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**NEW APPROACH TO INVESTIGATION OF RESONANT
VIBRATIONS OF NONCIRCULAR SHELLS BASED ON THE
THEORY OF COUPLED WAVEGUIDES**

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VICTOR V. KRYLOV AND VASIL B. GEORGIEV

In the present paper a new simple method of analytical description of resonant vibrations of finite noncircular cylindrical shells is developed. The method is based on the theory of coupled waveguides formed by quasiflat areas of the same noncircular shells having an infinite length (depth). The physical reason for guided wave propagation along quasiflat areas of such shells is the difference between flexural wave velocities in their quasiflat and curved areas, respectively. Using asymptotic expressions for flexural wave velocities in circular shells with different radii of curvature, approximate dispersion equations are derived for waves propagating in such waveguides and their corresponding coupling coefficients. After that, considering shells of finite length, the transition is made from the coupled guided modes to the coupled resonant vibrations of the shell. The obtained resonant frequencies and spatial distributions of the resulting vibration modes are in good agreement with the results of finite element calculations.
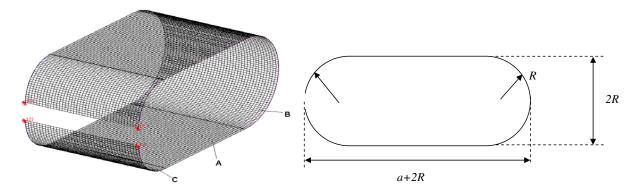
1. Introduction

Noncircular cylindrical shells are relatively simple structures that are widely used for their own sake as well as for describing physical processes in more complex structures, especially in aeronautical and oceanic engineering applications; see [Armenakas and Koumousis 1983; Kumar and Singh 1993; Suzuki et al. 1996; Ganapathi and Haboussi 2003]. Some recent applications of elliptical or oval-type noncircular shells include modeling structure-borne vehicle interior noise in simplified reduced-scale vehicle models, where they form flexible parts of the model structures [Krylov et al. 2003; 2004]. Finite element calculations of resonant vibrations of such noncircular cylindrical shells, as seen in Figure 1, show that normal vibration modes of such structures can be represented as symmetric and antisymmetric combinations of vibrations of their quasiflat parts taken separately [Georgiev et al. 2004]. The same conclusions follow also from analytical or semianalytical solutions currently available for noncircular shells; see, for example, [Kumar and Singh 1993].

Note that obtaining analytical solutions for noncircular cylindrical shells is a very complex task that normally involves solving a system of governing differential equations with variable coefficients that describe the effects of variable curvature. This can be done, for example, using power series expansion of the solution [Suzuki et al. 1996] or by means of its representation in terms of special functions, such as Bezier polynomials [Kumar and Singh 1993].

In the present paper we propose to describe resonant vibrations of noncircular cylindrical shells analytically in a simpler way using the so-called coupled-wave theory approach utilizing the concept of coupled-waveguide propagation in shells of the same noncircular shape, but having an infinite length
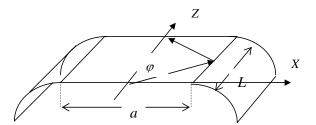
Figure 1. Three-dimensional view (left) and profile (right) of noncircular cylindrical shell used for experimental modeling of structure-borne vehicle interior noise [Krylov et al. 2003; 2004].

(depth). Note that the concept of coupled waveguides has been previously applied to studying Rayleigh wave propagation on the surfaces of elastic bodies having complex cross-sectional shapes [Krylov 1987]. This method simplifies the analysis of noncircular shells by the fact that the effects of variable curvature can be taken into account indirectly, via the local curvature-dependent velocity of flexural waves freely propagating at arbitrary angle in the circular shell of the same radius of curvature. The problem is then reduced to the wave propagation problem in an inhomogeneous medium that contains two or more coupled waveguides for flexural waves formed by quasiflat areas of the shell.

The physical reason for waveguide propagation along quasiflat areas of noncircular cylindrical shells is the difference between flexural wave velocities in their quasiflat and curved parts. In particular, for waveguide propagation to be possible it is necessary that the velocity of flexural waves in the quasiflat area is lower than their velocity in the adjacent curved areas. This is always the case for flexural wave propagation in near-axial directions of circular cylindrical shells.

In the following sections we first employ the approximate expressions for flexural wave velocities (wavenumbers) in circular shells with different radii of curvature to derive the dispersion equations for flexural waves propagating in the waveguides composed of infinitely long flat plates (strips) bounded by fragments of two circular shells. We then discuss the coupling between two neighboring waveguides and the coupled waveguide modes. After that we will consider shells of finite length making the transition from coupled guided modes to coupled resonant vibrations of quasiflat parts of the shells. It will be demonstrated that resonant frequencies and spatial distributions of the normal vibration modes are in good agreement with the results of finite element calculations [Georgiev et al. 2004].

The proposed coupled-wave theory approach can be useful for a quick prediction of resonant vibration modes in oval-type shells containing two coupled waveguides and in more complex shell structures that can be associated with combinations of several coupled waveguides. The advantage of this approach lies in the fact that it is much easier to estimate resonant frequencies and modal shapes of complex noncircular shells on the basis of understanding the behavior of a single quasiflat waveguide considered as their basic structural component.

Figure 2. Waveguide propagation of flexural waves in an infinitely long flat plate (strip) bounded by two circular cylindrical shells.

2. Waveguide propagation in a plate bounded by two circular cylindrical shells

Let us consider waveguide propagation of flexural waves in an infinitely long flat plate (strip) of thickness h and width a bounded by fragments of two cylindrical shells having equal radii R as shown in Figure 2. We assume that the shells are of the same thickness as the flat plate. Such a structure can be considered as a three-layered anisotropic medium for flexural waves, the middle layer (a flat isotropic strip) being characterised by a lower phase velocity of flexural waves in comparison with flexural wave velocities in the near-axial direction in the adjacent cylindrical shells. It is well known that the layers formed by adjacent cylindrical shells are anisotropic in respect of flexural wave propagation. We remind the reader that we assume initially that the aforementioned structure is infinite in the z-direction, that is, $L = \infty$.

Waveguide propagation in such a three-layered medium can be described by the general dispersion equation well-known from acoustics and optics [Brekhovskikh and Godin 1990] that is slightly modified for the case of shell-induced anisotropic side layers [Schmidt and Coldren 1975; Krylov 1987]:

$$\frac{a}{2}\big[k_{pl}{}^2 - \gamma^2\big]^{1/2} = m\frac{\pi}{2} + \tan^{-1}\left[\frac{\gamma^2 - k_{sh}{}^2(\varphi)}{k_{pl}{}^2 - \gamma^2}\right]^{1/2}. \tag{1}$$

Here γ is as of yet unknown wavenumber of a guided flexural wave propagating in our three-layered system, $k_{pl} = (\omega^2 \rho_s h / D)^{1/4}$ is the wavenumber of flexural waves in a flat plate, where ρ_s is the mass density of the plate material and D is plate flexural rigidity, $k_{sh}(\varphi)$ is the angle-dependent wavenumber of flexural waves in a circular cylindrical shell, and $m = 0, 1, 2, \dots$ Note that

$$\gamma = k_{pl} \cos\varphi, \tag{2}$$

where φ is the angle of propagation of two plane waves comprising a guided mode; see Figure 2. The guided wave fields propagating in the flat plate area and penetrating into the adjacent shells are described by, respectively,

$$w_{pl} = \cos\big[(k_{pl}^2 - \gamma^2)^{1/2}x\big]\exp(i\gamma z - i\omega t), \quad w_{sh} = C\exp\big[-\big(\gamma^2 - k_{sh}^2(\varphi(\gamma))\big)^{1/2}|x_{sh}|\big]\exp(i\gamma z - i\omega t). \tag{3}$$

Here x_{sh} is the surface curvilinear coordinate that extends the coordinate x from the plate to the shell area, and C is the constant determined from the continuity condition at the plate-shell boundary.

To find γ from the dispersion equation (1) one has to use appropriate analytical expressions for the wavenumber of flexural waves in the shell $k_{sh}(\varphi)$ propagating at arbitrary angle φ with respect to the axial direction of the shell. It is well known that propagation of flexural waves in shells is governed by

bending and membrane effects, which makes the expressions for flexural wavenumbers rather complex [Junger and Feit 1972; Fahy 1985; Leissa 1973]. Their functional appearance depends on the characteristic parameters of the shell, in particular on its ring frequency $\omega_r = c_l'/R$, where c_l' is the velocity of a quasilongitudinal wave in a thin flat plate (plate wave velocity), and R is the mean radius of the shell's curvature [Germogenova 1973; Tyutekin 2004]. In what follows we will use the relationships following from Kirchhoff–Love theory of thin elastic shells made of isotropic materials; see, for example, [Fahy 1985; Leissa 1973; Tyutekin 2004].

Let us first consider the case of wave propagation in a circular shell at frequencies ω that are essentially higher than the ring frequency ω_r, that is, consider the nondimensional parameter $\Omega = \omega/\omega_r \gg 1$.

Our starting point in this derivation is the simplified dispersion equation for flexural type wave propagation in a circular cylindrical shell; see [Fahy 1985, Equation (4.167)]. We can rewrite it in a more compact form using a slightly different notation as

$$\cos^4 \varphi + R^4 k_{sh}{}^4 \beta^2 = \Omega^2. \tag{4}$$

Here $k_{sh} = k_{sh}(\varphi)$ is the wavenumber of flexural waves in the shell propagating at arbitrary angle φ with respect to the axial direction of the shell, R is the mean radius of the shell, and $\beta = h/\sqrt{12}R$, where h is the shell thickness. The first term on the left hand side of Equation (4) is associated with membrane strain energy, while the second term is associated with strain energy of bending. Equation (4) is accurate for thin shells ($\beta \ll 10^{-1}$) with $k_{sh} R \sin \varphi > 2$.

Using the condition $\Omega \gg 1$ one easily derives the approximate expression for the wavenumber of a flexural wave propagating in such a shell

$$k_{sh}(\varphi) = k_{pl}\left[1 - \frac{1}{4\Omega^2} \cos^4 \varphi\right], \tag{5}$$

where $k_{pl} = (\Omega^{1/2}/R\beta^{1/2}) = 12^{1/4}\omega^{1/2}/(c_l')^{1/2}h^{1/2} = (\omega^2 \rho_s h/D)^{1/4}$ is the wavenumber of flexural waves in a flat plate of the same thickness h (see also above).

Keeping in mind that $\cos \varphi = 1 - \varphi^2/2$ for small φ and substituting Equations (2) and (5) into Equation (1), one can derive the following simplified equation for φ:

$$\frac{a}{2}k_{pl}\varphi = m\frac{\pi}{2} + \tan^{-1}\left[\frac{1}{2\Omega^2\varphi^2} - 1\right]^{1/2}.$$

Considering for simplicity the propagation of only the lowest order mode $m = 0$, this equation can be reduced to a biquadratic one. Assuming that $ak_{pl} \gg \Omega$, the approximate solution of this equation is $\varphi^2 = 2^{1/2}/\Omega k_{pl}a$, which describes a weak waveguide effect. Using Equation (2) and this solution, one can obtain the following expression for γ

$$\gamma = k_{pl}\left[1 - \frac{1}{2^{1/2}\Omega k_{pl}a}\right]. \tag{6}$$

As it could be expected for waveguide propagation in a three-layered system, the velocity of guided wave $c = \omega/\gamma$ in the case under consideration is higher than the velocity in the *slow region* (flat plate), but lower than the velocity in the adjacent *fast regions* (fragments of circular shell).

Let us now turn to waveguide propagation at frequencies lower than the ring frequency. In this case the expressions for $k_{sh}(\varphi)$ are generally too complex to be described analytically [Junger and Feit 1972;

Fahy 1985]. For illustration purposes we limit ourselves, as above, to the case of small wave propagation angles φ, for which $k_{sh}(\varphi)$ is known to be extremely small. To simplify things even further, we assume that $k_{sh}(\varphi) = 0$ for all φ in the range of interest. In this case the dispersion equation (1) becomes

$$\frac{a}{2}\left[k_{pl}{}^2 - \gamma^2\right]^{1/2} = m\frac{\pi}{2} + \tan^{-1}\left[\frac{\gamma^2}{k_{pl}{}^2 - \gamma^2}\right]^{1/2}.$$ (7)

Again using the fact that $\cos\varphi = 1 - \varphi^2/2$ for small φ and substituting Equation (2) into Equation (7), one can derive the following simplified equation for φ

$$\frac{a}{2}k_{pl}\varphi = m\frac{\pi}{2} + \tan^{-1}\left[\frac{1 - \varphi^2}{\varphi^2}\right]^{1/2}.$$

For small φ the last term in the right hand side of this equation is approximately equal to $\pi/2$, allowing us to reduce it to

$$\frac{a}{2}k_{pl}\varphi = (m+1)\frac{\pi}{2},$$

from which it follows that $\varphi = (m+1)\pi/ak_{pl}$. Substituting this solution into Equation (2), one can derive the following approximate expression for the wavenumbers of guided flexural waves propagating in the above system at frequencies below the ring frequency

$$\gamma = k_{pl}\left[1 - \frac{(m+1)^2\pi^2}{2a^2k_{pl}{}^2}\right].$$ (8)

In this case the waveguide effect is rather strong, and the energy of a guided wave is almost entirely concentrated in the flat plate area.

3. Resonant vibrations of a flat plate bounded by two circular shells

In this section we consider the plate/shell system of finite length L and assume for simplicity that at $z = 0, L$ the system is subject to simply supported boundary conditions. Then the distribution of the resulting elastic field along the z-axis formed by incident and reflected guided waves can be expressed in the form $\sin(\gamma z)$. Using the condition $\sin(\gamma L) = 0$, we see that $\gamma L = \pi n$, where $n \in \mathbb{N}$.

Let us first analyze resonant vibrations for the case of frequencies higher than the ring frequency ω_r. Then using Equation (6) subject to the condition $\gamma L = \pi n$ and expressing $k_{pl} = \omega/c_{pl}$, with $c_{pl} = \omega^{1/2}(D/\rho_s h)^{1/4}$, one can derive a simple equation for resonant frequencies ω_{0n}. Solving this equation by the perturbation method, one obtains the following expression for the resonant frequencies ω_{0n}, where we consider only modes with $m = 0$,

$$\omega_{0n} = \frac{\pi^2 n^2}{L^2}\left(\frac{D}{\rho_s h}\right)^{1/2}\left[1 + \frac{2^{1/2}c_l'\rho_s{}^{1/2}h^{1/2}L^3}{\pi^3 n^3 D^{1/2}Ra}\right].$$

Note that this equation describes the increase in resonant frequencies of the plate with waveguiding properties in comparison with the case of one-dimensional wave propagation in a flat plate of the same length L but with infinitely large value of the width a. We recall that the distribution of the guided wave field in the transverse direction is described by Equation (3).

Similarly for the case of frequencies lower than the ring one, one can obtain the equation for the system's resonant frequencies

$$\omega_{mn} = \left(\frac{D}{\rho_s h}\right)^{1/2} \left[\frac{\pi^2 (m+1)^2}{a^2} + \frac{\pi^2 n^2}{L^2}\right].$$ (9)

It is remarkable that Equation (9) coincides with the well-known expression for resonant frequencies of simply supported plates having the dimensions L and a (note that in the present paper we assume that $m = 0, 1, 2, 3, \ldots$, whereas in the plate theory usually $m = 1, 2, 3, \ldots$). This coincidence reflects the fact that at frequencies lower than the ring frequency the waveguide effect provided by two adjacent circular shells is very strong and almost the entirety of vibration energy is concentrated in the flat plate area. One should keep in mind, however, that Equation (9) is valid for $n \gg m$, which corresponds to low values of the propagation angle φ.

Note that, although in the above derivations we have used simply supported boundary conditions at $z = 0, L$, the approach we have developed remains valid for any other types of boundary conditions at the edges, such as, for example, free or clamped boundary conditions. In these cases, however, one has to take into account the presence of nonpropagating field components in the z-direction, which are important only near the edges, and the corresponding phase changes for the reflection coefficients of propagating waves.

4. Effect of coupling between the neighboring strip/shell waveguides

Let us now return to the full cylindrical noncircular shell shown in Figure 1 and consider it as a system of two coupled identical parallel strip/shell waveguides separated by the distance $d = a + \pi R$ between their central lines measured along the surface. If we assume that $u_n = u_n(z)$ is a slowly varying amplitude factor which characterises the field in the n-th waveguide, where $n = 1, 2$ in the case considered, then for a shell with a gap on one side, as shown in Figure 1, the coupling takes place on the opposite side only. In such a case the amplitude factors u_n should satisfy the system of two simultaneous coupled equations [Louisell 1960; Schmidt and Coldren 1975; Krylov 1987]:

$$\frac{\partial u_1}{\partial z} - i\gamma u_1 + i\kappa u_2 = 0, \qquad \frac{\partial u_2}{\partial z} - i\gamma u_2 + i\kappa u_1 = 0.$$ (10)

Here γ represents the wavenumber of any chosen guided mode in each waveguide taken separately (uncoupled waveguides), and κ is the wave coupling coefficient for this particular mode, which depends on the strength of mutual penetration of the vibration fields from the neighboring waveguides. In this case κ can be written as [Schmidt and Coldren 1975; Krylov 1987]

$$\kappa = \frac{\left(k_{pl}^2 - \gamma^2\right)\left(\gamma^2 - k_{sh}^2(\gamma)\right)}{\gamma \left[1 + \left(\gamma^2 - k_{sh}^2(\gamma)\right)^{1/2} a/2\right]\left(k_{pl}^2 - k_{sh}^2(\gamma)\right)} \exp\left[-\left(\gamma^2 - k_{sh}^2(\gamma)\right)^{1/2}(d-a)\right].$$ (11)

It is seen from Equation (11) that the coupling coefficient κ is determined primarily by the exponential factor that describes the decay of the field outside the waveguides.

Making an ansatz $u_n = A_n \exp(ikz)$ with $A_n = $ const, for the solution of the simultaneous coupled equations (10), one obtains two types of relationships between A_1 and A_2 that correspond to the wavenumber

mismatch $\Delta\gamma = k - \gamma$. $\Delta\gamma = \kappa$ corresponds to the antisymmetric mode of the coupled system $A_1 = -A_2$, while $\Delta\gamma = -\kappa$ corresponds to the symmetric mode of the coupled system $A_1 = A_2$.

If the gap on the left hand side of the shell shown in Figure 1 is bridged, then both waveguides interact with each other on two sides and the resulting coupling coefficient increases by a factor of two. The values of the mismatch $\Delta\gamma$ here can be obtained from the above expressions replacing κ by 2κ.

Considering propagation of antisymmetric and symmetric modes in the coupled system of finite length L, one can easily derive the expressions for resonant frequencies of our finite noncircular shell. Obviously, the coupling is stronger at frequencies higher than the ring frequency and weaker at frequencies below it, where there is a little energy escape from the flat plate area; see also Equation (11).

For the case of higher frequencies one can use Equation (6) for γ and derive the following expression for resonant frequencies of symmetric and antisymmetric modes, denoted by $+$ and $-$, respectively

$$\omega_{0n} = \left(\frac{\pi n}{L} \pm \kappa\right)^2 \left(\frac{D}{\rho_s h}\right)^{1/2} \left[1 + \frac{2^{1/2} c_t' \rho_s^{1/2} h^{1/2} L^3}{\pi^3 n^3 D^{1/2} Ra}\right].$$
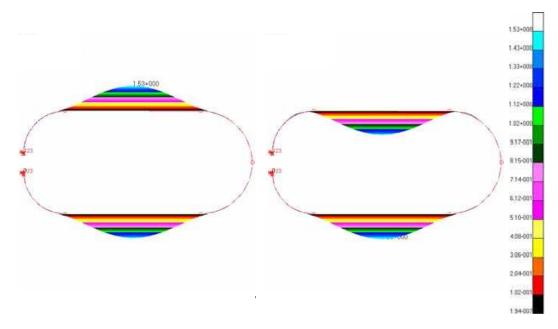
For the case of frequencies lower than the ring frequency, Equation (8) must be used, giving the expression for resonant frequencies of symmetric and antisymmetric modes, denoted by $+$ and $-$, respectively,

$$\omega_{mn} = \left(\frac{D}{\rho_s h}\right)^{1/2} \left[\frac{\pi^2 (m+1)^2}{a^2} + \left(\frac{\pi n}{L} \pm \kappa\right)^2\right]. \tag{12}$$

Note that in the latter case the values of κ are extremely small and can be ignored in Equation (12), since the energy of vibrations is almost entirely confined in the flat plate area. Nevertheless, the symmetric and antisymmetric coupled modes do exist even at negligibly small coupling, though their resonant frequencies are almost indistinguishable. This agrees well with the results of recent experiments [Krylov et al. 2003; 2004] and with the corresponding finite element calculations [Georgiev et al. 2004].

In particular, according to the finite element calculations, the resonant frequencies of symmetric and antisymmetric modes of the noncircular shells under consideration are practically the same. Moreover, they are close to the resonant frequencies of the simply-supported plates with same geometrical dimensions as the flat areas of the shells (Table 1), which agrees with the above theory, Equations (9) and (12), for the case of weak coupling. Furthermore, the numerical routine clearly identifies the lowest-order symmetric and antisymmetric modes in the calculated distributions of vibrations over the shell cross-section; see Figure 3. In the case of higher-order vibration modes, one can also clearly see the similarity of the modal patterns of higher-order modes with the corresponding modes of the simply supported plates; see Figure 4.
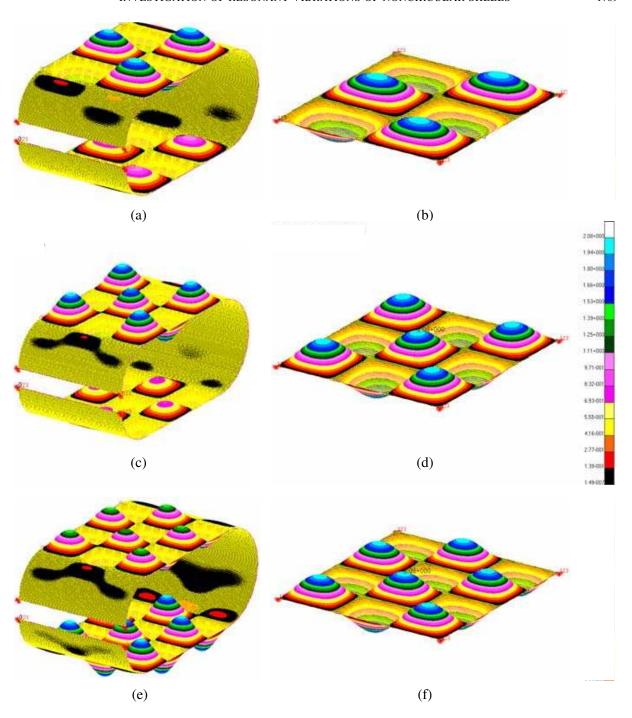
Although in the present paper we analyzed a noncircular shell composed of only two coupled plate areas, the results of the analysis can be easily extended to an arbitrary number of coupled components, in particular to structures with 3, 4 or more quasiflat surfaces. For example, in the case of a noncircular cylindrical shell shaped like a rounded rectangle as shown in Figure 5, one should consider the same quasiflat single waveguides that have been analyzed in the previous sections and then take into account wave coupling in the system of four equal waveguides. Moreover, one can analyze the case of coupled shell systems characterised by spatially changing signs of shell curvature, for example, periodically corrugated thin-walled structures. In this case the waveguide propagation will take place in the quasiflat shell areas around zero curvature, and all other calculations will be the same as above. The obvious
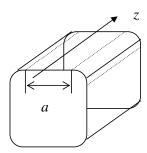
Figure 3. Numerically calculated distributions of the vibration fields for lowest order symmetric (left) and antisymmetric (right) modes of the noncircular cylindrical shell made of steel and having two quasiflat surfaces; see also Figure 1. Shell dimensions: radius $R = 125$ mm, total length $a + 2R = 550$ mm, width $L = 300$ mm, and shell thickness $h = 1.2$ mm; natural frequency is 67.039 Hz.

Simply supported plate			Noncircular shell
Plate mode	Analytical	FE	FE
(1,1)	59.04	59.18	67.04
(1,2)	140.54	140.89	146.22
(2,1)	154.68	155.10	157.67
(2,2)	236.18	236.62	239.15
(1,3)	276.36	277.12	264.51
(3,1)	314.08	314.99	315.33
(2,3)	372.00	372.62	364.40
(3,2)	395.58	396.29	394.69

Table 1. The first eight numerically calculated natural frequencies of noncircular cylindrical shell and of simply-supported rectangular plates of same dimensions as the shell's flat areas. Shell dimensions are as in Figures 3 and 4. Frequencies in Hz.

(a)

(b)

(c)

(d)

(e)

(f)

Figure 4. Numerically calculated higher-order structural modes of noncircular cylindrical shell (left) and of corresponding simply-supported rectangular plates (right); dimensions are as in Figure 3. (a) antisymmetric mode at 364.4 Hz, (b) 372.2 Hz, (c) symmetric mode at 521.45 Hz, (d) 531.95 Hz, (e) antisymmetric mode at 691.68 Hz, (f) 721.99 Hz.

Figure 5. Noncircular cylindrical shell with shape of rounded rectangle that can be considered as a system of four coupled strip/shell waveguides.

advantage of the developed coupled-wave theory approach lies in the fact that it is much easier to estimate resonant frequencies and modal shapes of complex noncircular shells on the basis of understanding the behavior of a single quasiflat waveguide considered as their basic structural component.

5. Conclusions

In the present paper we have demonstrated that resonant vibrations of cylindrical noncircular shells can be easily described analytically using the coupled-wave theory approach utilizing the concept of coupled-waveguide propagation in shells of the same noncircular shape, but having an infinite length. The physical reason for waveguide propagation along quasiflat parts of such shells is the difference between flexural wave velocities in their flat and curved areas.

Using simple approximations for wavenumbers of flexural waves propagating in circular shells with different radii of curvature, the approximate expressions for resonant frequencies have been derived for both uncoupled and coupled finite shell systems. In the case of very weak coupling, which is typical for vibrations at frequencies lower than the ring frequencies of adjacent circular shells, the values of the resultant resonant frequencies are almost entirely determined by resonant properties of the flat plate areas. This agrees well with the experiments and with the results of finite element calculations.

The developed coupled-wave theory approach to the description of resonant flexural vibrations of finite noncircular cylindrical shells can be used for quick estimations of resonant frequencies and normal modes in such shells. It also can be applied to analyzing resonant vibrations in more complex noncircular shells containing arbitrary numbers of quasiflat surfaces.

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