

LIST OF TUNNEL NUMBERS

11- and 12-crossing knots with tunnel number 1:

11aX with $X = 1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 21, 23, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 45, 46, 48, 50, 51, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 74, 75, 77, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 96, 98, 110, 111, 117, 118, 119, 120, 121, 140, 144, 145, 153, 154, 159, 161, 166, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 188, 190, 191, 192, 193, 195, 203, 204, 205, 206, 207, 208, 210, 211, 220, 221, 222, 224, 225, 226, 229, 230, 234, 235, 236, 238, 240, 241, 242, 243, 245, 246, 247, 257, 258, 259, 260, 306, 307, 308, 309, 310, 311, 333, 334, 335, 336, 337, 339, 341, 342, 343, 355, 356, 357, 358, 359, 360, 363, 364, 365, 367.$

11nX with $X = 1, 2, 3, 12, 13, 14, 15, 16, 17, 18, 19, 20, 28, 29, 30, 38, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 70, 79, 96, 102, 104, 111, 135, 143, 145.$

12aX with $X = 3, 9, 12, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 31, 32, 34, 35, 37, 38, 42, 56, 62, 77, 78, 81, 82, 83, 84, 85, 86, 87, 95, 96, 97, 98, 99, 104, 106, 110, 112, 118, 121, 124, 128, 130, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 158, 159, 160, 161, 165, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 183, 193, 194, 196, 197, 203, 204, 205, 206, 210, 212, 221, 226, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 251, 254, 255, 257, 258, 259, 299, 300, 302, 303, 304, 305, 306, 307, 329, 330, 345, 378, 379, 380, 384, 385, 406, 420, 421, 422, 423, 424, 425, 436, 437, 447, 454, 463, 471, 477, 482, 497, 498, 499, 500, 501, 502, 506, 507, 508, 510, 511, 512, 514, 517, 518, 519, 520, 521, 522, 528, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 544, 545, 549, 550, 551, 552, 579, 580, 581, 582, 583, 584, 585, 591, 595, 596, 597, 600, 601, 643, 644, 649, 650, 651, 652, 669, 681, 682, 684, 689, 690, 691, 713, 714, 715, 716, 717, 718, 720, 721, 722, 723, 724, 726, 727, 728, 729, 731, 732, 733, 736, 738, 740, 743, 744, 745, 757, 758, 759, 760, 761, 762, 763, 764, 772, 773, 774, 775, 789, 790, 791, 792, 794, 795, 796, 797, 800, 802, 803, 822, 823, 826, 827, 835, 836, 837, 838, 839, 840, 842, 843, 876, 877, 878, 879, 880, 881, 882, 937, 938, 1023, 1024, 1029, 1030, 1033, 1034, 1039, 1040, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1138, 1139, 1140, 1145, 1146, 1148, 1149, 1157, 1158, 1159, 1161, 1162, 1163, 1165, 1166, 1214, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1281, 1282, 1287.$

12nX with $X = 11, 12, 13, 25, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 54, 65, 77, 78, 79, 121, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 198, 199, 200, 218, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 248, 249, 250, 251, 288, 289, 293, 303, 304, 305, 306, 307, 308, 309, 310, 352, 370, 371, 374, 404, 433, 446, 464, 483, 487, 488, 500, 501, 502, 503, 552, 575, 579, 591, 594, 624, 650, 721, 722, 723, 724, 725, 726, 749, 851.$

11 and 12 crossing knots with tunnel number 2:

11aX with $X = 2, 3, 14, 15, 17, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 38, 43, 44, 47, 49, 52, 53, 54, 57, 66, 67, 68, 69, 70, 71, 72, 73, 76, 78, 79, 80, 81, 86, 87, 88, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 114, 115, 116, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138,$

139, 141, 142, 143, 146, 147, 148, 149, 150, 151, 152, 155, 156, 157, 158, 160, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 181, 187, 189, 194, 196, 197, 198, 199, 200, 201, 202, 209, 212, 213, 214, 215, 216, 217, 218, 219, 223, 227, 228, 231, 232, 233, 237, 239, 244, 248, 249, 250, 251, 252, 253, 254, 255, 256, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 338, 340, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 361, 362, 366.

11nX with $X = 4, 5, 6, 7, 8, 9, 10, 11, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 93, 97, 100, 101, 103, 105, 106, 107, 108, 109, 110, 114, 116, 120, 122, 124, 126, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 141, 147, 148, 151, 154, 157, 159, 160, 162, 164, 165, 166, 167, 172, 174, 175, 176, 177, 183, 184, 185.$

12aX with $X = 1, 2, 4, 5, 6, 7, 8, 10, 11, 13, 14, 15, 23, 29, 30, 33, 36, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 79, 80, 88, 89, 90, 91, 92, 93, 94, 100, 101, 102, 103, 105, 107, 108, 109, 111, 113, 114, 115, 116, 117, 119, 120, 122, 123, 125, 126, 127, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 150, 154, 155, 156, 157, 162, 163, 164, 166, 167, 177, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 195, 198, 199, 200, 201, 202, 207, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 227, 228, 229, 230, 231, 232, 233, 244, 245, 248, 249, 250, 252, 253, 256, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 301, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 453, 455, 456, 457, 458, 459, 460, 461, 462, 464, 465, 466, 467, 468, 469, 470, 472, 473, 474, 475, 476, 478, 479, 480, 481, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 503, 504, 505, 509, 513, 515, 516, 523, 524, 525, 526, 527, 529, 530, 531, 542, 543, 546, 547, 548, 553, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 586, 587, 588, 589, 590, 592, 593, 594, 598, 599, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 645, 646, 647, 648, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 683, 685, 686, 687, 688, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708,$

709, 710, 711, 712, 719, 725, 730, 734, 735, 737, 739, 741, 742, 746, 747, 748, 749, 751, 752, 753, 754, 755, 756, 765, 766, 767, 768, 769, 770, 771, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 793, 798, 799, 801, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 824, 825, 828, 829, 830, 831, 832, 833, 834, 841, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1031, 1032, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1137, 1141, 1142, 1143, 1144, 1147, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1160, 1164, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1280, 1283, 1284, 1285, 1286, 1288.

$12nX$ with $X = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 201, 202, 203, 205, 206, 208, 209, 210, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 224, 225, 226, 227, 229, 231, 232, 252, 253, 254, 255, 256, 257, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 281, 286, 290, 291, 292, 294, 295, 296, 297, 298, 300, 301, 302, 312, 313, 315, 316, 317, 318, 319, 320, 322, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343,$

344, 345, 346, 347, 348, 349, 350, 351, 353, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 372, 376, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 393, 394, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 424, 427, 428, 429, 430, 431, 434, 435, 436, 437, 440, 442, 444, 447, 448, 454, 455, 456, 457, 460, 461, 462, 463, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 484, 485, 491, 492, 493, 494, 495, 496, 497, 498, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 520, 522, 523, 526, 528, 529, 530, 531, 532, 533, 534, 536, 539, 540, 541, 545, 546, 547, 549, 550, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 576, 577, 578, 581, 582, 583, 584, 586, 588, 589, 590, 592, 593, 597, 598, 599, 600, 601, 602, 604, 605, 606, 607, 608, 609, 611, 614, 616, 617, 618, 619, 620, 621, 622, 623, 626, 627, 630, 631, 633, 634, 635, 636, 637, 640, 641, 643, 644, 645, 646, 647, 648, 649, 651, 652, 654, 655, 656, 657, 658, 659, 660, 661, 662, 665, 666, 669, 670, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 693, 694, 695, 696, 697, 698, 699, 701, 702, 703, 704, 705, 706, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 719, 720, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 742, 745, 746, 747, 748, 752, 753, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 778, 779, 780, 781, 782, 783, 784, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 806, 812, 813, 816, 817, 819, 826, 827, 828, 831, 834, 836, 837, 838, 839, 840, 841, 842, 843, 844, 846, 847, 848, 852, 853, 854, 857, 858, 859, 861, 862, 863, 864, 865, 866, 869, 870, 871, 872, 873, 874, 876, 877, 878, 879, 881, 883, 884, 885, 887, 888.

11 and 12 crossing knots with tunnel number 3:

12aX with $X = 554, 750$.

12nX with $X = 553, 554, 555, 556, 642$.

11 and 12 crossing knots with tunnel number $\in \{1, 2\}$:

11nX with $X = 45, 65, 66, 86, 92, 94, 95, 98, 99, 112, 113, 115, 117, 118, 119, 121, 123, 125, 127, 136, 142, 144, 146, 149, 150, 152, 153, 155, 156, 158, 161, 163, 168, 169, 170, 171, 173, 178, 179, 180, 181, 182$.

12nX with $X = 16, 52, 53, 88, 156, 157, 158, 204, 207, 211, 228, 230, 245, 246, 247, 258, 260, 279, 280, 282, 283, 284, 285, 287, 299, 311, 314, 321, 328, 329, 330, 354, 373, 375, 377, 383, 391, 392, 395, 411, 416, 417, 425, 426, 432, 438, 439, 441, 443, 445, 449, 450, 451, 452, 453, 458, 459, 481, 482, 486, 489, 490, 499, 511, 519, 521, 524, 525, 527, 535, 537, 538, 542, 543, 544, 548, 551, 580, 585, 587, 595, 596, 603, 610, 612, 613, 615, 625, 628, 629, 632, 638, 639, 653, 663, 664, 667, 668, 671, 691, 692, 700, 707, 718, 739, 740, 741, 743, 744, 750, 751, 754, 759, 777, 785, 801, 803, 804, 805, 807, 808, 809, 810, 811, 814, 815, 818, 820, 821, 822, 823, 824, 825, 829, 830, 832, 833, 835, 845, 849, 850, 855, 856, 860, 867, 868, 875, 880, 882, 886$.