $e = 64$; $K_{4,16}$
1 graph : $n = 20$; $e = 72$; $(9, 6)$ subgraph of $K_{8,12}$
1 graph : $n = 20$; $e = 75$; $K_{5,15}$
2 graphs : $n = 20$; $e = 80$; $(8, 8)$ subgraph of $K_{10,10}$; $Wreath(5, 4)$
1 graph : $n = 20$; $e = 84$; $K_{6,14}$
2 graphs : $n = 20$; $e = 90$; $K_{10,10} - PM$; 6-tetrahedral (Johnson) graph
1 graph : $n = 20$; $e = 91$; $K_{7,13}$
1 graph : $n = 20$; $e = 96$; $K_{8,12}$
1 graph : $n = 20$; $e = 99$; $K_{9,11}$
1 graph : $n = 20$; $e = 100$; $K_{10,10}$
2 graphs : $n = 20$; $e = 120$; $\overline{K_5 \times K_4}$; G_3
1 graph : $n = 20$; $e = 150$; $K_{5,5,5,5}$
1 graph : $n = 20$; $e = 160$; $K_{4,4,4,4,4}$
1 graph : $n = 20$; $e = 180$; $K_{20} - PM$
1 graph : $n = 20$; $e = 190$; K_{20}

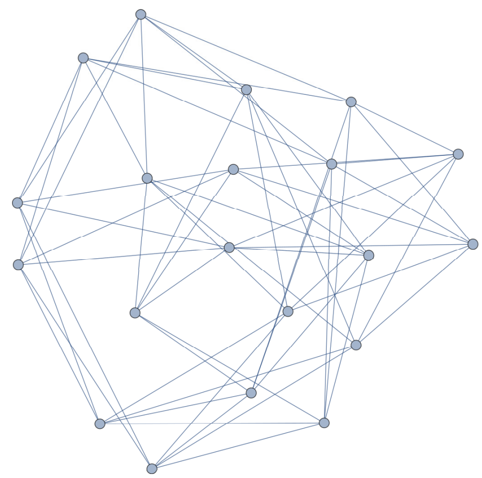

Figure 5: G_1.

Mathematica Code: Graph[{0<->1,0<->2,0<->3,0<->4,0<->5,0<->6,1<->7,1<->8,1<->9,1<->10,1<->11,2<->7,2<->8,2<->9,2<->10,2<->11, 3<->7,3<->12,3<->13,3<->14,3<->15,4<->7,4<->12,4<->13,4<->14,4<->15,5<->7,5<->16,5<->17,5<->18,5<->19,6<->7,6<->16,6<->17,6<->18,6<->19,8<->12,8<->13,8<->16,8<->17,9<->12,9<->13,9<->16,9<->17, 10<->14,10<->15,10<->18,10<->19,11<->14,11<->15,11<->18,11<->19,12<->18,12<->19,13<->18,13<->19,14<->16,14<->17,15<->16,15<->17}]

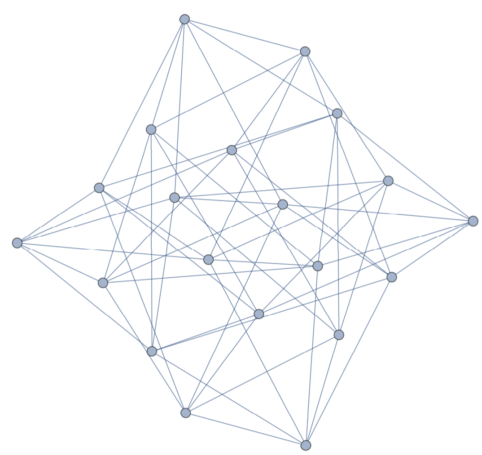

Figure 6: G_2.

Mathematica Code: Graph[{0<->1,0<->2,0<->3,0<->4,0<->5,0<->6,1<->2,1<->7,1<->8,1<->13,1<->14,2<->9,2<->10,2<->15,2<->16,3<->5,3<->7,3<->12,3<->15,3<->17,4<->6,4<->8,4<->12,4<->16,4<->18,5<->10,5<->11,5<->14,5<->18,

6<->9,6<->11,6<->13,6<->17,7<->10,7<->13,7<->17,7<->19,8<->9,8<->14,8<->18,8<->19,9<->15,9<->17,9<->19,10<->16,10<->18,10<->19,11<->12,11<->13,11<->14,11<->19,12<->15,12<->16,12<->19,13<->15,13<->18,14<->16,14<->17,15<->18,16<->17}]

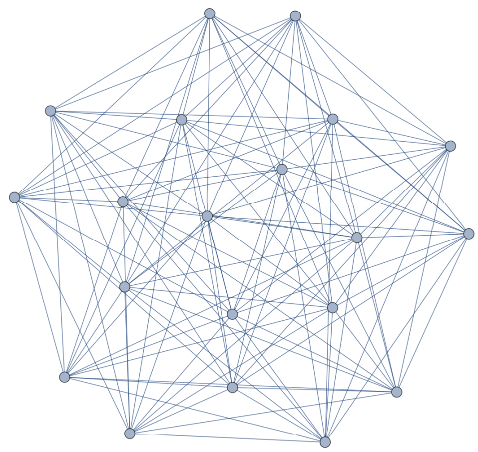

Figure 7: G_3.

Mathematica Code: Graph[{0<->1,0<->2,0<->3,0<->4,0<->5,0<->6,0<->7,0<->8,0<->9,0<->10,0<->11,0<->12,1<->2,1<->3,1<->4,1<->5,1<->6,1<->7,1<->13,1<->14,1<->15,1<->16,1<->17,2<->3,2<->4,2<->8,2<->9,2<->10,2<->13,2<->14,2<->15,2<->18,2<->19,

3<->5,3<->8,3<->11,3<->13,3<->16,3<->17,3<->18,3<->19,3<->12,4<->5,4<->8,4<->11,4<->12,4<->13,4<->16,4<->17,4<->18,4<->19,5<->8,5<->9,5<->10,5<->13,5<->14,5<->15,5<->18,5<->19,6<->8,6<->9,6<->10,6<->11,6<->12,6<->13,6<->14,6<->15,6<->16,6<->17,

7<->8,7<->9,7<->10,7<->11,7<->12,7<->13,7<->14,7<->15,7<->16,7<->17,8<->13,8<->14,8<->15,8<->16,8<->17,9<->11,9<->12,9<->13,9<->14,9<->15,9<->18,9<->19,10<->11,10<->12,10<->13,10<->14,10<->15,10<->18,10<->19,

11<->13,11<->16,11<->17,11<->18,11<->19,12<->13,12<->16,12<->17,12<->18,12<->19,14<->16,14<->17,14<->18,14<->19,15<->16,15<->17,15<->18,15<->19,16<->18,16<->19,17<->18,17<->19}]