Journal of

Mechanics of

Materials and Structures

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Volume 2, № 1

January 2007

mathematical sciences publishers

JOURNAL OF MECHANICS OF MATERIALS AND STRUCTURES
Vol. 2, No. 1, 2007

SHEAR WAVES AT A CORRUGATED INTERFACE BETWEEN TWO DISSIMILAR FIBER-REINFORCED ELASTIC HALF-SPACES

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A problem of reflection and transmission of shear waves (SH waves) at a corrugated interface between two distinct fiber-reinforced elastic half-spaces has been analyzed. Rayleigh's method of approximation is used to determine the reflection and transmission coefficients. We find that (i) these coefficients are functions of the angle of incidence and the elastic parameters of the media, (ii) the coefficients corresponding to irregularly reflected and transmitted waves are proportional to the amplitude of the corrugated interface, and (iii) reflection and transmission coefficients of the regularly reflected and transmitted waves are greater than those of irregularly reflected and transmitted waves. The energy ratios of reflected and transmitted waves are also presented. Numerical computations are performed and the results obtained are presented graphically. Some earlier results by other workers are recovered by our treatment.

1. Introduction

Problems of wave propagation in elastic media have applications in various fields, such as engineering, geophysics, and seismology. When elastic waves are transmitted through one medium to another medium of different characteristics, the phenomena of reflection and transmission take place. These phenomena depend not only upon the characteristics of the media but are also influenced by the shape of the interface between the two media. Thus, while investigating the problems of reflection and transmission of elastic waves from a corrugated interface, one must take into account the shape of the corrugated interface. Rayleigh [1893] was the first who attempted to solve a problem of wave scattering of sound waves and electromagnetic waves from a rough surface. He gave an approximate method of solving this problem for a sinusoidal surface with a small amplitude, restricting himself to the case of normal wave incidence. In his method, the amplitude and slope of the interface which is expressed in Fourier series are assumed to be very small. By using the boundary conditions of the problem, the unknown coefficients in the solutions are determined for any order of approximation. Rayleigh used this method, in his paper "On the dynamical theory of grating" [1907], and later, researchers in various fields applied his method to explain reflection and transmission phenomena of waves from irregular boundary surfaces. Using different techniques, many problems of reflection and refraction of elastic waves from irregular boundary surfaces have appeared in the open literature, such as [Abubakar 1962; Asano 1960; 1961; 1966; Dunkin and Eringen 1962; Tomar and Saini 1997; Okamoto and Takenaka 1999; Tomar et al. 2002; Gupta 1987; Kumar et al. 2003; Tomar and Kaur 2003; Kaur and Tomar 2004; Kaur et al. 2005].

Keywords: SH waves, fiber reinforcement, Rayleigh's method of approximation, apparent velocity, reflection coefficient, transmission coefficient.

The authors are thankful to Council of Scientific and Industrial Research, New Delhi, for providing financial assistance through Grant No. 25 (0134) /04 /EMR-II to complete this study.

Chattopadhyay and Choudhury [1990] studied propagation, reflection and transmission of magnetoelastic shear waves in a self-reinforced medium. Later, Chattopadhyay and Choudhury [1995] studied magnetoelastic shear waves in an infinite self-reinforced elastic plate. Sengupta and Nath [2001] investigated surface waves (Rayleigh, Love and Stoneley types) in anisotropic fiber-reinforced solid elastic media. Pradhan et al. [2003] studied the dispersion of Love waves in a self-reinforced layer over an elastic non-homogeneous half-space. Singh and Singh [2004] studied the propagation of plane waves in fiber-reinforced elastic media and showed that the phase velocities of quasi P and SV waves depended on the angle between direction of propagation and the direction of reinforcement. They also discussed the reflection of these elastic waves from the free surface of a fiber reinforced elastic half-space. In this paper, a problem of an SH wave striking obliquely at a corrugated interface between two dissimilar fiber-reinforced elastic half-spaces has been discussed. The amplitude and slope of the corrugated interface are assumed to be very small and Rayleigh's method of approximation has been used to explain the reflection and transmission coefficients for first and second order approximation of the corrugation. For a special type of interface, that is, where the corrugated interface is a simple harmonic interface given by $\zeta = d \cos npx$, we have obtained the formulae of reflection and transmission coefficients of regularly and irregularly reflected and transmitted waves in closed form for the first order approximation of the corrugation. Partitioning of energy due to reflected and refracted waves at the corrugated interface is also presented. Numerically, the effects of corrugation and frequency parameters on these coefficients are studied for a particular model and the results obtained are shown graphically. In the present work, if we neglect the reinforcement parameters, we reduce to the case of a problem in an isotropic medium. In this case, the problem of Asano [1960] can be recovered by setting $\alpha = \beta = 0$ and $\mu_L = \mu_T$. The expressions of energy ratios of regularly and irregularly reflected and transmitted waves are obtained and their variations are depicted graphically with respect to the angle of incidence.

2. Basic relations and equations

The constitutive relations for a fiber-reinforced linear elastic medium, as given in [Belfield et al. 1983], are

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha\big(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j\big) + 2(\mu_L - \mu_T)\big(a_i a_k e_{kj} + a_j a_k e_{ki}\big) + \beta a_k a_m e_{km} a_i a_j, \quad (i, j, k, m = 1, 2, 3), \tag{1}$$

where τ_{ij} is the stress tensor, e_{ij} is the strain tensor, μ_T and λ are elastic constants, α, β, and $(\mu_L - \mu_T)$ are fiber-reinforcement parameters having the dimensions of stress, and a_i are the components of a unit vector $\boldsymbol{a}$ that gives the direction of fiber-reinforcement. Spencer [1974] has shown that if the preferred direction of $\boldsymbol{a}$ is chosen along the x-axis then μ_T can be identified as the shear modulus in transverse shear across the preferred direction and μ_L as the shear modulus in longitudinal shear in the preferred direction. He also established some relations among the elastic constants and given their physical meaning. It can be seen that if $\alpha = \beta = 0$ and $\mu_L = \mu_T$, then (1) reduces to the generalized Hooke's law for an isotropic medium. The strain tensor e_{ij} in terms of displacement components u_i is given by

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{2}$$

The equations of motion in a fiber-reinforced medium without body forces are

$$\frac{\partial \tau_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (i, j = 1, 2, 3), \tag{3}$$

where ρ is the density of the medium.

Let the direction of reinforcement be along the x-axis: $\boldsymbol{a} = (1, 0, 0)$. For an SH wave propagating in the x_1x_2 plane and having displacement along x_3 axis, we have $\partial/\partial x_3 \equiv 0$. Using the notations $u_3 \equiv u$, $\partial/\partial x_1 \equiv \partial/\partial x$, and $\partial/\partial x_2 \equiv \partial/\partial y$, and substituting Equations (1) and (2) into (3), we obtain

$$\mu_L \frac{\partial^2 u}{\partial x^2} + \mu_T \frac{\partial^2 u}{\partial y^2} = \rho \frac{\partial^2 u}{\partial t^2}.$$

This is the equation of motion for SH wave propagation in a fiber-reinforced elastic medium.

3. Problem and boundary conditions

Let the x and z axes of a Cartesian coordinate system be on the horizontal plane and the y axis be pointing vertically downward. Let the equation of the corrugated interface separating the two different homogeneous fiber-reinforced elastic half-spaces, namely $L_1[-\infty < y \le \zeta(x)]$ and $L_2[\zeta(x) \le y < \infty]$, be given by $y = \zeta(x)$. The geometry of the problem is shown in Figure 1.

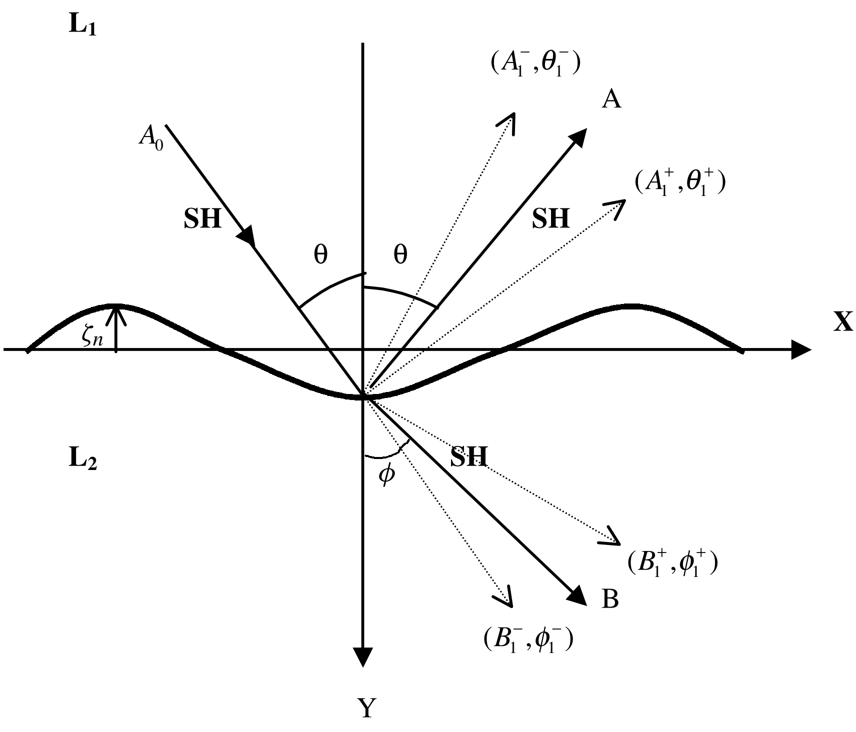

Figure 1. Geometry of the problem.

We shall take both the half-spaces as homogeneous such that the length scale of the reinforcement (that is, the cross section of the fiber, or the separation between fibers) is small compared to the wavelength. We denote the elastic parameters and density in the medium L_l $(l = 1, 2)$ by the quantities μ_{L_l}, μ_{T_l}, and ρ_l, respectively. Fourier series representation of the function $\zeta(x)$ is given by

$$\zeta(x) = \sum_{n=1}^{\infty} \left(\zeta_n e^{inpx} + \zeta_{-n} e^{-inpx}\right), \tag{4}$$

where $\zeta(x)$ is a periodic function of x and independent of y whose mean value is zero, ζ_n and ζ_{-n} are Fourier series coefficients, p is the wave number, n is the series expansion order and $i = \sqrt{-1}$. Introducing the constants d, c_n and s_n such that $\zeta_1 = \zeta_{-1} = \frac{1}{2}d$, $\zeta_n = \frac{1}{2}(c_n - is_n)$, $\zeta_{-n} = \frac{1}{2}(c_n + is_n)$, and $n = 2, 3, 4, \ldots$ into (4), we obtain

$$\zeta = d \cos px + \sum_{n=2}^{\infty} \left[c_n \cos npx + s_n \sin npx\right].$$

If the coefficients $\zeta_n = \zeta_{-n}$ vanish for $n = 2, 3, 4, \ldots$ then the equation of the corrugated interface reduces to the simple harmonic interface $\zeta = d \cos px$, where d is the amplitude of the corrugation and $2\pi/p$ is the wavelength of corrugation.

The equation of motion for SH wave propagation in the fiber-reinforced elastic half-spaces L_l $(l = 1, 2)$ are

$$\mu_{L_l} \frac{\partial^2 u_l}{\partial x^2} + \mu_{T_l} \frac{\partial^2 u_l}{\partial y^2} = \rho_l \frac{\partial^2 u_l}{\partial t^2}.$$

An incident plane SH wave at the corrugated interface, after propagating through the medium L_1, will give rise to regularly reflected and regularly transmitted waves as well as irregularly reflected and irregularly transmitted waves [Asano 1960]. The irregularly reflected and transmitted waves are due to the corrugation of the interface. Thus, the system of waves which arises due to corrugation on both sides of the regularly reflected waves are called *irregularly reflected waves*. Similarly, the system of waves which arises on both sides of regularly transmitted wave are called *irregularly transmitted waves*. These waves propagate with different amplitudes but with the same velocity as the regular waves. The n-th component of the spectrum form of an irregularly reflected wave is given by

$$u_1^{irr} = A_n^+ \exp\left(\frac{i\omega}{c_1}\left(c_1 t - x \sin\theta_n^+ + \eta_n^+ y\right)\right) + A_n^- \exp\left(\frac{i\omega}{c_1}\left(c_1 t - x \sin\theta_n^- + \eta_n^- y\right)\right),$$

where A_n^+ and A_n^- are the amplitude constants of the irregularly reflected SH waves propagating at angles of reflection θ_n^+ and θ_n^- respectively, and where

$$\eta_n^{\pm} = \sqrt{\frac{\rho_1 c_1^2}{\mu_{T_1}} - \frac{\mu_{L_1}}{\mu_{T_1}} \sin^2 \theta_n^{\pm}}.$$

The total displacement u_1 in the medium L_1 will contain the displacements due to the incident wave, the regularly reflected wave, and all irregularly reflected SH waves:

$$u_1 = A_0 \exp\Big(\frac{i\omega}{c_1}\big(c_1 t - x\sin\theta - \eta y\big)\Big) + A\exp\Big(\frac{i\omega}{c_1}\big(c_1 t - x\sin\theta + \eta y\big)\Big) + \sum_{n=1}^{\infty}\bigg(A_n^+ \exp\Big(\frac{i\omega}{c_1}\big(c_1 t - x\sin\theta_n^+ + \eta_n^+ y\big)\Big) + A_n^- \exp\Big(\frac{i\omega}{c_1}\big(c_1 t - x\sin\theta_n^- + \eta_n^- y\big)\Big)\bigg), \quad (5)$$

where A_0 is the amplitude of the incident wave, θ is the angle of incidence, ω is the angular frequency, $c_1 = \sqrt{\mu_{L_1}/\rho_1}$ is the speed of SH wave along x-axis in medium L_1, A is the amplitude of the regularly reflected SH wave with the angle of reflection θ, and

$$\eta = \sqrt{\frac{\rho_1 c_1^2}{\mu_{T_1}} - \frac{\mu_{L_1}}{\mu_{T_1}}\sin^2\theta}.$$

Similarly, the n-th component of the spectrum form of the irregularly transmitted wave is given by

$$u_2^{irr} = B_n^+ \exp\Big(\frac{i\omega}{c_2}\big(c_2 t - x\sin\phi_n^+ - \eta_{0n}^+ y\big)\Big) + B_n^- \exp\Big(\frac{i\omega}{c_2}\big(c_2 t - x\sin\phi_n^- - \eta_{0n}^- y\big)\Big),$$

where B_n^+ and B_n^- are the amplitudes of the irregularly transmitted waves with transmitted angles ϕ_n^+ and ϕ_n^-, and

$$\eta_{0n}^{\pm} = \sqrt{\frac{\rho_2 c_2^2}{\mu_{T_2}} - \frac{\mu_{L_2}}{\mu_{T_2}}\sin^2\phi_n^{\pm}}.$$

Thus, the displacement u_2 in the medium L_2 will contain the displacements due to regularly transmitted waves and due to all irregularly transmitted SH waves as

$$u_2 = B\exp\Big(\frac{i\omega}{c_2}\big(c_2 t - x\sin\phi - \eta_0 y\big)\Big) + \sum_{n=1}^{\infty}\bigg(B_n^+ \exp\Big(\frac{i\omega}{c_2}\big(c_2 t - x\sin\phi_n^+ - \eta_{0n}^+ y\big)\Big) + B_n^- \exp\Big(\frac{i\omega}{c_2}\big(c_2 t - x\sin\phi_n^- - \eta_{0n}^- y\big)\Big)\bigg), \quad (6)$$

where $c_2 = \sqrt{\mu_{L_2}/\rho_2}$ is the speed of the SH wave along the x axis in medium L_2, B is the amplitude of the regularly transmitted wave, ϕ is the angle which the transmitted wave makes with the normal, and

$$\eta_0 = \sqrt{\frac{\rho_2 c_2^2}{\mu_{T_2}} - \frac{\mu_{L_2}}{\mu_{T_2}}\sin^2\phi}.$$

The angles of the regularly reflected and regularly transmitted waves are related by Snell's law:

$$\frac{\sin\theta}{c_1} = \frac{\sin\phi}{c_2} = \frac{1}{c}, \quad (7)$$

where c is the apparent velocity. The relation between the angles of the regular wave and the corresponding irregular waves is given by the Spectrum theorem [Abubakar 1962; Asano 1960]:

$$\sin\theta_n^{\pm} - \sin\theta = \pm\frac{npc_1}{\omega}, \qquad \sin\phi_n^{\pm} - \sin\phi = \pm\frac{npc_2}{\omega}, \quad (8)$$

where the $\pm$ signs on both sides of each equality are matched.

The appropriate boundary conditions are the continuity of displacements and traction at the corrugated interface. Mathematically, at $y=\zeta(x)$, these boundary conditions are

$$[u_1]_{L_1}=[u_2]_{L_2}, \tag{9}$$

$$[\tau_{32}-\zeta'\tau_{31}]_{L_1}=[\tau_{32}-\zeta'\tau_{31}]_{L_2}, \tag{10}$$

where

$$\zeta'=\sum_{n=1}^{\infty}(\zeta_n e^{inpx}-\zeta_{-n}e^{-inpx})inp.$$

The boundary condition given in (10) can be expressed in terms of displacements as

$$\mu_{T_1}\frac{\partial u_1}{\partial y}-\zeta'\mu_{L_1}\frac{\partial u_1}{\partial x}=\mu_{T_2}\frac{\partial u_2}{\partial y}-\zeta'\mu_{L_2}\frac{\partial u_2}{\partial x}. \tag{11}$$

Substituting Equations (5)–(8) into the boundary conditions (9) and (11), we obtain

$$\begin{aligned}A_0\exp\Big(-i\eta\frac{\omega\zeta}{c_1}\Big)+A\exp\Big(i\eta\frac{\omega\zeta}{c_1}\Big)+\sum_{n=1}^{\infty}\Big(A_n^+\exp\Big(i\eta_n^+\frac{\omega\zeta}{c_1}\Big)e^{-inpx}+A_n^-\exp\Big(i\eta_n^-\frac{\omega\zeta}{c_1}\Big)e^{inpx}\Big)\\ =B\exp\Big(-i\eta_0\frac{\omega\zeta}{c_2}\Big)+\sum_{n=1}^{\infty}\Big(B_n^+\exp\Big(-i\eta_{0n}^+\frac{\omega\zeta}{c_2}\Big)e^{-inpx}+B_n^-\exp\Big(-i\eta_{0n}^-\frac{\omega\zeta}{c_2}\Big)e^{inpx}\Big)\end{aligned} \tag{12}$$

and

$$\begin{aligned}&-A_0\mu_{T_1}\frac{\eta}{c_1}\exp\Big(-i\eta\frac{\omega\zeta}{c_1}\Big)+A\eta\frac{\mu_{T_1}}{c_1}\exp\Big(i\eta\frac{\omega\zeta}{c_1}\Big)\\ &+\sum_{n=1}^{\infty}\frac{\mu_{T_1}}{c_1}\Big(A_n^+\eta_n^+\exp\Big(i\eta_n^+\frac{\omega\zeta}{c_1}\Big)e^{-inpx}+A_n^-\eta_n^-\exp\Big(i\eta_n^-\frac{\omega\zeta}{c_1}\Big)e^{inpx}\Big)\\ &+\zeta'\frac{\mu_{L_1}}{c_1}\Big(\sin\theta\Big(A_0\exp\Big(-i\eta\frac{\omega\zeta}{c_1}\Big)+A\exp\Big(i\eta\frac{\omega\zeta}{c_1}\Big)\Big)\\ &\qquad+\sum_{n=1}^{\infty}\Big(A_n^+\Big(\sin\theta+\frac{npc_1}{\omega}\Big)\exp\Big(i\eta_n^+\frac{\omega\zeta}{c_1}\Big)e^{-inpx}+A_n^-\Big(\sin\theta-\frac{npc_1}{\omega}\Big)\exp\Big(i\eta_n^-\frac{\omega\zeta}{c_1}\Big)e^{inpx}\Big)\Big)\\ &=\frac{\mu_{T_2}}{c_2}\Big(-B\eta_0\exp\Big(-i\eta_0\frac{\omega\zeta}{c_2}\Big)-\sum_{n=1}^{\infty}\Big(B_n^+\eta_{0n}^+\exp\Big(-i\eta_{0n}^+\frac{\omega\zeta}{c_2}\Big)e^{-inpx}+B_n^-\eta_{0n}^-\exp\Big(-i\eta_{0n}^-\frac{\omega\zeta}{c_2}\Big)e^{inpx}\Big)\Big)\\ &+\zeta'\frac{\mu_{L_2}}{c_2}\Big(B\sin\phi\exp\Big(-i\eta_0\frac{\omega\zeta}{c_2}\Big)\\ &\qquad+\sum_{n=1}^{\infty}\Big(B_n^+\Big(\sin\phi+\frac{npc_2}{\omega}\Big)\exp\Big(-i\eta_{0n}^+\frac{\omega\zeta}{c_2}\Big)e^{-inpx}+B_n^-\Big(\sin\phi-\frac{npc_2}{\omega}\Big)\exp\Big(-i\eta_{0n}^-\frac{\omega\zeta}{c_2}\Big)e^{inpx}\Big)\Big).\end{aligned} \tag{13}$$

Equations (12) and (13) provide the reflection and transmission coefficients for any order of approximation of corrugation.

4. Solution for the first order approximation

Since we assume that the corrugation and slope of the interface are small, the first order approximation to the exponential term containing ζ can be written as

$$\exp\Big(\pm i\eta\frac{\omega\zeta}{c_1}\Big) = 1 \pm i\eta\frac{\omega\zeta}{c_1}, \text{ etc.} \tag{14}$$

Substituting Equations (4) and (14) into the boundary conditions (12) and (13), and comparing the term independent of x and ζ in both sides of the equations, we have

$$-\frac{A}{A_0}+\frac{B}{A_0}=1, \qquad \mu_{T_1}\eta\frac{A}{A_0c_1}+\mu_{T_2}\eta_0\frac{B}{A_0c_2}=\frac{\eta\mu_{T_1}}{c_1}. \tag{15}$$

Comparing the coefficients of e^{-inpx} for A_n^+ and B_n^+ on both sides of the equations, we get

$$\frac{A_n^+}{A_0}-\frac{B_n^+}{A_0}=i\zeta_{-n}\omega\bigg(\Big(1-\frac{A}{A_0}\Big)\frac{\eta}{c_1}-\frac{B\eta_0}{c_2A_0}\bigg), \tag{16}$$

$$\mu_{T_1}\eta_n^+\frac{A_n^+}{A_0c_1}+\mu_{T_2}\eta_{0n}^+\frac{B_n^+}{A_0c_2}$$
$$=i\zeta_{-n}\bigg(\mu_{L_1}np\frac{\sin\theta}{c_1}-\mu_{T_1}\omega\frac{\eta^2}{c_1^2}+\Big(\mu_{L_1}np\frac{\sin\theta}{c_1}-\mu_{T_1}\omega\frac{\eta^2}{c_1^2}\Big)\frac{A}{A_0}+\Big(-\mu_{L_2}np\frac{\sin\theta}{c_1}+\mu_{T_2}\omega\frac{\eta_0^2}{c_2^2}\Big)\frac{B}{A_0}\bigg). \tag{17}$$

Similarly, comparing the coefficients of e^{inpx} for A_n^- and B_n^-, we obtain

$$\frac{A_n^-}{A_0}-\frac{B_n^-}{A_0}=i\zeta_n\omega\bigg(\Big(1-\frac{A}{A_0}\Big)\frac{\eta}{c_1}-\frac{B\eta_0}{c_2A_0}\bigg), \tag{18}$$

$$\mu_{T_1}\eta_n^-\frac{A_n^-}{A_0c_1}+\mu_{T_2}\eta_{0n}^-\frac{B_n^-}{A_0c_2}$$
$$=i\zeta_n\bigg(-\mu_{L_1}np\frac{\sin\theta}{c_1}-\mu_{T_1}\omega\frac{\eta^2}{c_1^2}-\Big(\mu_{L_1}np\frac{\sin\theta}{c_1}+\mu_{T_1}\omega\frac{\eta^2}{c_1^2}\Big)\frac{A}{A_0}+\Big(\mu_{L_2}np\frac{\sin\theta}{c_1}+\mu_{T_2}\omega\frac{\eta_0^2}{c_2^2}\Big)\frac{B}{A_0}\bigg). \tag{19}$$

Solving the system of equations (15), we obtain the reflection and transmission coefficients of the regularly reflected and transmitted SH waves as

$$\frac{A}{A_0}=\frac{1-M}{1+M}, \qquad \frac{B}{A_0}=\frac{2}{1+M}, \tag{20}$$

where $M=(\mu_{T_2}\eta_0c_1)/(\mu_{T_1}\eta c_2)$. These are the reflection and transmission coefficients of the SH wave at a plane interface between two different fiber-reinforced elastic half-spaces.

Solving the systems (16)–(17) and (18)–(19), we obtain

$$\frac{A_n^+}{A_0}=\frac{\Delta_{A_n^+}}{\Delta_n^+}, \qquad \frac{B_n^+}{A_0}=\frac{\Delta_{B_n^+}}{\Delta_n^+}, \qquad \frac{A_n^-}{A_0}=\frac{\Delta_{A_n^-}}{\Delta_n^-}, \qquad \frac{B_n^-}{A_0}=\frac{\Delta_{B_n^-}}{\Delta_n^-}, \tag{21}$$

where

$$\Delta_{A_n^+} = i\zeta_{-n}\Bigg(\mu_{T_2}\omega\frac{\eta_{on}^+\eta}{c_1c_2} + \mu_{L_1}np\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} + \left(\mu_{L_1}np\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} - \omega\mu_{T_2}\frac{\eta_{on}^+\eta}{c_1c_2}\right)\frac{A}{A_0} + \left(-\mu_{L_2}np\frac{\sin\theta}{c_1} + \mu_{T_2}\omega\frac{\eta_0^2}{c_2^2} - \omega\mu_{T_2}\frac{\eta_{on}^+\eta_0}{c_2^2}\right)\frac{B}{A_0}\Bigg),$$

$$\Delta_{B_n^+} = i\zeta_{-n}\Bigg(\mu_{T_2}\omega\frac{\eta_{on}^+\eta}{c_1c_2} + \mu_{L_1}np\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} - \mu_{T_1}\omega\frac{\eta\eta_n^+}{c_1^2} - \mu_{T_2}\omega\frac{\eta\eta_{0n}^+}{c_1c_2} + \left(\mu_{L_1}np\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} - \omega\mu_{T_2}\frac{\eta_{on}^+\eta}{c_1c_2} + \omega\frac{\eta}{c_1}\left(\frac{\eta_n^+\mu_{T_1}}{c_1} + \frac{\eta_{0n}^+\mu_{T_2}}{c_2}\right)\right)\frac{A}{A_0} + \left(-\mu_{L_2}np\frac{\sin\theta}{c_1} + \mu_{T_2}\omega\frac{\eta_0^2}{c_2^2} - \omega\mu_{T_2}\frac{\eta_{on}^+\eta_0}{c_2^2} + \omega\frac{\eta_0}{c_2}\left(\frac{\eta_n^+\mu_{T_1}}{c_1} + \frac{\eta_{0n}^+\mu_{T_2}}{c_2}\right)\right)\frac{B}{A_0}\Bigg),$$

$$\Delta_{A_n^-} = i\zeta_n\Bigg(\mu_{T_2}\omega\frac{\eta_{on}^-\eta}{c_1c_2} - \mu_{L_1}np\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} - \left(\mu_{L_1}np\frac{\sin\theta}{c_1} + \mu_{T_1}\omega\frac{\eta^2}{c_1^2} + \omega\mu_{T_2}\frac{\eta_{on}^-\eta}{c_1c_2}\right)\frac{A}{A_0} + \left(\mu_{L_2}np\frac{\sin\theta}{c_1} + \mu_{T_2}\omega\frac{\eta_0^2}{c_2^2} - \omega\mu_{T_2}\frac{\eta_{on}^-\eta_0}{c_2^2}\right)\frac{B}{A_0}\Bigg),$$

$$\Delta_{B_n^-} = i\zeta_n\Bigg(\mu_{T_2}\omega\frac{\eta_{on}^-\eta}{c_1c_2} - \mu_{L_1}np\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} - \mu_{T_1}\omega\frac{\eta\eta_n^-}{c_1^2} - \mu_{T_2}\omega\frac{\eta\eta_{0n}^-}{c_1c_2} - \left(\mu_{L_1}np\frac{\sin\theta}{c_1} + \mu_{T_1}\omega\frac{\eta^2}{c_1^2} + \omega\mu_{T_2}\frac{\eta_{on}^-\eta}{c_1c_2} + \omega\frac{\eta}{c_1}\left(\frac{\eta_n^-\mu_{T_1}}{c_1} + \frac{\eta_{0n}^-\mu_{T_2}}{c_2}\right)\right)\frac{A}{A_0} + \left(\mu_{L_2}np\frac{\sin\theta}{c_1} + \mu_{T_2}\omega\frac{\eta_0^2}{c_2^2} - \omega\mu_{T_2}\frac{\eta_{on}^-\eta_0}{c_2^2} + \omega\frac{\eta_0}{c_2}\left(\frac{\eta_n^-\mu_{T_1}}{c_1} + \frac{\eta_{0n}^-\mu_{T_2}}{c_2}\right)\right)\frac{B}{A_0}\Bigg),$$

$$\Delta_n^+ = \frac{\eta_n^+\mu_{T_1}}{c_1} + \frac{\eta_{0n}^+\mu_{T_2}}{c_2}, \qquad \Delta_n^- = \frac{\eta_n^-\mu_{T_1}}{c_1} + \frac{\eta_{0n}^-\mu_{T_2}}{c_2}.$$

The formulae in Equation (21) give reflection and transmission coefficients of irregularly reflected and transmitted waves for the first order approximations. Note that these coefficients depend on the elastic parameter of the medium, angle of incidence, corrugation parameter, and frequency of the incident wave.

5. Solution for second order approximation

For the second order approximation, we assume that the corrugation of the interface is so small that we can neglect the term containing the third and higher powers of ζ:

$$\exp\left(\pm i\eta\frac{\omega\zeta}{c_1}\right) = 1 \pm i\eta\frac{\omega\zeta}{c_1} - \left(\eta\frac{\omega\zeta}{c_1}\right)^2, \text{ etc.} \tag{22}$$

Substituting Equations (4) and (22) into the boundary conditions (12) and (13), and comparing the term independent of x, the coefficients of e^{-inpx}, and those of e^{inpx} separately on both sides of the resulting equations, one obtains six equations in six unknowns (see Appendix). The reflection and transmission coefficients of the reflected and transmitted waves at the corrugated interfaces for the second order approximation can be obtained by solving these equations for any value of n.

6. The case of a simple harmonic interface

We now obtain the reflection and transmission coefficients of incident plane SH waves at an interface given by $\zeta = d\cos px$. This equation for the interface can be obtained by setting $\zeta_1 = \zeta_{-1} = \frac{1}{2}d$ and $\zeta_n = \zeta_{-n} = 0$ for $n = 2, 3, \ldots$ in Equation (4). In this case, $2\pi/p$ is the wavelength and d is the amplitude of corrugation. Thus, the reflection and transmission coefficients for the first order approximation of the corrugation can be obtained by setting $n = 1$ in Equation (21), and we obtain

$$\frac{A_1^+}{A_0} = \frac{\Delta_{A_1^+}}{\Delta_1^+}, \qquad \frac{B_1^+}{A_0} = \frac{\Delta_{B_1^+}}{\Delta_1^+}, \qquad \frac{A_1^-}{A_0} = \frac{\Delta_{A_1^-}}{\Delta_1^-}, \qquad \frac{B_1^-}{A_0} = \frac{\Delta_{B_1^-}}{\Delta_1^-}, \tag{23}$$

where

$$\begin{aligned}\Delta_{A_1^+} = \frac{id}{2}\Bigg(&\mu_{T_2}\omega\frac{\eta_{o1}^+\eta}{c_1c_2} + \mu_{L_1}p\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} \\ &+ \left(\mu_{L_1}p\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} - \omega\mu_{T_2}\frac{\eta_{o1}^+\eta}{c_1c_2}\right)\frac{A}{A_0} + \left(-\mu_{L_2}p\frac{\sin\theta}{c_1} + \mu_{T_2}\omega\frac{\eta_0^2}{c_2^2} - \omega\mu_{T_2}\frac{\eta_{01}^+\eta_0}{c_2^2}\right)\frac{B}{A_0}\Bigg),\end{aligned}$$

$$\begin{aligned}\Delta_{B_1^+} = \frac{id}{2}\Bigg(&\mu_{T_2}\omega\frac{\eta_{o1}^+\eta}{c_1c_2} + \mu_{L_1}p\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} - \mu_{T_1}\omega\frac{\eta\eta_1^+}{c_1^2} - \mu_{T_2}\omega\frac{\eta\eta_{01}^+}{c_1c_2} \\ &+ \left(\mu_{L_1}p\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} - \omega\mu_{T_2}\frac{\eta_{o1}^+\eta}{c_1c_2} + \omega\frac{\eta}{c_1}\left(\frac{\eta_1^+\mu_{T_1}}{c_1} + \frac{\eta_{01}^+\mu_{T_2}}{c_2}\right)\right)\frac{A}{A_0} \\ &+ \left(-\mu_{L_2}p\frac{\sin\theta}{c_1} + \mu_{T_2}\omega\frac{\eta_0^2}{c_2^2} - \omega\mu_{T_2}\frac{\eta_{o1}^+\eta_0}{c_2^2} + \omega\frac{\eta_0}{c_2}\left(\frac{\eta_1^+\mu_{T_1}}{c_1} + \frac{\eta_{01}^+\mu_{T_2}}{c_2}\right)\right)\frac{B}{A_0}\Bigg),\end{aligned}$$

$$\begin{aligned}\Delta_{A_1^-} = \frac{id}{2}\Bigg(&\mu_{T_2}\omega\frac{\eta_{o1}^-\eta}{c_1c_2} - \mu_{L_1}p\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} \\ &- \left(\mu_{L_1}p\frac{\sin\theta}{c_1} + \mu_{T_1}\omega\frac{\eta^2}{c_1^2} + \omega\mu_{T_2}\frac{\eta_{01}^-\eta}{c_1c_2}\right)\frac{A}{A_0} + \left(\mu_{L_2}p\frac{\sin\theta}{c_1} + \mu_{T_2}\omega\frac{\eta_0^2}{c_2^2} - \omega\mu_{T_2}\frac{\eta_{01}^-\eta_0}{c_2^2}\right)\frac{B}{A_0}\Bigg),\end{aligned}$$

$$\begin{aligned}\Delta_{B_1^-} = \frac{id}{2}\Bigg(&\mu_{T_2}\omega\frac{\eta_{01}^-\eta}{c_1c_2} - \mu_{L_1}p\frac{\sin\theta}{c_1} - \mu_{T_1}\omega\frac{\eta^2}{c_1^2} - \mu_{T_1}\omega\frac{\eta\eta_1^-}{c_1^2} - \mu_{T_2}\omega\frac{\eta\eta_{01}^-}{c_1c_2} \\ &- \left(\mu_{L_1}p\frac{\sin\theta}{c_1} + \mu_{T_1}\omega\frac{\eta^2}{c_1^2} + \omega\mu_{T_2}\frac{\eta_{o1}^-\eta}{c_1c_2} + \omega\frac{\eta}{c_1}\left(\frac{\eta_1^-\mu_{T_1}}{c_1} + \frac{\eta_{01}^-\mu_{T_2}}{c_2}\right)\right)\frac{A}{A_0} \\ &+ \left(\mu_{L_2}p\frac{\sin\theta}{c_1} + \mu_{T_2}\omega\frac{\eta_0^2}{c_2^2} - \omega\mu_{T_2}\frac{\eta_{o1}^-\eta_0}{c_2^2} + \omega\frac{\eta_0}{c_2}\left(\frac{\eta_1^-\mu_{T_1}}{c_1} + \frac{\eta_{01}^-\mu_{T_2}}{c_2}\right)\right)\frac{B}{A_0}\Bigg),\end{aligned}$$

$$\Delta_1^+ = \frac{\eta_1^+\mu_{T_1}}{c_1} + \frac{\eta_{01}^+\mu_{T_2}}{c_2}, \qquad \Delta_1^- = \frac{\eta_1^-\mu_{T_1}}{c_1} + \frac{\eta_{01}^-\mu_{T_2}}{c_2}.$$

7. Energy equation

The expression of the energy flux for SH waves is obtained by multiplying the total energy per unit volume, which is twice the mean kinetic energy density, by the velocity of the propagation and the area of the wave front involved. The area of the wave front is proportional to the cosine of the angle between the wave normal and the vertical. The modulus of energy ratio of the regularly and irregularly reflected and transmitted SH waves are expressed as

$$E_{\mathrm{RF}} = \frac{|A|^2}{|A_0|^2}, \qquad E^{+}_{\mathrm{RF}-n} = \frac{|A_n^+|^2 \cos\theta_n^+}{|A_0|^2 \cos\theta}, \qquad E^{-}_{\mathrm{RF}-n} = \frac{|A_n^-|^2 \cos\theta_n^-}{|A_0|^2 \cos\theta},$$

$$E_{\mathrm{TR}} = \frac{|B|^2}{|A_0|^2}\frac{\rho_2 c_2^2 \tan\theta}{\rho_1 c_1^2 \tan\phi}, \qquad E^{+}_{\mathrm{TR}-n} = \frac{|B_n^+|^2}{|A_0|^2}\frac{\rho_2 c_2 \cos\phi_n^+}{\rho_1 c_1 \cos\theta},$$

$$E^{-}_{\mathrm{TR}-n} = \frac{|B_n^-|^2}{|A_0|^2}\frac{\rho_2 c_2 \cos\phi_n^-}{\rho_1 c_1 \cos\theta},$$

where E_{RF} is the ratio of the energy of regularly reflected wave to the energy of the incident wave, $E^{\pm}_{\mathrm{RF}-n}$ are the ratios of the energy of an irregularly reflected wave for the n-th spectrum to the energy of an incident wave, E_{TR} is the ratio of the energy of a regularly transmitted wave to the energy of an incident wave, and $E^{\pm}_{\mathrm{TR}-n}$ are ratios of the energy of an irregularly transmitted wave for n-th spectrum to the energy of an incident wave. Thus, the energy partitioning equation at the corrugated interface is given by

$$\left|\frac{A}{A_0}\right|^2 + \sum_{n=1}^{\infty}\left(\frac{\cos\theta_n^+}{\cos\theta}\left|\frac{A_n^+}{A_0}\right|^2 + \frac{\cos\theta_n^-}{\cos\theta}\left|\frac{A_n^-}{A_0}\right|^2\right) + \frac{\rho_2 c_2^2 \tan\theta}{\rho_1 c_1^2 \tan\phi}\left|\frac{B}{A_0}\right|^2 + \sum_{n=1}^{\infty}\left(\frac{\rho_2 c_2 \cos\phi_n^+}{\rho_1 c_1 \cos\theta}\left|\frac{B_n^+}{A_0}\right|^2 + \frac{\rho_2 c_2 \cos\phi_n^-}{\rho_1 c_1 \cos\theta}\left|\frac{B_n^-}{A_0}\right|^2\right) = 1. \tag{24}$$

When $n = 1$, Equation (24) reduces to

$$\left|\frac{A}{A_0}\right|^2 + \left|\frac{A_1^+}{A_0}\right|^2 \frac{\cos\theta_1^+}{\cos\theta} + \left|\frac{A_1^-}{A_0}\right|^2 \frac{\cos\theta_1^-}{\cos\theta} + \left|\frac{B}{A_0}\right|^2 \frac{\rho_2 c_2^2 \tan\theta}{\rho_1 c_1^2 \tan\phi} + \left|\frac{B_1^+}{A_0}\right|^2 \frac{\rho_2 c_2 \cos\phi_1^+}{\rho_1 c_1 \cos\theta} + \left|\frac{B_1^-}{A_0}\right|^2 \frac{\rho_2 c_2 \cos\phi_1^-}{\rho_1 c_1 \cos\theta} = 1.$$

Thus, the sum of energy ratios of the reflected and transmitted waves at the interface $\zeta = d\cos px$ must be equal to unity.

8. Particular case

When the fiber-reinforced elastic half-spaces L_1 and L_2 are reduced to isotropic half-spaces, we have $\mu_{T_1} = \mu_{L_1} = \mu_1$, $c_1^2 = \mu_1/\rho_1$, $\mu_{T_2} = \mu_{L_2} = \mu_2$, $c_2^2 = \mu_2/\rho_2$, $\eta = \cos\theta$ and $\eta_0 = \cos\phi$. With these values, the reflection and transmission coefficients at the plane interface between two uniform elastic half-spaces can be obtained from Equation (20), with a modified value M given by $M = (\mu_2 c_1 \cos\phi)/(\mu_1 c_2 \cos\theta)$. This result perfectly matches those given in Achenbach [1976]. (For the relevant problem, refer to Equations (5.77) and (5.78) on page 184).

Moreover, in this case, the values of η_1^+, η_1^-, η_{01}^+ and η_{01}^- reduce to $\eta_1^+ = \cos\theta_1^+$, $\eta_1^- = \cos\theta_1^-$, $\eta_{01}^+ = \cos\phi_1^+$, $\eta_{01}^- = \cos\phi_1^-$. The reflection and transmission coefficients for the first-order approximation of corrugation are given by Equation (23), with the modified values

$$\Delta_{A_1^+} = \frac{id}{2}\Bigg(\mu_2\omega\frac{\cos\phi_1^+\cos\theta}{c_1c_2} + \mu_1 p\frac{\sin\theta}{c_1} - \mu_1\omega\frac{\cos^2\theta}{c_1^2} + \left(\mu_1 p\frac{\sin\theta}{c_1} - \mu_1\omega\frac{\cos^2\theta}{c_1^2} - \omega\mu_2\frac{\cos\phi_1^+\cos\theta}{c_1c_2}\right)\frac{A}{A_0} + \left(-\mu_2 p\frac{\sin\theta}{c_1} + \mu_2\omega\frac{\cos^2\phi}{c_2^2} - \omega\mu_2\frac{\cos\phi_1^+\cos\phi}{c_2^2}\right)\frac{B}{A_0}\Bigg),$$

$$\Delta_{B_1^+} = \frac{id}{2}\Bigg(\mu_2\omega\frac{\cos\phi_1^+\cos\theta}{c_1c_2} + \mu_1 p\frac{\sin\theta}{c_1} - \mu_1\omega\frac{\cos^2\theta}{c_1^2} - \mu_1\omega\frac{\cos\theta\cos\theta_1^+}{c_1^2} - \mu_2\omega\frac{\cos\theta\cos\phi_1^+}{c_1c_2} + \left(\mu_1 p\frac{\sin\theta}{c_1} - \mu_1\omega\frac{\cos^2\theta}{c_1^2} - \omega\mu_2\frac{\cos\phi_1^+\cos\theta}{c_1c_2} + \omega\frac{\cos\theta}{c_1}\left(\frac{\cos\theta_1^+\mu_1}{c_1} + \frac{\cos\phi_1^+\mu_2}{c_2}\right)\right)\frac{A}{A_0} + \left(-\mu_2 p\frac{\sin\theta}{c_1} + \mu_2\omega\frac{\cos^2\phi}{c_2^2} - \omega\mu_2\frac{\cos\phi_1^+\cos\phi}{c_2^2} + \omega\frac{\cos\phi}{c_2}\left(\frac{\cos\theta_1^+\mu_1}{c_1} + \frac{\cos\phi_1^+\mu_2}{c_2}\right)\right)\frac{B}{A_0}\Bigg),$$

$$\Delta_{A_1^-} = \frac{id}{2}\Bigg(\mu_2\omega\frac{\cos\phi_1^-\cos\theta}{c_1c_2} - \mu_1 p\frac{\sin\theta}{c_1} - \mu_1\omega\frac{\cos^2\theta}{c_1^2} - \left(\mu_1 p\frac{\sin\theta}{c_1} + \mu_1\omega\frac{\cos^2\theta}{c_1^2} + \omega\mu_2\frac{\cos\phi_1^-\cos\theta}{c_1c_2}\right)\frac{A}{A_0} + \left(\mu_2 p\frac{\sin\theta}{c_1} + \mu_2\omega\frac{\cos^2\phi}{c_2^2} - \omega\mu_2\frac{\cos\phi_1^-\cos\phi}{c_2^2}\right)\frac{B}{A_0}\Bigg),$$

$$\Delta_{B_1^-} = \frac{id}{2}\Bigg(\mu_2\omega\frac{\cos\phi_1^-\cos\theta}{c_1c_2} - \mu_1 p\frac{\sin\theta}{c_1} - \mu_1\omega\frac{\cos^2\theta}{c_1^2} - \mu_1\omega\frac{\cos\theta\cos\theta_1^-}{c_1^2} - \mu_2\omega\frac{\cos\theta\cos\phi_1^-}{c_1c_2} - \left(\mu_1 p\frac{\sin\theta}{c_1} + \mu_1\omega\frac{\cos^2\theta}{c_1^2} + \omega\mu_2\frac{\cos\phi_1^-\cos\theta}{c_1c_2} + \omega\frac{\cos\theta}{c_1}\left(\frac{\cos\theta_1^-\mu_1}{c_1} + \frac{\cos\phi_1^-\mu_2}{c_2}\right)\right)\frac{A}{A_0} + \left(\mu_2 p\frac{\sin\theta}{c_1} + \mu_2\omega\frac{\cos^2\phi}{c_2^2} - \omega\mu_2\frac{\cos\phi_1^-\cos\phi}{c_2^2} + \omega\frac{\cos\phi}{c_2}\left(\frac{\cos\theta_1^-\mu_1}{c_1} + \frac{\cos\phi_1^-\mu_2}{c_2}\right)\right)\frac{B}{A_0}\Bigg),$$

$$\Delta_1^+ = \frac{\cos\theta_1^+\mu_1}{c_1} + \frac{\cos\phi_1^+\mu_2}{c_2}, \qquad \Delta_1^- = \frac{\cos\theta_1^-\mu_1}{c_1} + \frac{\cos\phi_1^-\mu_2}{c_2}.$$

For the normal incidence, that is, when $\theta = \phi = 0$ and from spectrum theorem given by Equation (8), we have $\cos\theta_1^+ = \cos\theta_1^-$, $\cos\phi_1^+ = \cos\phi_1^-$. In this case of normal incidence, we see that $A_1^+/A_0 = A_1^-/A_0$ and $B_1^+/A_0 = B_1^-/A_0$. These are the same results as obtained by Asano [1960] for the relevant problem.

9. Numerical results and discussion

To study the effect of various parameters on reflection and transmission coefficients, we computed the latter for a specific model having a simple cosine law interface, $\zeta = d\cos px$. We used the following relevant elastic parameters in the fiber-reinforced media:

In medium L_1, $\mu_{L_1} = 4.4\times 10^9$ N/m^2, $\mu_{T_1} = 1.89\times\ 10^9$ N/m^2 and $\rho_1 = 5.60\times\ 10^3$ kg/m^3. In medium L_2, $\mu_{L_2} = 5.66\times\ 10^9$ N/m^2, $\mu_{T_2} = 2.46\times\ 10^9$ N/m^2, and $\rho_2 = 7.80\times\ 10^3$ kg/m^3. Unless otherwise specified, $\omega d/c_1 = 0.12$, $pd = 0.00155$ and $\theta = 25°$.

Figures 2 and 3 show the variation of the modulus of reflection and transmission coefficients and energy ratios of reflected and transmitted waves with angle of incidence θ. Figure 2, top, shows that the reflection coefficient A/A_0 decreases, while the transmission coefficient B/A_0 increases with increasing θ. It is also clear that the reflection and transmission coefficients at plane interface between two fiber-reinforced half-spaces possess reverse behavior with angle of incidence.

In Figure 2, bottom, the reflection coefficients A_1^+/A_0 and A_1^-/A_0 of irregularly reflected waves at angles θ_1^+ and θ_1^- start from a certain value which increases up to $\theta = 12°$, and thereafter decrease with increasing angle of incidence. The transmission coefficient B_1^+/A_0 of irregularly transmitted waves at angle ϕ_1^+ increases with increasing angle of incidence, while the transmission coefficient B_1^-/A_0 of irregularly transmitted waves at angle ϕ_1^- starts from a certain value which increases up to $\theta = 14°$ and thereafter decreases with angle of incidence. Figure 3 shows the variation of energy ratios with θ. Note that E_{RF} increases and E_{TR} decreases with the angle of incidence. However, the rate of increase or decrease is very slow up to $\theta = 60°$ but at a very fast rate thereafter. At normal incidence, E_{TR} has maximum value and E_{RF} has minimum value. The energy ratios $E^+_{\mathrm{RF}-1}$ and $E^-_{\mathrm{RF}-1}$ of irregularly reflected waves at angles θ_1^+ and θ_1^-, respectively, start from a certain value, increase slightly up to $\theta = 13°$, and thereafter decrease with θ, achieving minimum values near grazing incidence. The energy ratio $E^+_{\mathrm{TR}-1}$ increases with increase in the angle of incidence in an almost similar pattern as that of E_{RF} with θ. The energy ratio $E^-_{\mathrm{TR}-1}$ starts from a certain value at normal incidence and then increases until $\theta = 16°$. It then decreases to certain value and again starts increasing, achieving maximum value at $\theta = 83°$. Beyond this point, the energy ratio decreases with the angle of incidence. These figures show that for the incident energy, the maximum amount of energy is carried by regularly reflected and transmitted waves, and a very small amount by irregular waves. The sum of energy ratios is very close to unity, which shows that there is no dissipation during transmission.

Figures 4 and 5 show the variation of the modulus of reflection and transmission coefficients and energy ratios with the frequency parameter $(\omega d/c_1)$, when the SH wave is incident at $\theta = 25^0$. We see that the reflection and transmission coefficients of regular waves and their corresponding energy ratios are not influenced by the frequency parameter. The reflection coefficients A_1^+/A_0, and A_1^-/A_0, and the transmission coefficients B_1^+/A_0, and B_1^-/A_0 of the irregular waves increase linearly with the increase in the frequency parameter. Thus the reflection and transmission coefficients of irregular waves are influenced by the frequency parameter. Figure 5, bottom, shows that the energy ratios of irregular waves increase with increasing frequency parameter.

Figures 6 and 7 show the variation of the modulus of reflection and transmission coefficients and energy ratios with the corrugation parameter pd. The top part of Figures 6 and 7 show that reflection and transmission coefficients of regular waves and their corresponding energy ratios are not influenced by

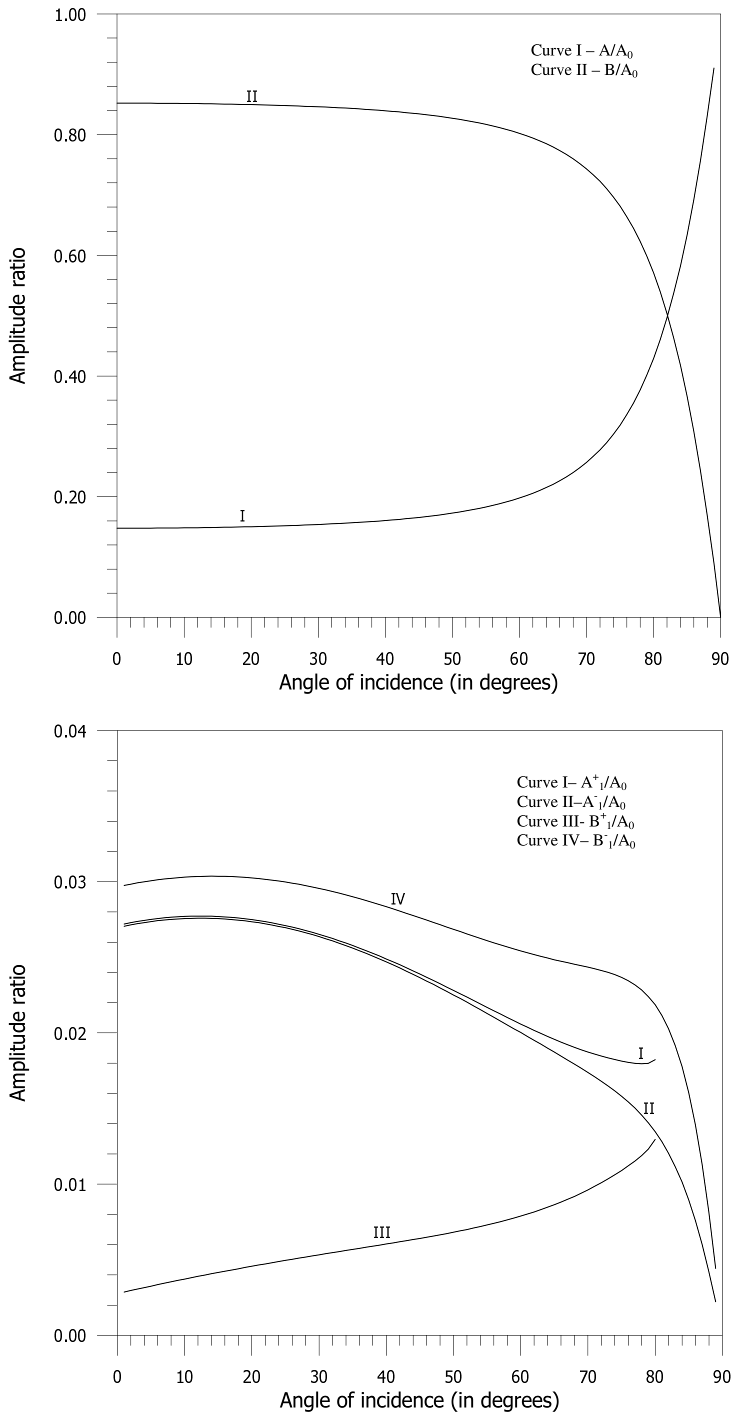

Figure 2. Variation of the modulus of reflection and transmission coefficients of regular (top) and irregular (top) waves with angle of incidence, when $pd = 1.55 \times 10^{-3}$ and $\omega d/c_1 = 0.12$.

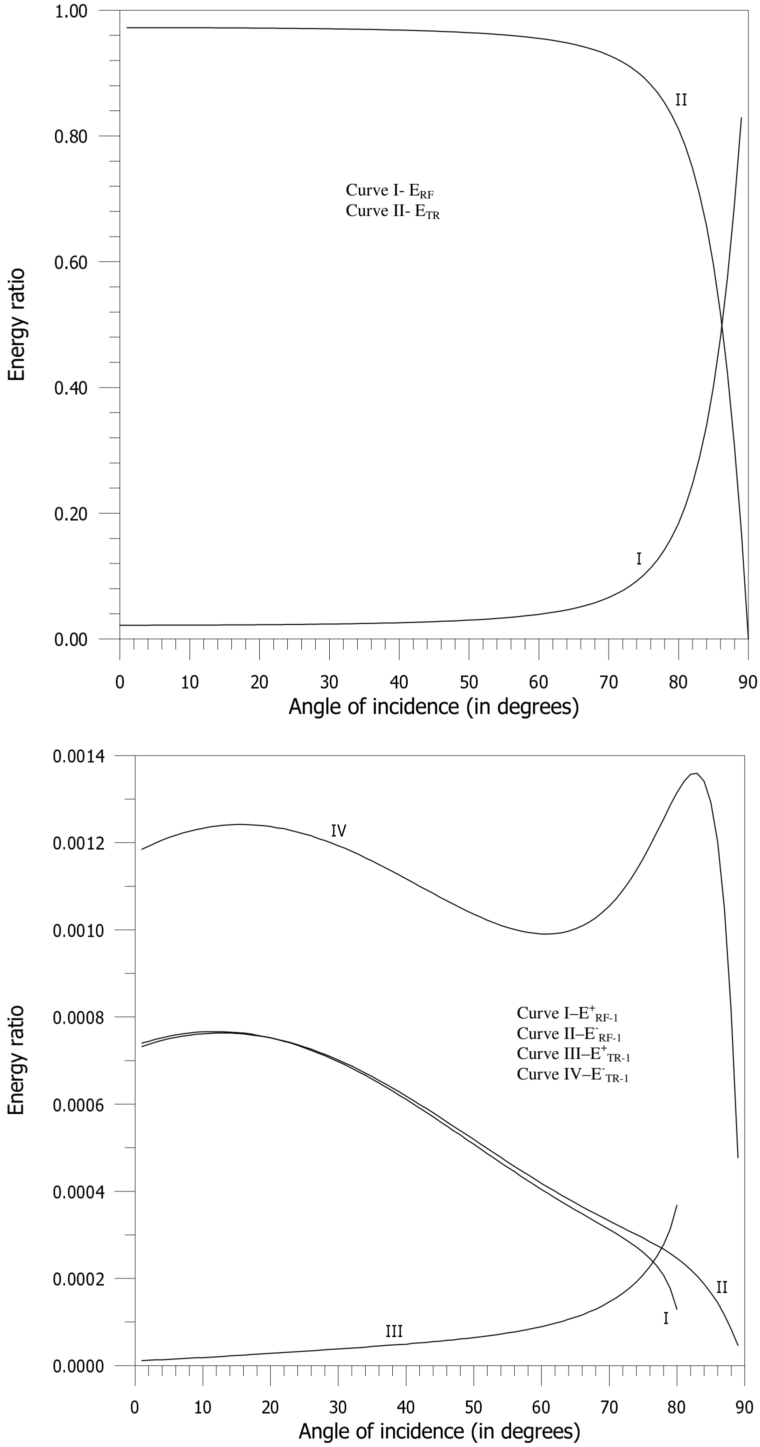

Figure 3. Variation of the modulus of the energy ratios of regularly (top) and irregularly (bottom) reflected and transmitted SH waves, when $pd = 1.55 \times 10^{-3}$ and $\omega d/c_1 = 0.12$.

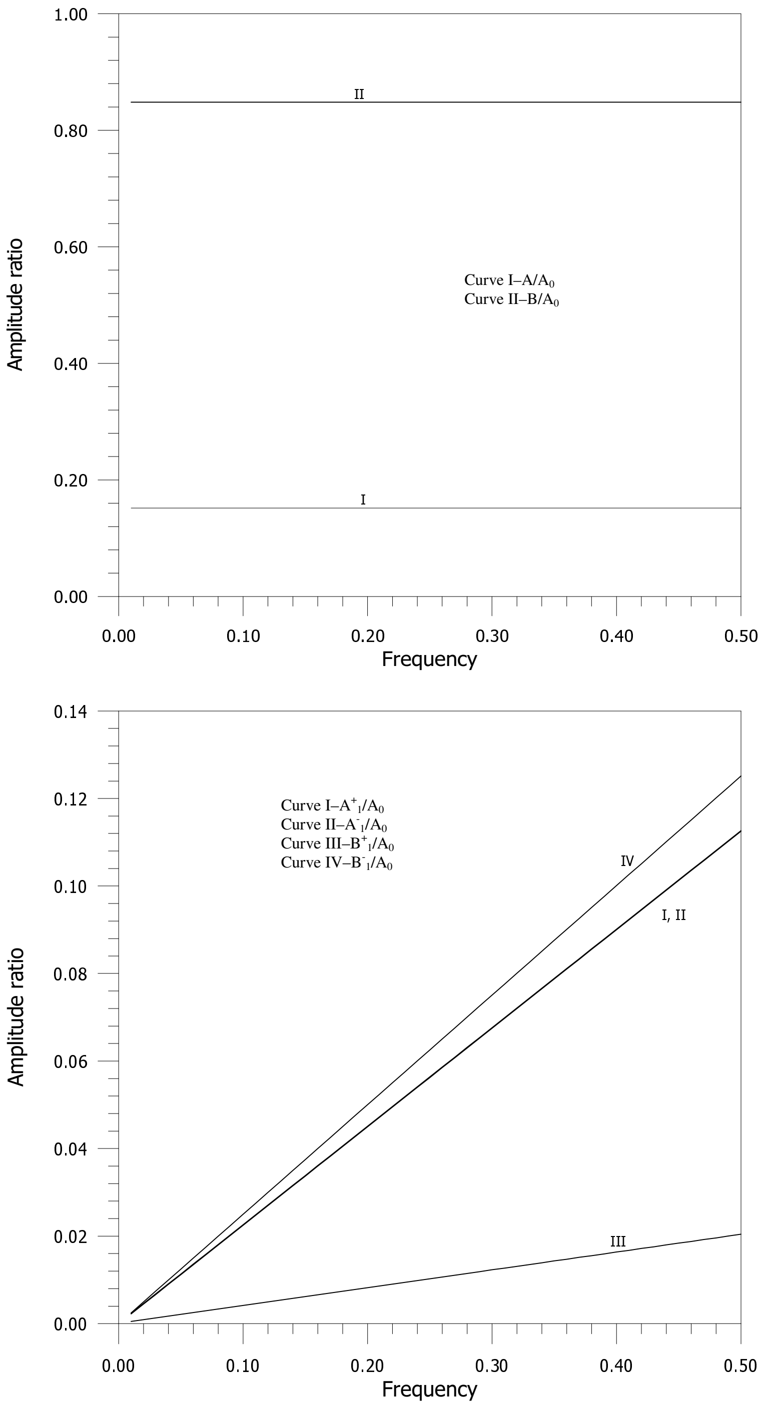

Figure 4. Variation of the modulus of reflection and transmission coefficients of regularly (top) and irregularly (bottom) reflected and transmitted SH waves with frequency $\omega d/c_1$, when $pd = 1.55 \times 10^{-3}$ and $\theta = 25°$.

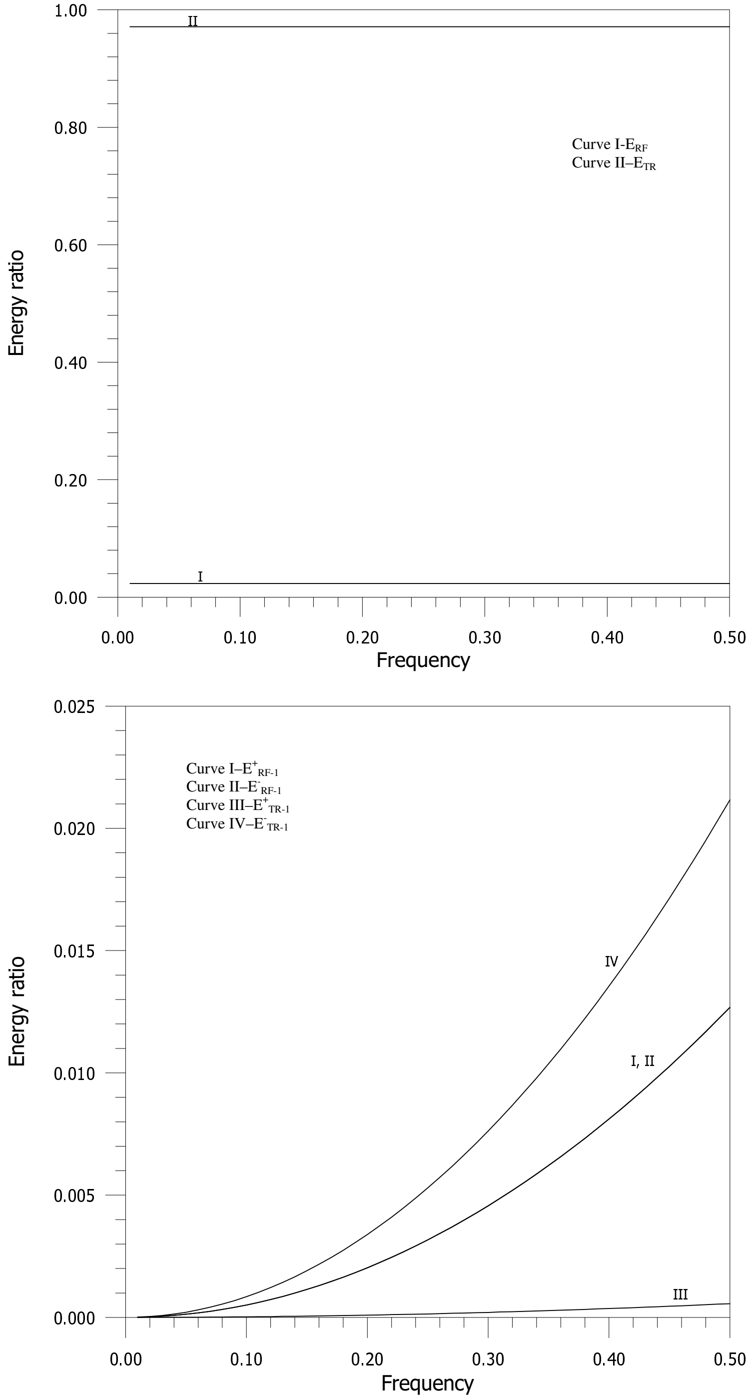

Figure 5. Variation of modulus of energy ratios of regularly (top) and irregularly (bottom) reflected and transmitted SH waves with frequency $\omega d/c_1$, when $pd = 1.55 \times 10^{-3}$ and $\theta = 25°$.

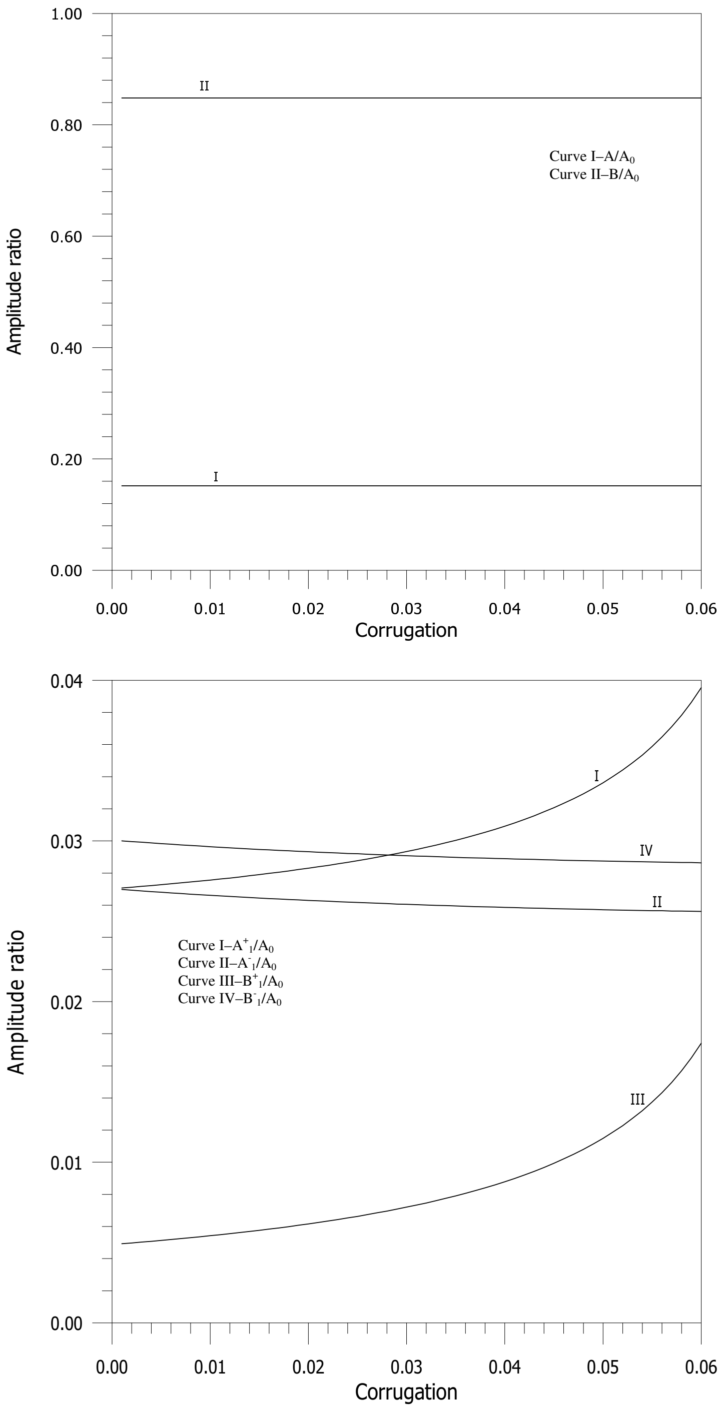

Figure 6. Variation of the modulus of reflection and transmission coefficients of regularly (top) and irregularly (bottom) reflected and transmitted SH waves with corrugation parameter pd, when $\omega d/c_1 = 0.12$ and $\theta = 25°$.

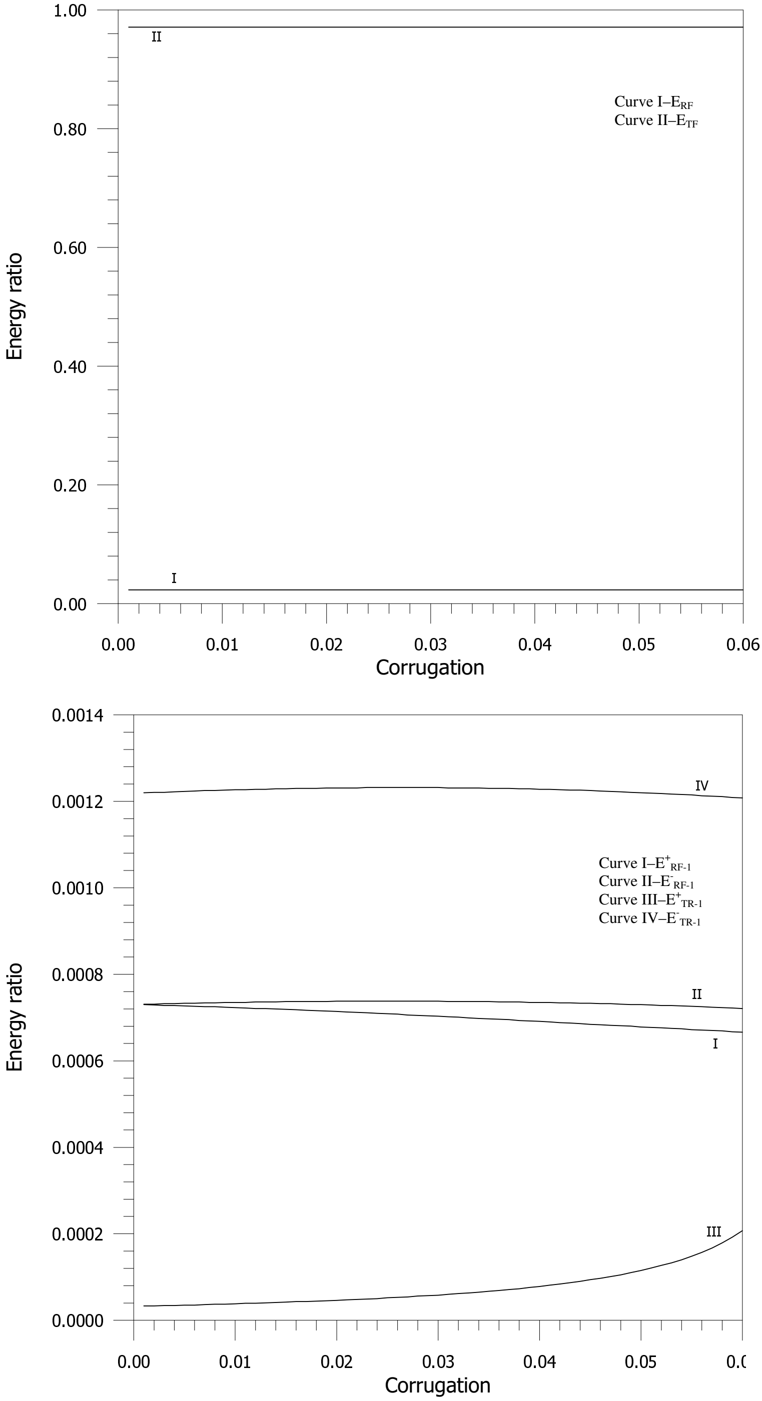

Figure 7. Variation of the modulus of energy ratios of regularly (top) and irregularly (bottom) reflected and transmitted SH waves with corrugation parameter pd, when $\omega d/c_1 = 0.12$ and $\theta = 25°$.

the corrugation parameter, as was expected. Figure 6, bottom, shows that the reflection coefficient A_1^+/A_0 and the transmission coefficient B_1^+/A_0 increase as the corrugation parameter pd increases, while the reflection coefficient A_1^-/A_0 and the transmission coefficient B_1^-/A_0 decrease as pd increases. Hence, the coefficients corresponding to irregular waves are found to be influenced by the corrugation parameter pd. In Figure 7, bottom, we see that as the corrugation parameter pd increases, the energy ratios $E^+_{\mathrm{RF}-1}$ and $E^-_{\mathrm{RF}-1}$ decrease at a very small rate, the energy ratio $E^+_{\mathrm{TR}-1}$ increases, and the energy ratio $E^-_{\mathrm{TR}-1}$ decreases but at very small rate.

10. Conclusions

The reflection and transmission phenomena of an incident SH wave at a corrugated interface between two dissimilar elastic fiber-reinforced half-spaces are studied. It is assumed that amplitude and slope of the corrugated interface are small and the formulae for reflection and transmission coefficients for the first and second order approximations of corrugation are presented using Rayleigh's method of approximation. These coefficients are expressed in the closed form for the first order approximation of corrugation, and for a special type of interface. The energy partition equation at a corrugated interface is also obtained. Numerically, these coefficients and energy ratios are calculated for a specific model and the results obtained are shown graphically. We conclude that

(i) The reflection and transmission coefficients are functions of elastic parameters and the angle of incidence. Moreover, the coefficients of irregularly reflected and transmitted waves, and hence the energy ratios, are functions of the corrugation parameters and frequency of the incident wave.

(ii) The reflection and transmission coefficients of regularly reflected and transmitted SH waves are independent of the corrugation and frequency parameters. But there is a significant effect of corrugation and frequency on the reflection and transmission coefficients of irregularly reflected and transmitted waves. Reflection and transmission coefficients of irregularly reflected and transmitted waves increase as the normalized frequency $\omega d/c_1$ and corrugation parameter pd increase.

(iii) The reflection and transmission coefficients of regular waves are greater than those of irregular waves. It is also noted that the energy ratio of regular waves is greater than the energy ratios of irregular waves,

(iv) The coefficients and energy ratios of irregular waves increase with increasing frequency and corrugation parameter. The sum of the energy ratios of reflected and transmitted waves for first order approximation of corrugation is found to be very close to unity.

Acknowledgements

The authors are grateful to the unknown reviewers for their help and suggestions, which have led to an improvement in the paper.

Appendix: Equations for the second-order approximation

$$\left(1-2\zeta_{-n}\zeta_n\frac{\omega^2\eta^2}{c_1^2}\right)(A_0+A)+i\zeta_n\eta_n^+\frac{\omega}{c_1}A_n^+ +i\zeta_{-n}\eta_n^-\frac{\omega}{c_1}A_n^- = \left(1-2\zeta_{-n}\zeta_n\frac{\omega^2\eta_0^2}{c_2^2}\right)B-i\zeta_n\eta_{0n}^+\frac{\omega}{c_2}B_n^+ -i\zeta_{-n}\eta_{0n}^-\frac{\omega}{c_2}B_n^-,$$

$$\mu_{T_1}\frac{\eta}{c_1}\left(1-2\zeta_{-n}\zeta_n\frac{\omega^2\eta^2}{c_1^2}\right)(A-A_0)+\mu_{T_2}\frac{\eta_0}{c_2}\left(1-2\zeta_{-n}\zeta_n\frac{\omega^2\eta_0^2}{c_2^2}\right)B + i\zeta_n\left(\mu_{T_1}(\eta_n^+)^2\frac{\omega}{c_1^2}+\mu_{L_1}np\frac{\sin\theta_n^+}{c_1}\right)A_n^+ + i\zeta_{-n}\left(\mu_{T_1}(\eta_n^-)^2\frac{\omega}{c_1^2}-\mu_{L_1}np\frac{\sin\theta_n^-}{c_1}\right)A_n^- = i\zeta_n\left(\mu_{T_2}(\eta_{0n}^+)^2\frac{\omega}{c_2^2}+\mu_{L_2}np\frac{\sin\phi_n^+}{c_2}\right)B_n^+ + i\zeta_{-n}\left(\mu_{T_2}(\eta_{0n}^-)^2\frac{\omega}{c_2^2}-\mu_{L_2}np\frac{\sin\phi_n^-}{c_2}\right)B_n^-,$$

$$i\eta\zeta_{-n}\frac{\omega}{c_1}(A_0+A)-\left(1-2\zeta_{-n}\zeta_n\frac{(\omega\eta_n^+)^2}{c_1^2}\right)A_n^+ +\zeta_{-n}^2\frac{(\omega\eta_n^-)^2}{c_1^2}A_n^- = i\zeta_{-n}\eta_0\frac{\omega B}{c_2}-\left(1-2\zeta_{-n}\zeta_n\frac{(\omega\eta_{0n}^+)^2}{c_2^2}\right)B_n^+ +\zeta_{-n}^2\frac{(\omega\eta_{0n}^-)^2}{c_2^2}B_n^-,$$

$$i\zeta_{-n}\left(\mu_{T_1}\eta^2\frac{\omega}{c_1^2}-\mu_{L_1}np\frac{\sin\theta}{c_1}\right)(A_0+A)-i\zeta_{-n}\left(\mu_{T_2}\frac{\omega\eta_0^2}{c_2^2}-\mu_{L_2}np\frac{\sin\theta}{c_1}\right)B + \mu_{T_1}\frac{\eta_n^+}{c_1}\left(1-2(\eta_n^+)^2\zeta_n\zeta_{-n}\frac{\omega^2}{c_1^2}\right)A_n^+ +\zeta_{-n}^2\left(\mu_{T_1}(\eta_n^-)^3\frac{\omega^2}{c_1^3}-\mu_{L_1}np\eta_n^-\frac{\sin\theta_n^-}{c_1^2}\right)A_n^- = -\mu_{T_2}\frac{\eta_{0n}^+}{c_2}\left(1-2\zeta_{-n}\zeta_n\frac{(\omega\eta_{0n}^+)^2}{c_2^2}\right)B_n^+ +\zeta_{-n}^2\frac{\omega\eta_{0n}^-}{c_2^2}\left(\mu_{T_2}(\eta_{0n}^-)^2\frac{\omega}{c_2}-\mu_{L_2}np\sin\phi_n^-\right)B_n^-,$$

$$i\eta\zeta_n\frac{\omega}{c_1}(A_0+A)+\zeta_n^2\frac{(\omega\eta_n^+)^2}{c_1^2}A_n^+ -\left(1-2\zeta_{-n}\zeta_n\frac{(\omega\eta_n^-)^2}{c_1^2}\right)A_n^- = i\zeta_n\eta_0\frac{\omega B}{c_2}+\zeta_n^2\frac{(\omega\eta_{0n}^+)^2}{c_2^2}B_n^+ -\left(1-2\zeta_{-n}\zeta_n\frac{(\omega\eta_{0n}^-)^2}{c_2^2}\right)B_n^-,$$

$$i\zeta_n\left(\mu_{T_1}\frac{\omega\eta^2}{c_1^2}+\mu_{L_1}np\frac{\sin\theta}{c_1}\right)(A_0+A)-i\zeta_n\left(\mu_{T_2}\frac{\omega\eta_0^2}{c_2^2}+\mu_{L_2}np\frac{\sin\theta}{c_1}\right)B - \frac{\omega\zeta_n^2}{c_1^2}\eta_n^+\left(\mu_{T_1}\frac{\omega(\eta_n^+)^2}{c_1}+\mu_{L_1}np\sin\theta_n^+\right)A_n^+ +\mu_{T_1}\frac{\eta_n^-}{c_1}\left(1-2(\eta_n^-)^2\zeta_n\zeta_{-n}\frac{\omega^2}{c_1^2}\right)A_n^- = \zeta_n^2\left(\mu_{T_2}\frac{\omega^2(\eta_{0n}^+)^3}{c_2^3}+\mu_{L_2}\eta_{0n}^+np\frac{\omega\sin\phi_n^+}{c_2^2}\right)B_n^+ -\mu_{T_2}\frac{\eta_{0n}^-}{c_2}\left(1-2\zeta_{-n}\zeta_n\frac{(\omega\eta_{0n}^-)^2}{c_2^2}\right)B_n^-.$$

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Received 14 Feb 2006. Revised 21 May 2006. Accepted 1 Jul 2006.

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