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DYNAMIC RESPONSE OF MULTILAYER CYLINDERS: THREE-DIMENSIONAL ELASTICITY THEORY

ALEXANDER SHUPIKOV AND NATALIYA DOLGOPOLOVA

We suggest an analytic-numerical approach to solving the problem of vibrations of multilayer cylinders under impulse loading. The behavior of the cylinder is described by dynamic equations of three-dimensional elasticity theory. The number of layers in the pack and the thickness and mechanical characteristics of each layer are selected arbitrarily. The possibilities of the approach are proposed, and the validity of results obtained is demonstrated by numerical examples.

1. Introduction

Structural elements in the form of multilayer plates and shells are used extensively in different branches of machine building and civil engineering. The stressed-strained state (SSS) of real objects can be described most simply using the finite-element method, but the development of analytic and hybrid calculation methods is the focus of much current work.

In works where analytic and hybrid methods are used, the SSS is investigated most often by using different 2-dimensional discrete, continuous and discrete-continuous theories [Grigoliuk and Kogan 1972; Grigoliuk and Kulikov 1988; Reddy 1989; 1993; Noor and Rarig 1974; Noor and Burton 1989; Noor and Burton 1990a; 1996; Smetankina et al. 1995; Shupikov and Ugrimov 1997; Shupikov et al. 2004]. Using these theories, obtaining numerical results is relatively straightforward. In the process, one is faced with the complex problem of determining the limits within which these theories adequately describe the behavior of the object being investigated with a prescribed accuracy. At impulse and other nonstationary short-time actions, the solution of this problem is yet more challenging.

The behavior of a multilayer structure can be investigated most effectively in terms of three-dimensional elasticity theory. Pagano [1969] was one of the first to investigate this problem. He studied the cylindrical bending of a simply supported orthotropic infinite laminated strip under static loading. Later the exact solution of the bending problem for a finite-dimension plate under static loading was obtained [Pagano 1970a; 1970b; Little 1973; Noor and Burton 1990b].

Keywords: three-dimensional theory, elasticity theory, cylindrical shell, multilayer shells, dynamics, equations of motion.

Several other works concerned with the behavior of cylindrical shells under static loading can be mentioned. Thus, Ren [1987] studied the plane strain deformation of an infinitely long cylindrical shell subjected to a radial load changing harmonically along the circumference. In terms of three-dimensional elasticity theory, solutions were obtained for finite-length, cross-ply cylindrical shells, simply supported at both ends and subjected to transverse sinusoidal loading [Varadan and Bhaskar 1991], and for shell panels [Ren 1989] subjected to a transverse load that changes both in the axial and circumferential directions. Subsequent works [Bhaskar and Varadan 1993; 1994; Bhaskar and Ganapathysaran 2002; 2003] dealt with the three-dimensional analysis of cylindrical shells subjected to different kinds of loads acting both in the axial and circumferential directions. Liu [2000] presented static analyses of thick rectangular plane-view laminated plates, carried out in terms of the three-dimensional theory of elasticity using the differential quadrature element method.

Attention has also been given to the study of vibrations of multilayer cylindrical shells in terms of three-dimensional elasticity theory.

Noor and Rarig [1974] obtained equations of free vibrations of a simply supported laminated orthotropic circular cylinder based on linear three-dimensional elasticity theory. Kang and Leissa [2000] suggested a three-dimensional analysis method for determining the free vibrations and the form of the segment of a variable-thickness spherical shell. The displacement components in the meridional, normal, and circumferential directions were taken to be sinusoidal with respect to time, periodic in the circumferential direction, and were expanded into algebraic polynomials in the meridional and normal directions.

Weingarten and Reismann [1974] gave solutions for nonaxisymmetrical nonstationary vibrations of a uniform finite-length cylinder. They considered vibrations of uniform cylindrical shells within the framework of the three-dimensional theory of elasticity, and compared the results obtained with those given by other theories of shells. They showed that none of the two-dimensional theories could describe satisfactorily the wave-like character of the initial strain phase. Philippov et al. [1978] solved a similar problem, providing an analytic solution to the problem of axisymmetrical vibrations of a uniform infinite-length cylinder subjected to an impulse load.

Shupikov and Ugrimov [1999] have suggested an analytic-numerical method for solving the three-dimensional problem in the elasticity theory of nonstationary vibrations of multilayer plates subjected to impulse loads. The displacements in the tangential direction are expanded into a double Fourier series, and the partial derivatives in the transverse coordinate are replaced by their finite-difference version. As a result of these transformations, the problem of nonstationary vibration of a multilayer plate under the action of an impulse load is reduced to integrating a system of ordinary differential equations with constant coefficients.

The objective of this work is to develop and generalize further the approach suggested by [Shupikov and Ugrimov 1999], and to investigate the vibrations of a thick multilayer closed cylindrical shell subjected to an impulse load.

2. Problem formulation

We consider a multilayer cylindrical shell of finite length A and radius R_0, which is composed of I uniform constant-thickness isotropic layers. The shell is referenced to the right-hand system of orthogonal curvilinear coordinates z, θ, r.

The coordinate surface is linked to the outer surface of the first layer, R_0 is the radius of inner surface of the shell (Figure 1). Contact between layers prevents their delamination and mutual slipping.

The behavior of each layer is described by Lamé's equations [Novatsky 1975]:

$$\mu^i\left(\nabla^2 u_r^i - \frac{u_r^i}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right) + (\lambda^i + \mu^i)\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r u_r^i) + \frac{1}{r}\frac{\partial u_\theta^i}{\partial \theta} + \frac{\partial u_z^i}{\partial z}\right) - \rho^i \frac{\partial^2 u_r^i}{\partial t^2} = 0,$$

$$\mu^i\left(\nabla^2 u_\theta^i - \frac{u_\theta^i}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right) + (\lambda^i + \mu^i)\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{r}\frac{\partial}{\partial r}(r u_r^i) + \frac{1}{r}\frac{\partial u_\theta^i}{\partial \theta} + \frac{\partial u_z^i}{\partial z}\right) - \rho^i \frac{\partial^2 u_\theta^i}{\partial t^2} = 0,$$

$$\mu^i \nabla^2 u_z^i + (\lambda^i + \mu^i)\frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial}{\partial r}(r u_r^i) + \frac{1}{r}\frac{\partial u_\theta^i}{\partial \theta} + \frac{\partial u_z^i}{\partial z}\right) - \rho^i \frac{\partial^2 u_z^i}{\partial t^2} = 0, \tag{1}$$

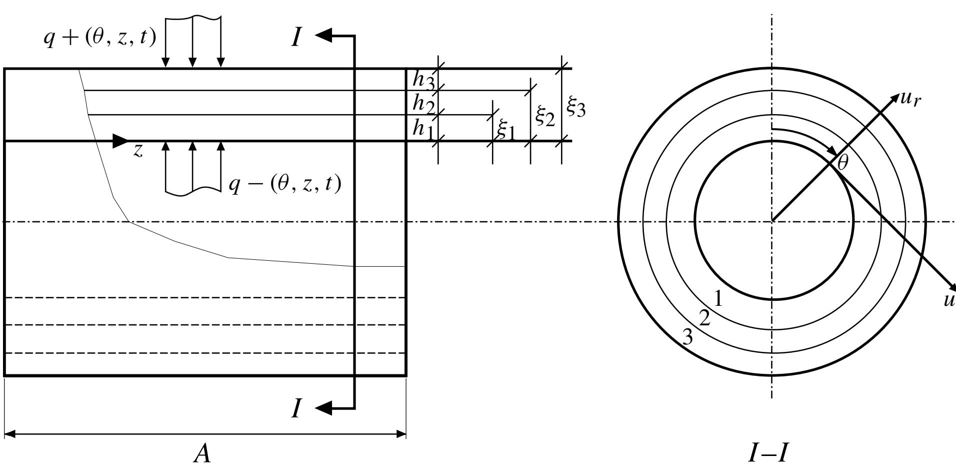


Figure 1. Multilayer cylindrical shell.

where $\nabla^2 = \partial^2/\partial r^2 + (1/r)\partial/\partial r + (1/r^2)\partial^2/\partial\theta^2 + \partial^2/\partial z^2$.

This system of equations is solved with the boundary conditions on the external surfaces of the first and I-th layers, namely

$$\begin{aligned} &\sigma_{zr}^1 = \sigma_{r\theta}^1 = 0, \quad \sigma_{rr}^1 = -q^- \quad \text{for } r = R_0, \\ &\sigma_{zr}^I = \sigma_{r\theta}^I = 0, \quad \sigma_{rr}^I = -q^+ \quad \text{for } r = R_0 + \xi^I,\ \xi^i = \sum_{j=1}^{i} h^j; \end{aligned} \tag{2}$$

the boundary conditions at the ends,

$$\sigma_{zz}^i = u_r^i = u_\theta^i = 0 \quad \text{for } z = 0, L,\ i = \overline{1, I}; \tag{3}$$

the contact conditions at adjacent layers,

$$\begin{aligned} &u_r^i = u_r^{i+1}, \quad u_\theta^i = u_\theta^{i+1}, \quad u_z^i = u_z^{i+1}, \\ &\sigma_{rr}^i = \sigma_{rr}^{i+1}, \quad \sigma_{r\theta}^i = \sigma_{r\theta}^{i+1}, \quad \sigma_{rz}^i = \sigma_{rz}^{i+1} \qquad \text{for } r = R_0 + \xi^i,\ i = \overline{1, I-1}; \end{aligned} \tag{4}$$

and the initial conditions

$$\mathbf{u}^i(r, \theta, z, 0) = \frac{\partial \mathbf{u}^i(r, \theta, z, 0)}{\partial t} = 0 \quad \text{for } i = \overline{1, I}. \tag{5}$$

Here i is the layer number, λ^i, μ^i are Lamé's coefficients, ρ^i is the specific density, and $\mathbf{u}^i = \{u_r^i,\ u_\theta^i,\ u_z^i\}$ is the displacement vector of a point in the i-th layer.

The stress tensor components are calculated from

$$\sigma_{jk}^i = 2\mu^i \varepsilon_{jk}^i + \lambda^i \delta_{jk} \Delta^i, \tag{6}$$

where δ is Kronecker's delta,

$$\Delta^i = \varepsilon_{rr}^i + \varepsilon_{\theta\theta}^i + \varepsilon_{zz}^i \quad \text{for } i = \overline{1, I},$$

and the components of the strain tensor have the form

$$\begin{aligned} &\varepsilon_{rr}^i = \frac{\partial u_r^i}{\partial r}, \quad \varepsilon_{\theta\theta}^i = \frac{1}{r_i}\left(\frac{\partial u_i^\theta}{\partial \theta} + u_r^i\right), \quad \varepsilon_{zz}^i = \frac{\partial u_z^i}{\partial z}, \quad \varepsilon_{r\theta}^i = \frac{1}{2}\left(\frac{1}{r_i}\frac{\partial u_r^i}{\partial \theta} + \frac{\partial u_\theta^i}{\partial r} - \frac{u_\theta^i}{r_i}\right), \\ &\varepsilon_{\theta z}^i = \frac{1}{2}\left(\frac{\partial u_\theta^i}{\partial z} + \frac{1}{r_i}\frac{\partial u_z^i}{\partial \theta}\right), \quad \varepsilon_{rz}^i = \frac{1}{2}\left(\frac{\partial u_z^i}{\partial r} + \frac{\partial u_r^i}{\partial z}\right), \quad \text{for } i = \overline{1, I}. \end{aligned} \tag{7}$$

Lamé's coefficients are linked to Young's modulus E^i and Poisson's coefficient ν^i by the relations

$$\lambda^i = \frac{\nu^i E^i}{(1+\nu^i)(1-2\nu^i)}, \quad \mu^i = \frac{E^i}{2(1+\nu^i)}.$$

3. Solution method

The displacements and the external load are expanded into double Fourier series with respect to the complete scheme of orthogonal functions satisfying the boundary conditions (3):

$$
\begin{aligned}
u_r^i &= \sum_{m=1}^{M}\sum_{n=0}^{N} \Phi_{1mn}^i(r,t) B_{1mn}(z,\theta), & u_\theta^i &= \sum_{m=1}^{M}\sum_{n=1}^{N} \Phi_{2mn}^i(r,t) B_{2mn}(z,\theta),\\
u_z^i &= \sum_{m=1}^{M}\sum_{n=0}^{N} \Phi_{3mn}^i(r,t) B_{3mn}(z,\theta), & q^{(\pm)} &= \sum_{m=1}^{M}\sum_{n=0}^{N} q_{mn}^{(\pm)}(t) B_{3mn}(z,\theta),
\end{aligned} \tag{8}
$$

for $i = \overline{1, I}$, where $B_{1mn}(z,\theta) = \sin \dfrac{m\pi z}{A} \cos n\theta$, $B_{2mn}(z,\theta) = \sin \dfrac{m\pi z}{A} \sin n\theta$, and

$$B_{3mn}(z,\theta) = \cos \frac{m\pi z}{A} \cos n\theta.$$

The partial derivatives of the functions $\Phi_{kmn}^i(r,t)$, $k = 1, 2, 3$, with respect to the coordinate r are replaced with their finite-difference presentations. For this we build a regular grid in each layer:

$$r^{i(l)} = R_0 + \xi^{i-1} + l\Delta^i, \qquad l = \overline{0, L^i},\ \Delta^i = \frac{h^i}{L^i},\ i = \overline{1, I}.$$

Here L^i is the number of nodes in the finite-difference grid in the i-th layer, $i = \overline{1, I}$.

The number of series terms M, N retained in expansion (8), and the number of nodes in the finite-difference grid in each of the layers L^i, is selected so as to ensure convergence of numeric results.

We set

$$\Phi_{kmn}^{i(l)} = \Phi_{kmn}^i(r^{i(l)}, t).$$

For approximation of partial derivatives, a three-point template is used [Forsythe and Wasov 1960]

$$\frac{\partial \Phi_{kmn}^{i(l)}}{\partial r} = \frac{\Phi_{kmn}^{i(l+1)} - \Phi_{kmn}^{i(l-1)}}{2\Delta^i}, \qquad \frac{\partial^2 \Phi_{kmn}^{i(l)}}{\partial r^2} = \frac{\Phi_{kmn}^{i(l+1)} - 2\Phi_{kmn}^{i(l)} + \Phi_{kmn}^{i(l-1)}}{(\Delta^i)^2}. \tag{9}$$

As a result of these transforms, for $n = 0$ and $m = \overline{1, M}$ system (1) takes the form

$$
\begin{aligned}
&(2\mu^i + \lambda^i)\left(\frac{1}{(\Delta^i)^2} - \frac{1}{r^{i(l)}}\frac{1}{2\Delta^i}\right)\Phi_{1m0}^{i(l-1)} + \frac{m\pi}{A}\frac{\lambda^i + \mu^i}{2\Delta^i}\Phi_{3m0}^{i(l-1)}\\
&\quad + \left(-(2\mu^i + \lambda^i)\left(\frac{2}{(\Delta^i)^2} + \frac{1}{(r^{i(l)})^2}\right) - \mu^i\frac{m^2\pi^2}{A^2}\right)\Phi_{1m0}^{i(l)}\\
&\quad + (2\mu^i + \lambda^i)\left(\frac{1}{(\Delta^i)^2} + \frac{1}{r^{i(l)}}\frac{1}{2\Delta^i}\right)\Phi_{1m0}^{i(l+1)} - \frac{m\pi}{A}\frac{\lambda^i + \mu^i}{2\Delta^i}\Phi_{3m0}^{i(l+1)} = \rho^i \frac{d^2\Phi_{1m0}^{i(l)}}{dt^2},
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
-\frac{m\pi}{A}\frac{\lambda^i+\mu^i}{2\Delta^i}\Phi_{1m0}^{i(l-1)}+\mu^i\left(\frac{1}{(\Delta^i)^2}-\frac{1}{r^{i(l)}}\frac{1}{2\Delta^i}\right)\Phi_{3m0}^{i(l-1)}& \\
+\frac{m\pi}{A}\frac{\lambda^i+\mu^i}{r^{i(l)}}\Phi_{1m0}^{i(l)}-\left(\mu^i\frac{2}{(\Delta^i)^2}+\frac{m^2\pi^2}{A^2}(2\mu^i+\lambda^i)\right)\Phi_{3m0}^{i(l)}& \qquad (11)\\
+\frac{m\pi}{A}\frac{\lambda^i+\mu^i}{2\Delta^i}\Phi_{1m0}^{i(l+1)}+\mu^i\left(\frac{1}{(\Delta^i)^2}+\frac{1}{r^{i(l)}}\frac{1}{2\Delta^i}\right)\Phi_{3m0}^{i(l+1)}&=\rho^i\frac{d^2\Phi_{3m0}^{i(l)}}{dt^2}.
\end{aligned}
$$

Conditions (2), (4), and (5) become

$$
(2\mu^1+\lambda^1)\frac{\Phi_{1m0}^{1(1)}-\Phi_{1m0}^{1(-1)}}{2\Delta^1}+\lambda^1\left(-\frac{m\pi}{A}\Phi_{3m0}^{1(0)}+\frac{1}{r^{1(0)}}\Phi_{1m0}^{1(0)}\right)=-q_{m0}^-, \qquad (12)
$$

$$
(2\mu^I+\lambda^I)\frac{\Phi_{1m0}^{I(L^I+1)}-\Phi_{1m0}^{I(L^I-1)}}{2\Delta^I}+\lambda^I\left(-\frac{m\pi}{A}\Phi_{3m0}^{I(L^I)}+\frac{1}{r^{I(L^I)}}\Phi_{1m0}^{I(L^I)}\right)=-q_{m0}^+,
$$

$$
\frac{\Phi_{3m0}^{1(1)}-\Phi_{3m0}^{1(-1)}}{2\Delta^1}+\frac{m\pi}{A}\Phi_{1m0}^{1(0)}=0, \qquad \frac{\Phi_{3m0}^{I(L^I+1)}-\Phi_{3m0}^{I(L^I-1)}}{2\Delta^I}+\frac{m\pi}{A}\Phi_{1m0}^{I(L^I)}=0,
$$

$$
\Phi_{1m0}^{i(L^i)}=\Phi_{1m0}^{i+1(0)}, \qquad \Phi_{3m0}^{i(L^i)}=\Phi_{3m0}^{i+1(0)},
$$

$$
\begin{aligned}
&\frac{2\mu^i+\lambda^i}{2\Delta^i}\left(\Phi_{1m0}^{i(L^i+1)}-\Phi_{1m0}^{i(L^i-1)}\right)+\lambda^i\left(-\frac{m\pi}{A}\Phi_{3m0}^{i(L^i)}+\frac{1}{r^{i(L^i)}}\Phi_{1m0}^{i(L^i)}\right)\\
&\quad=\frac{2\mu^{i+1}+\lambda^{i+1}}{2\Delta^{i+1}}\left(\Phi_{1m0}^{i+1(1)}-\Phi_{1m0}^{i+1(-1)}\right)+\lambda^{i+1}\left(-\frac{m\pi}{A}\Phi_{3m0}^{i+1(0)}+\frac{1}{r^{i+1(0)}}\Phi_{1m0}^{i+1(0)}\right),
\end{aligned}
$$

$$
\mu^i\left(\frac{\Phi_{3m0}^{i(L^i+1)}-\Phi_{3m0}^{i(L^i-1)}}{2\Delta^i}+\frac{m\pi}{A}\Phi_{1m0}^{i(L^i)}\right)=\mu^{i+1}\left(\frac{\Phi_{3m0}^{i+1(1)}-\Phi_{3m0}^{i+1(-1)}}{2\Delta^{i+1}}+\frac{m\pi}{A}\Phi_{1m0}^{i+1(0)}\right),
$$

$$
\Phi_{1m0}^{i(s)}(0)=\frac{d\Phi_{1m0}^{i(s)}(0)}{dt}=\Phi_{2m0}^{i(s)}(0)=\frac{d\Phi_{2m0}^{i(s)}(0)}{dt}=\Phi_{3m0}^{i(s)}(0)=\frac{d\Phi_{3m0}^{i(s)}(0)}{dt}=0,
$$

$$
\text{all for } i=\overline{1,\,I}.
$$

For pairs (m,n) with $n=\overline{1,\,N}$ and $m=\overline{1,\,M}$, we have instead the following form for system (1):

$$
\begin{aligned}
&(2\mu^i+\lambda^i)\left(\frac{1}{(\Delta^i)^2}-\frac{1}{r^{i(l)}}\frac{1}{2\Delta^i}\right)\Phi_{1mn}^{i(l-1)}-\frac{n}{r^{i(l)}}\frac{\lambda^i+\mu^i}{2\Delta^i}\Phi_{2mn}^{i(l-1)}+\frac{m\pi}{A}\frac{\lambda^i+\mu^i}{2\Delta^i}\Phi_{3mn}^{i(l-1)}\\
&\quad-\left((2\mu^i+\lambda^i)\left(\frac{2}{(\Delta^i)^2}+\frac{1}{(r^{i(l)})^2}\right)+\mu^i\left(\frac{n^2}{(r^{i(l)})^2}+\frac{m^2\pi^2}{A^2}\right)\right)\Phi_{1mn}^{i(l)}\\
&\quad+\left(-\frac{n}{r^{i(l)}}(3\mu^i+\lambda^i)\right)\Phi_{2mn}^{i(l)}+(2\mu^i+\lambda^i)\left(\frac{1}{(\Delta^i)^2}+\frac{1}{r^{i(l)}}\frac{1}{2\Delta^i}\right)\Phi_{1mn}^{i(l+1)}\\
&\quad+\left(\frac{n}{r^{i(l)}}\frac{\lambda^i+\mu^i}{2\Delta^i}\right)\Phi_{2mn}^{i(l+1)}-\frac{m\pi}{A}\frac{\lambda^i+\mu^i}{2\Delta^i}\Phi_{3mn}^{i(l+1)}=\rho^i\frac{d^2\Phi_{1mn}^{i(l)}}{dt^2},\\
&\frac{n}{r^{i(l)}}\frac{\lambda^i+\mu^i}{2\Delta^i}\Phi_{1mn}^{i(l-1)}+\mu^i\left(\frac{1}{(\Delta^i)^2}-\frac{1}{r^{i(l)}}\frac{1}{2\Delta^i}\right)\Phi_{2mn}^{i(l-1)}-\frac{n}{(r^{i(l)})^2}(3\mu^i+\lambda^i)\Phi_{1mn}^{i(l)}\\
&\quad-\left(\mu^i\left(\frac{2}{(\Delta^i)^2}+\frac{m^2\pi^2}{A^2}+\frac{1}{(r^{i(l)})^2}\right)+(2\mu^i+\lambda^i)\frac{n^2}{(r^{i(l)})^2}\right)\Phi_{2mn}^{i(l)}\\
&\quad+\frac{m\pi}{A}\frac{n}{r^{i(l)}}(\lambda^i+\mu^i)\Phi_{3mn}^{i(l)}-\frac{n}{r^{i(l)}}\frac{\lambda^i+\mu^i}{2\Delta^i}\Phi_{1mn}^{i(l+1)}+\mu^i\left(\frac{1}{(\Delta^i)^2}+\frac{1}{r^{i(l)}}\frac{1}{2\Delta^i}\right)\Phi_{2mn}^{i(l+1)}\\
&\quad=\rho^i\frac{d^2\Phi_{2mn}^{i(l)}}{dt^2},\\
&-\frac{m\pi}{A}\frac{\lambda^i+\mu^i}{2\Delta^i}\Phi_{1mn}^{i(l-1)}+\mu^i\left(\frac{1}{(\Delta^i)^2}-\frac{1}{r^{i(l)}}\frac{1}{2\Delta^i}\right)\Phi_{3mn}^{i(l-1)}+\frac{m\pi}{A}\frac{\lambda^i+\mu^i}{r^{i(l)}}\Phi_{1mn}^{i(l)}\\
&\quad+\frac{m\pi}{A}\frac{n}{r^{i(l)}}(\lambda^i+\mu^i)\Phi_{2mn}^{i(l)}-\left(\mu^i\left(\frac{2}{(\Delta^i)^2}+\frac{n^2}{(r^{i(l)})^2}\right)+\frac{m^2\pi^2}{A^2}(2\mu^i+\lambda^i)\right)\Phi_{3mn}^{i(l)}\\
&\quad+\frac{m\pi}{A}\frac{\lambda^i+\mu^i}{2\Delta^i}\Phi_{1mn}^{i(l+1)}+\mu^i\left(\frac{1}{(\Delta^i)^2}+\frac{1}{r^{i(l)}}\frac{1}{2\Delta^i}\right)\Phi_{3mn}^{i(l+1)}=\rho^i\frac{d^2\Phi_{3mn}^{i(l)}}{dt^2}.
\end{aligned}
\tag{13}
$$

For the same pairs (n, m), conditions (2), (4), and (5) become

$$
\begin{aligned}
&(2\mu^1+\lambda^1)\frac{\Phi_{1mn}^{1(1)}-\Phi_{1mn}^{1(-1)}}{2\Delta^1}+\lambda^1\left(\frac{1}{r^{1(0)}}\left(n\Phi_{2mn}^{1(0)}+\Phi_{1mn}^{1(0)}\right)-\frac{m\pi}{A}\Phi_{3mn}^{1(0)}\right)=-q_{mn}^-,\\
&\frac{\Phi_{2mn}^{1(1)}-\Phi_{2mn}^{1(-1)}}{2\Delta^1}+\frac{1}{r^{1(0)}}\left(-n\Phi_{1mn}^{1(0)}-\Phi_{2mn}^{1(0)}\right)=0,\qquad \frac{\Phi_{3mn}^{1(1)}-\Phi_{3mn}^{1(-1)}}{2\Delta^1}+\frac{m\pi}{A}\Phi_{1mn}^{1(0)}=0,\\
&(2\mu^I+\lambda^I)\frac{\Phi_{1mn}^{I(L^I+1)}-\Phi_{1mn}^{I(L^I-1)}}{2\Delta^I}+\lambda^I\left(\frac{1}{r^{I(L^I)}}\left(n\Phi_{2mn}^{I(L^I)}+\Phi_{1mn}^{I(L^I)}\right)-\frac{m\pi}{A}\Phi_{3mn}^{I(L^I)}\right)\\
&\quad=-q_{mn}^+,
\end{aligned}
\tag{14}
$$

$$\frac{\Phi_{2mn}^{I(L^I+1)}-\Phi_{2mn}^{I(L^I-1)}}{2\Delta^I}-\frac{1}{r^{I(L^I)}}\left(n\Phi_{1mn}^{I(L^I)}+\Phi_{2mn}^{I(L^I)}\right)=0, \tag{15}$$

$$\frac{\Phi_{3mn}^{I(L^I+1)}-\Phi_{3mn}^{I(L^I-1)}}{2\Delta^I}+\frac{m\pi}{A}\Phi_{1mn}^{I(L^I)}=0,$$

$$\Phi_{kmn}^{i(L^i)}=\Phi_{kmn}^{i+1(0)} \quad \text{for } k=1,2,3,$$

$$\begin{aligned}\frac{(2\mu^i+\lambda^i)}{2\Delta^i}\left(\Phi_{1mn}^{i(L^i+1)}-\Phi_{1mn}^{i(L^i-1)}\right)+\lambda^i\left(\frac{1}{r^{i(L^i)}}\left(n\Phi_{2mn}^{i(L^i)}+\Phi_{1mn}^{i(L^i)}\right)-\frac{m\pi}{A}\Phi_{3mn}^{i(L^i)}\right)\\=\frac{(2\mu^{i+1}+\lambda^{i+1})}{2\Delta^{i+1}}\left(\Phi_{1mn}^{i+1(1)}-\Phi_{1mn}^{i+1(-1)}\right)\\+\lambda^{i+1}\left(\frac{1}{r^{i+1(0)}}\left(n\Phi_{2mn}^{i+1(0)}+\Phi_{1mn}^{i+1(0)}\right)-\frac{m\pi}{A}\Phi_{3mn}^{i+1(0)}\right),\end{aligned}$$

$$\begin{aligned}\mu^i\left(\frac{\Phi_{2mn}^{i(L^i+1)}-\Phi_{2mn}^{i(L^i-1)}}{2\Delta^i}-\frac{1}{r^{i(L^i)}}\left(n\Phi_{1mn}^{i(L^i)}+\Phi_{2mn}^{i(L^i)}\right)\right)\\=\mu^{i+1}\left(\frac{\Phi_{2mn}^{i+1(1)}-\Phi_{2mn}^{i+1(-1)}}{2\Delta^{i+1}}-\frac{1}{r^{i+1(0)}}\left(n\Phi_{1mn}^{i+1(0)}+\Phi_{2mn}^{i+1(0)}\right)\right),\end{aligned}$$

$$\mu^i\left(\frac{\Phi_{3mn}^{i(L^i+1)}-\Phi_{3mn}^{i(L^i-1)}}{2\Delta^i}+\frac{m\pi}{A}\Phi_{1mn}^{i(L^i)}\right)=\mu^{i+1}\left(\frac{\Phi_{3mn}^{i+1(1)}-\Phi_{3mn}^{i+1(-1)}}{2\Delta^{i+1}}+\frac{m\pi}{A}\Phi_{1mn}^{i+1(0)}\right)$$

$$\text{for } i=\overline{1,I-1},$$

with

$$\Phi_{kmn}^{i(s)}(0)=\frac{d\Phi_{kmn}^{i(s)}(0)}{dt}=0 \quad \text{for } k=1,2,3,\ i=\overline{1,I}.$$

The boundary conditions at the ends of the cylinder (3) are satisfied exactly by selecting the coordinate functions B_{kmn} of (8), which correspond to simply supported conditions.

Conditions (12) allow us to exclude the values $\Phi_{km0}^{i(-1)}$ and $\Phi_{km0}^{i(L^i+1)}$ ($i=\overline{1,I}$, $k=1,3,\ n=0,\ m=\overline{1,M}$) of the sought-for functions in “extra-contour” points from system (10)–(11); and conditions (14)–(15) allow us to exclude the values $\Phi_{kmn}^{i(-1)}$ and $\Phi_{kmn}^{i(L^i+1)}$ ($i=\overline{1,I},\ k=1,2,3,\ n=\overline{1,N},\ m=\overline{1,M}$) of the sought-for functions in “extra-contour” points from system (13).

Hence, the solution of problems (1)–(5) on oscillations of a multilayer cylindrical shell subjected to an impulse load is reduced for each pair of values (m,n) to integrating a system of ordinary differential equations with constant coefficients. In this paper, the system obtained is integrated by Taylor expansion [Bakhvalov 1975; Shupikov et al. 2004] as described in the Appendix.

4. Numerical results

We illustrate with examples the method's feasibility and the validity of its results.

Consider an infinite uniform cylinder with $R_0 = 0.08\,\text{m}$, $h_1 = 0.04\,\text{m}$, $E_1 = 2.06 \cdot 10^8\,\text{kPa}$, $\rho_1 = 7.9 \cdot 10^3\,\text{kg/m}^3$, and $\nu_1 = 0.25$, subject to an impulse load applied to the inner surface, the load being a uniformly distributed pressure changing with respect to time according to the law

$$q^-(\theta, z, t) = q_0^- \exp(-t/\tau), \quad q^+(\theta, z, t) = 0,$$

with loading intensity $q_0^- = 1.49 \cdot 10^8$ Pa and load action time $\tau = 14.2 \cdot 10^{-6}$ s.

Figures 2 and 3 show data obtained using the exact solution from [Philippov

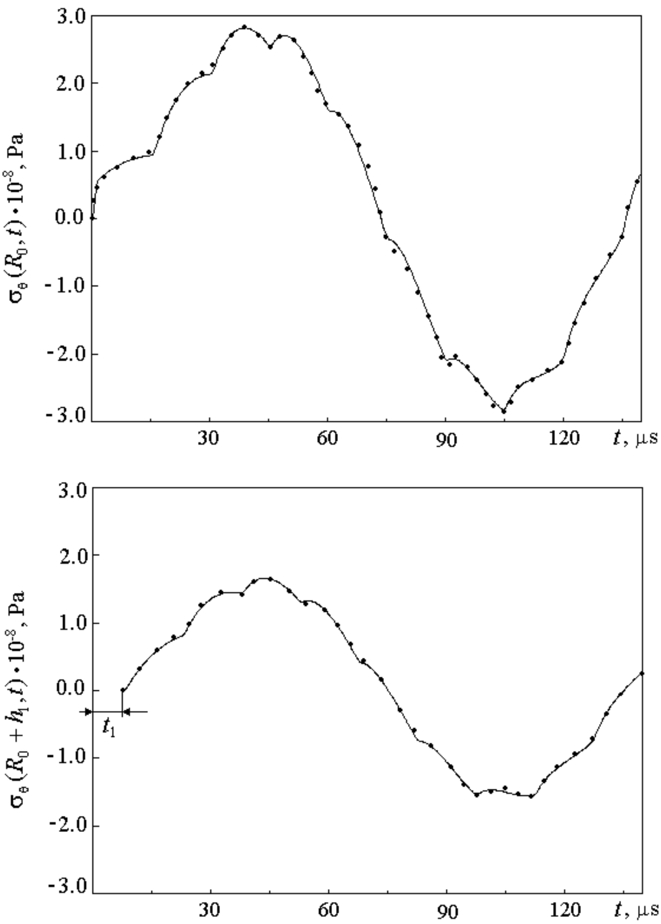


Figure 2. Circumferential stresses of infinite cylinder under impulse loading: dots, exact solution; solid line, present method.

et al. 1978], together with the results obtained with the method of this paper. For the calculations we took $L^1 = 160$. We observe surges at times of stress $\sigma_\theta^1(R^*, t)$ and $\sigma_r^1(R^*, t)$. For radial stresses $\sigma_r^1(R^*, t)$, such surges are more prominent than for circumferential ones $\sigma_\theta^1(R^*, t)$, and they have a dramatic impact on both the absolute values of the stresses $\sigma_r^1(R^*, t)$ and on their change in sign. Stress surges in time correspond to instances of arrival of waves reflected from the outer surface ($r = R_0 + h_1$) to the surface considered with the coordinate $r = R^* = 0.085\,\mathrm{m}$. The interval t_1 corresponds to the time required for the wave to travel the distance $s_1 = R^* - R_0 = 5\,\mathrm{mm}$, and it corresponds to the same value obtained from the

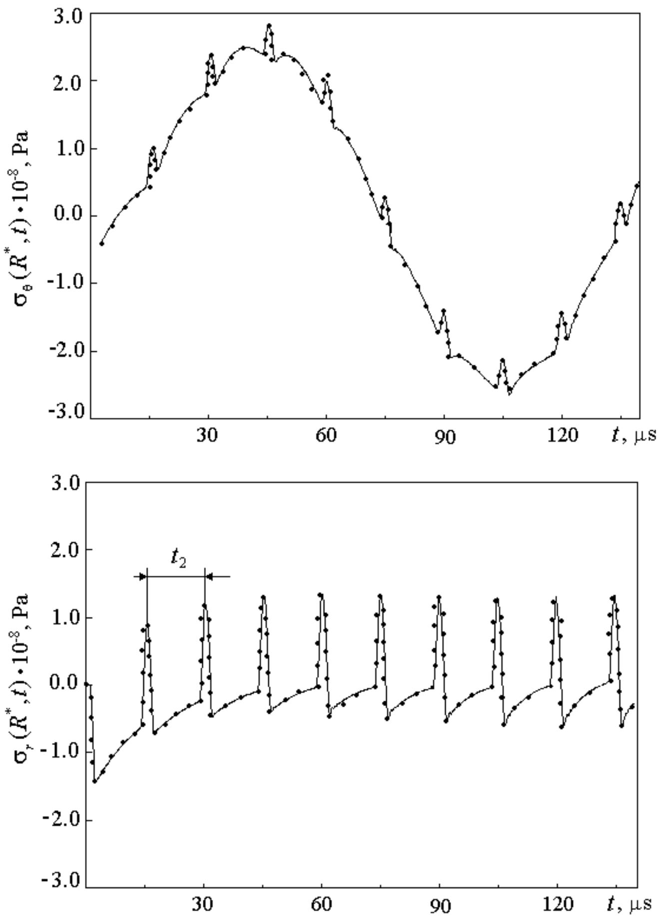


Figure 3. Radial stresses of infinite cylinder under impulse loading: dots, exact solution; solid line, present method.

exact formula

$$t_1 = \frac{s_1}{V} \approx 0.9\,\mu\text{s},$$

where V is the expansion wave [Novatsky 1975]

$$V = \sqrt{\frac{\lambda^1 + \mu^1}{\rho^1}} \approx 5.52 \cdot 10^3\ \text{m/s}.$$

Interval t_2 corresponds to the time required for the wave to travel the distance $s_2 = 2h_1 - (R^* - R_0) = 7.5\,\text{mm}$, and it corresponds to the same value obtained from the exact formula

$$t_2 = \frac{s_2}{V} \approx 1.36 \cdot 10^{-6}\ \text{s}.$$

In the problem considered, the uniform shell was presented in the form of a one-, two-, and three-ply shell. In all cases, we observed a satisfactory matching of results obtained with the help of the given technique and the analytic solution [Philippov et al. 1978].

For a uniform finite-length cylinder with parameters $A = 0.5\,\text{m}$, $R_0 = 0.095\,\text{m}$, $h^1 = 0.01\,\text{m}$, $E^1 = 6.67 \cdot 10^4\,\text{MPa}$, $\rho^1 = 2.5 \cdot 10^3\,\text{kg/m}^3$, and $\nu^1 = 0.3$, subjected to an external radially directed load applied instantaneously to the outer surface, we give a comparison of the results obtained by the analytic method in [Weingarten and Reismann 1974], and those obtained by implementing the given approach.

A load with intensity q_0^+ is applied instantaneously radially outside the cylinder, and distributed over a small area on the outer surface of the shell. The dimensions of the loading areas are ε rad in the direction of axis θ, and λ in the direction of axis z. The centre of the loading area has the coordinates $\theta = 0$, $z = L/2$, i.e., the load is distributed symmetrically with respect to the circumferential coordinate, and it has the following form:

$$q^+(\theta, z, t) = q_0^+ f(\theta) g(z) H(t), \qquad q^-(\theta, z, t) = 0,$$

$$f(\theta) = \begin{cases} 0, & |\theta| > \varepsilon/2 \\ 1, & |\theta| \le \varepsilon/2 \end{cases}, \qquad f(\theta + 2\pi) = f(\theta),$$

$$g(z) = \begin{cases} 0, & 0 \le z < (L - \lambda)/2 \\ 1, & (L - \lambda)/2 \le z \le (L + \lambda)/2, \\ 0, & (L + \lambda)/2 < z \le \lambda \end{cases}$$

$$\lambda = 0.5\,R, \qquad R = 0.1\,\text{m}, \qquad \varepsilon = 0.5\,\text{rad}.$$

Here q_0^+ is the loading intensity (0.1 MPa); $f(\theta)$ is the load distribution over coordinate θ; $g(z)$ is the load distribution over coordinate z, and $H(t)$ is Heaviside's function.

Figure 4 shows the cylinder's response to dynamic loading. The dots show the analytic solution, and the lines represent the solution obtained by using the given analytic-numerical method. For the calculations we took $L^1 = 150$. The top graph shows the change in radial displacement on the median surface as a function of time. The bottom graph shows the change in circumferential stresses on the outer surface of the shell as a function of time. The abscissa is dimensionless time, which is normed by the value of the time of travel of the shear wave over the shell radius, $\tau^* = tV/R_0$,

$$V = \sqrt{\frac{\mu^1}{\rho^1}} \approx 3.203 \cdot 10^3 \text{ m/s}.$$

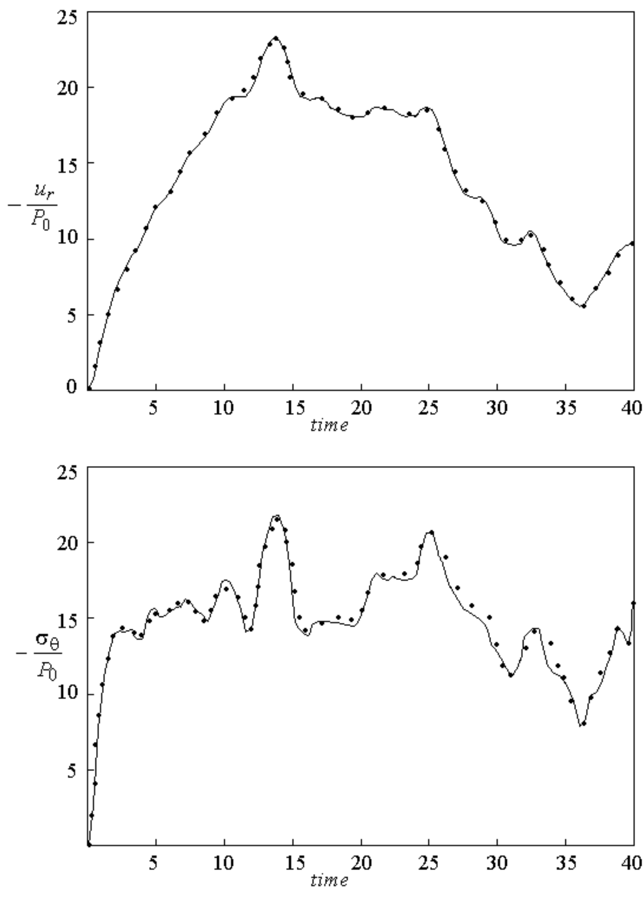


Figure 4. Response of a finite-length cylinder to dynamic loading: dots, analytic solution; solid line, present method.

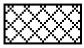
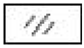
$R_0(m)$	i	Designation in Fig.	$h^i(m)$	E^i (MPa)	ν^i	ρ^i (kg/m^3)
0.08	1		0.01	$5.59 \cdot 10^3$	0.38	$1.2 \cdot 10^3$
	2		0.02	$6.67 \cdot 10^4$	0.22	$2.5 \cdot 10^3$

Table 1. Parameters of multilayer shell.

These examples show good agreement between the results obtained with the present method and analytic solutions obtained by other authors.

The process of propagation of the disturbance during impulse loading has been investigated. An infinite two-ply cylindrical shell, whose parameters are given in Table 1, is considered.

The load applied to the inner surface is a uniformly distributed pressure that changes with time according to the law

$$q^-(\theta, z, t) = q_0^- H(t), \quad q^+(\theta, z, t) = 0,$$

where $q_0^- = 1.49 \cdot 10^8$ Pa and $H(t)$ is Heaviside's function.

Figure 5 shows the distribution diagrams for stresses $\sigma_r^i(r, t)$ at different times. The symbols

 and 

show the direction of propagation of stress waves. For the calculations we took $L_1 = 70$ and $L_2 = 100$. The figures presented demonstrate the wavelike pattern of the process. Besides, one can see the effect of wave reflection from the boundary between the layers and external surfaces.

Hence, we have shown the possibility of investigating wave processes in thick uniform and multilayer cylindrical shells. Such an approach can be practical for evaluating the area of applicability of two-dimensional theories when it is necessary to investigate the process of propagation of elastic waves, and when the SSS of the object being investigated has an essentially three-dimensional character.

5. Conclusions

The present work suggests an analytic-numerical method of investigating vibrations in a multilayer closed cylindrical shell in terms of the three-dimensional theory of elasticity. The given method allows investigating the behavior of uniform and multilayer cylindrical shells subjected to impulse loading.

The examples given for different kinds of loading (Figures 2–4) show good agreement between the results obtained with the present method and analytic solutions obtained by other authors.

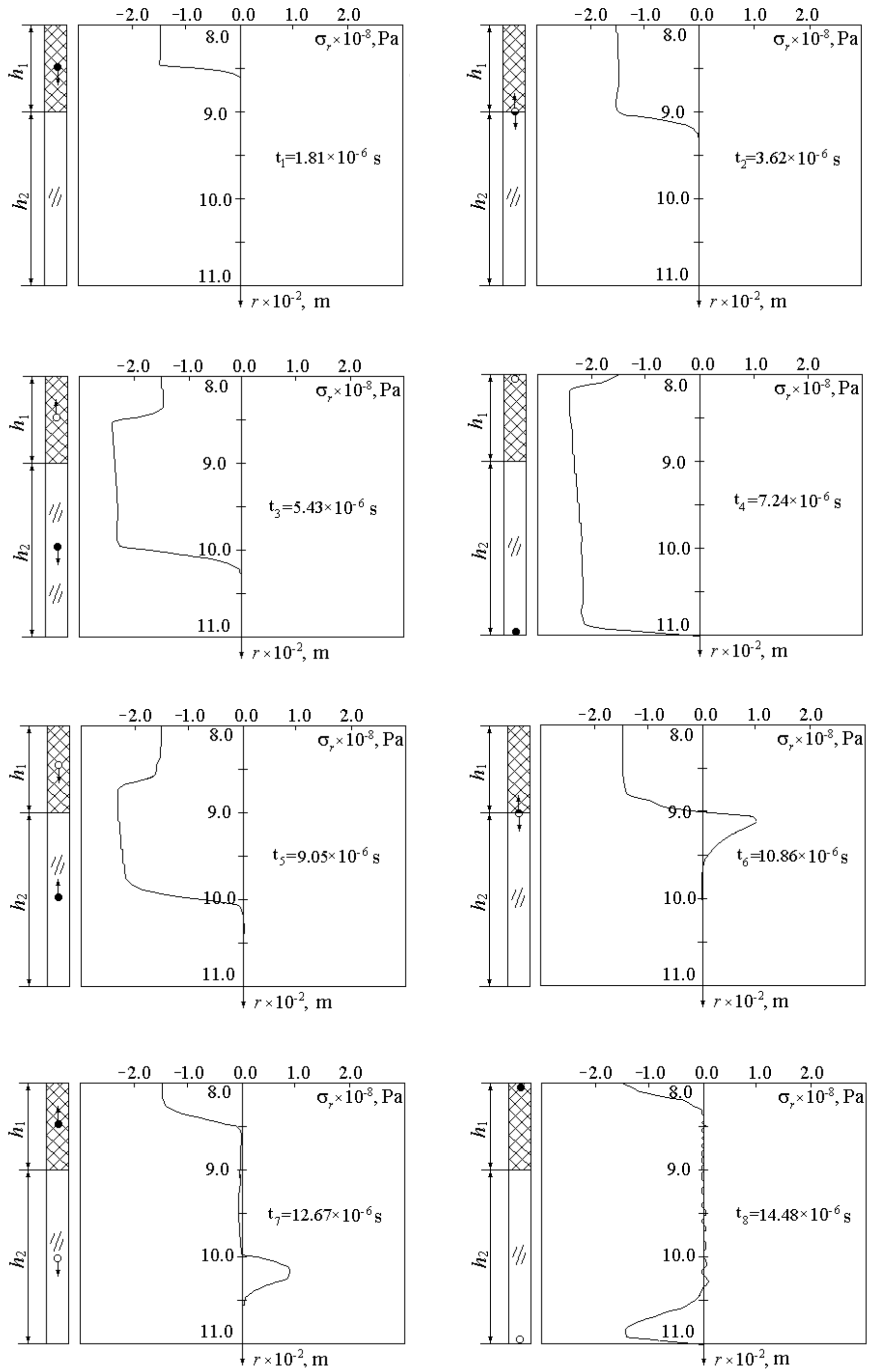


Figure 5. The propagation of stress waves of a two-layer cylindrical shell.

The plots in Figures 2, 3 and 5 demonstrate the possibility of investigating wave processes in thick uniform and multilayer cylindrical shells.

Hence, the given method can be used for verifying the validity of results based on different two-dimensional theories applied to analyzing vibrations of multilayer cylindrical shells, as well as for investigating wave processes in cylindrically shaped elastic bodies.

Appendix: A modified method of solution expansion into Taylor series

A modified method of solution expansion in Taylor series is applied to integrate a system of ordinary differential equations with constant coefficients.

The initial system of differential equations is written in the form

$$[\Omega^{mn}]\ddot{\overline{\Phi}}_{mn} + [\Lambda^{mn}]\overline{\Phi}_{mn} = \overline{Q}_{mn}.$$

By replacing variables and straightforward transforms, it is reduced to the form

$$\dot{\overline{G}}_{mn} = [R^{mn}]\dot{\overline{G}}_{mn} + \overline{H}_{mn}. \tag{A.1}$$

The integration interval $[0, t]$ is divided into s sections, each with a length of Δt so that $t = s\Delta t$. We denote $\overline{G}_{mn}(s\Delta t) = \overline{G}_{mns}$.

At each integration step Δt, the solution is represented as a Taylor series:

$$\overline{G}_{mns} = \overline{G}_{mns-1} + \frac{\dot{\overline{G}}_{mns-1}}{1!}\Delta t + \frac{\ddot{\overline{G}}_{mns-1}}{2!}\Delta t^2 + \cdots . \tag{A.2}$$

It is assumed that, within the integration step,

$$\overline{H}_{mn}(t) = \overline{H}_{mns}, \qquad (s-1)\Delta t \le t \le s\Delta t. \tag{A.3}$$

In this case, given (A.1) and (A.3), the derivatives in series (A.2) can be presented as

$$\overset{(k)}{\overline{G}}_{mn} = [R^{mn}]^k\overline{G}_{mn} + [R^{mn}]^{k-1}\overline{H}_{mn}. \tag{A.4}$$

An example of calculating the derivatives is given here:

$$\ddot{\overline{G}}_{mn} = \frac{d}{dt}\dot{\overline{G}}_{mn} = \frac{d}{dt}\big([R^{mn}]\overline{G}_{mn} + \overline{H}_{mn}\big) = [R^{mn}]\dot{\overline{G}}_{mn} = \\ = [R^{mn}]\big([R^{mn}]\overline{G}_{mn} + \overline{H}_{mn}\big) = [R^{mn}]^2\overline{G}_{mn} + [R^{mn}]\overline{H}_{mn}.$$

Substituting the expressions for derivatives (A.4) into (A.2), we obtain the following expressions for solving the system at the s-th step:

$$\overline{G}_{mns} = [K^{mn}]\overline{G}_{mns-1} + [T^{mn}]\overline{H}_{mns}. \tag{A.5}$$

Here

$$[K_{mn}] = [E] + \frac{[R^{mn}]}{1!}\Delta t + \frac{[R^{mn}]^2}{2!}\Delta t^2 + \cdots,$$
$$[T_{mn}] = \frac{[E]}{1!}\Delta t + \frac{[R^{mn}]}{2!}\Delta t^2 + \frac{[R^{mn}]^2}{3!}\Delta t^3 + \cdots,$$

where $[E]$ is the unit matrix.

To refine the solution, the interval $[t_{s-1}, t_s]$ is divided into r sections with the length of $\Delta\tau = \Delta t/r$. In each section, the function $\overline{G}_{mn}$ is calculated using (A.5):

$$\overline{G}_{mn}\Big(t_{s-1} + \frac{\Delta t}{r}\Big) = [\hat{K}^{mn}]\overline{G}_{mns-1} + [\hat{T}^{mn}]\overline{H}_{mns},$$
$$\vdots$$
$$\overline{G}_{mn}\Big(t_{s-1}+\frac{i\,\Delta t}{r}\Big) = [\hat{K}^{mn}]^i\,\overline{G}_{mns-1} + \big([\hat{K}^{mn}]^{i-1} + [\hat{K}^{mn}]^{i-2} + \cdots + [E]\big)[\hat{T}^{mn}]\overline{H}_{mns}$$
$$\text{for } i = \overline{1, r}.$$

The matrices $[\hat{K}^{mn}]$ and $[\hat{T}^{mn}]$ are derived from matrices $[K^{mn}]$ and $[T^{mn}]$ by replacing Δt with $\Delta\tau$.

At $i = r$, $\overline{G}_{mn}(t_{s-1} + \Delta t) = \overline{G}_{mns}$, the system solution takes its final form

$$\overline{G}_{mns} = [M^{mn}]\overline{G}_{mns-1} + [J^{mn}]\overline{H}_{mns},$$

where

$$[M^{mn}] = [\hat{K}^{mn}]^r;$$
$$[J^{mn}] = \big([\hat{K}^{mn}]^{r-1} + [\hat{K}^{mn}]^{r-2} + \cdots + [E]\big)[\hat{T}^{mn}].$$

Hence, integrating a system of ordinary differential equations is reduced to calculating the load vector $\overline{H}_{mns}$ and multiplying matrices by vectors. The matrices are calculated once for each pair of values m and n.

In this paper, to ensure stability of the process of numerically integrating a system of ordinary differential equations, the step of integration with respect to time Δt is taken to be equal to the time of strain wave travel between adjacent nodes of the finite-difference grid according to the Courant–Hilbert conditions.

References

[Bakhvalov 1975] N. S. Bakhvalov, *Numerical methods*, Nauka, Moskow, 1975.

[Bhaskar and Ganapathysaran 2002] K. Bhaskar and N. Ganapathysaran, "Three-dimensional analysis of a cross-ply cylindrical shell subjected to a localized circumferential shear force", *Int. J. Pressure Vessels Piping* **79**:7 (2002), 519–524.

[Bhaskar and Ganapathysaran 2003] K. Bhaskar and N. Ganapathysaran, "Elasticity solutions for laminated orthotropic cylindrical shells subjected to localized longitudinal and circumferential moments", *J. Pressure Vessel Technol.(Trans. ASME)* **125**:1 (2003), 26–35.

[Bhaskar and Varadan 1993] K. Bhaskar and T. K. Varadan, "Exact elasticity solution for laminated anisotropic cylindrical shells", *J. Appl. Mech. (Trans. ASME)* **60**:1 (1993), 41–47.

[Bhaskar and Varadan 1994] K. Bhaskar and T. K. Varadan, "Benchmark elasticity solution for locally loaded laminated orthotropic cylindrical shells", *AIAA J.* **32**:3 (1994), 627–632.

[Forsythe and Wasov 1960] G. E. Forsythe and W. R. Wasov, *Finite-difference methods for partial differential equations*, Wiley, New York, 1960.

[Grigoliuk and Kogan 1972] E. I. Grigoliuk and F. A. Kogan, "Modern state of the theory of multilayer shells", *Prikladnaya Mekhanika* **8**:3 (1972), 3–17. In Russian.

[Grigoliuk and Kulikov 1988] E. I. Grigoliuk and G. M. Kulikov, "General direction of development of the theory of multilayer shells", *Mekhanika Kompozitnykh Materialov* **24**:2 (1988), 287–298. In Russian; translation in *Mech. Compos. Materials* **24**:2 (1988), 231–241.

[Kang and Leissa 2000] J.-H. Kang and A. W. Leissa, "Three-dimensional vibrations of thick spherical shell segments with variable thickness", *Int. J. Solids Struct.* **37**:35 (2000), 4811–4823.

[Little 1973] R. W. Little, *Elasticity*, Prentice-Hall, Englewood Cliffs, N. J., 1973.

[Liu 2000] F.-L. Liu, "Static analysis of thick rectangular laminated plates: three-dimensional elasticity solutions via differential quadrature element method", *Int. J. Solids Struct.* **37**:51 (2000), 7671–7688.

[Noor and Burton 1989] A. K. Noor and W. S. Burton, "Assessment of shear deformation for multilayered composite plates", *Appl. Mech. Rev.* **42**:1 (1989), 1–12.

[Noor and Burton 1990a] A. K. Noor and W. S. Burton, "Assessment of computational models for multilayered anisotropic plates", *Compos. Struct.* **14**:3 (1990), 233–265.

[Noor and Burton 1990b] A. K. Noor and W. S. Burton, "Three-dimensional solutions for antisymmetrically laminated anisotropic plates", *J. Appl. Mech. (Trans. ASME)* **57** (1990), 182–188.

[Noor and Rarig 1974] A. K. Noor and P. L. Rarig, "Three-dimensional solutions of laminated cylinders", *Comput. Methods Appl. Mech. Eng.* **3**:3 (1974), 319–334.

[Noor et al. 1996] A. K. Noor, W. S. Burton, and C. W. Bert, "Computational models for sandwich panels and shells", *Appl. Mech. Rev.* **49**:3 (1996), 155–199.

[Novatsky 1975] V. Novatsky, *Theory of elasticity*, Mir, Moskow, 1975.

[Pagano 1969] N. J. Pagano, "Exact solutions for composite laminates in cylindrical bending", *J. Compos. Mater.* **3** (1969), 398–411.

[Pagano 1970a] N. J. Pagano, "Exact solutions for rectangular bidirectional composites and sandwich plates", *J. Compos. Mater.* **4** (1970), 20–34.

[Pagano 1970b] N. J. Pagano, "Influence of shear coupling in cylindrical bending of anisotropic laminates", *J. Compos. Mater.* **4** (1970), 330–343.

[Philippov et al. 1978] A. P. Philippov, S. S. Kohmaniuk, and Y. Yaniutin, *Strain in structural elements subjected to shock and impulse loads*, Naukova Dumka, Kyiv, 1978.

[Reddy 1989] J. N. Reddy, "On the generalization of displacement-based laminate theories", *Appl. Mech. Rev.* **42**:8 (1989), 213–222.

[Reddy 1993] J. N. Reddy, "An evaluation of equivalent-single-layer and layerwise theories of composite laminates", *Compos. Struct.* **25**:1–4 (1993), 21–35.

[Ren 1987] J. G. Ren, "Exact solutions for laminated cylindrical shells in cylindrical bending", *Compos. Sci. Technol.* **29**:3 (1987), 169–187.

[Ren 1989] J. G. Ren, "Analysis of simply-supported laminated circular cylindrical shell roofs", *Compos. Struct.* **11**:4 (1989), 277–292.

[Shupikov and Ugrimov 1997] A. N. Shupikov and S. V. Ugrimov, "High-order theory for multilayer plates", pp. 103–109 in *Current problems of strength and optimization of structures and machines*, Nauka i obrazovaniye, Dnepropetrovsk, 1997.

[Shupikov and Ugrimov 1999] A. N. Shupikov and S. V. Ugrimov, "Vibrations of multilayer plates under the effect of impulse loads, Three-dimensional theory", *Int. J. Solids Struct.* **36**:22 (1999), 3391–3402.

[Shupikov et al. 2004] A. N. Shupikov, N. N. Buzko, N. V. Smetankina, and S. V. Ugrimov, *Non-stationary vibrations of multilayer plates and shells and their optimization*, Publishing House KNEU, Kharkov, 2004.

[Smetankina et al. 1995] N. V. Smetankina, S. Y. Sotrikhin, and A. N. Shupikov, "Theoretical and experimental investigation of vibration of multilayer plates under the action of impulse and impact loads", *Int. J. Solids Struct.* **32**:8-9 (1995), 1247–1258.

[Varadan and Bhaskar 1991] T. K. Varadan and K. Bhaskar, "Bending of laminated orthotropic cylindrical shells: an elasticity approach", *Compos. Struct.* **17**:2 (1991), 141–156.

[Weingarten and Reismann 1974] L. I. Weingarten and H. Reismann, "Forced motion of cylindrical shells: a comparison of shell theory with elasticity theory", *Z. Angew. Math. Mech.* **54** (1974), 181–191.

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