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## ON NONLINEAR KINETIC EFFECTS IN THE VORTEX ARRAY IN SUPERCONDUCTORS

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The motion of vortices in a type II superconductor is accompanied by a heat flux coming from the vortices themselves. It leads to such thermogalvanomagnetic effects like the Nernst, Ettingshausen and Righi-Leduc effects. Moreover, besides the linear thermoelectric Seebeck and Peltier effects, the Hall effect also occurs. That situation seems to be very interesting because it does not take place during common electric conductivity processes but during diffusion and/or creep of magnetic vortices in superconductors. It is known that each vortex line carries a quantum of magnetic field and around it a supercurrent flows. But inside the vortex core a normal current exists. Therefore, the above kinetic linear and nonlinear effects are possible in the vortex array. The paper aims at the formulation of an unconventional thermodynamical model of the above kinetic phenomena including their relaxation properties. As a result we have obtained forms of the constitutive laws related to those processes.

### 1. Introduction

Magnetic flux can penetrate a type II superconductor in the form of Abrikosov vortices (also called flux lines, flux tubes or fluxons) each carrying a quantum of magnetic flux [Tilley 1974; Tinkham 1975; Orlando and Delin 1991; Cyrot and Pavuna 1992; Blatter et al. 1994; Brant 1995]. These tiny vortices tend to arrange themselves in a triangular or quadratic flux-line lattice, which is more or less perturbed by material inhomogeneities that can pin those flux lines. Pinning is caused by imperfections of a crystal lattice of a superconducting material, such as dislocations, point effects, grain boundaries, etc. Hence a honeycomb or quadratic pattern of the vortex array presents some mechanical properties. They come mainly from force interactions observed in the field of vortices. Indeed, the vortices are created by the applied magnetic field which penetrates the superconductor. Now, around each vortex the supercurrent flows, so there are Lorentz-like force interactions among those lines. Such a situation is a cause of the previously mentioned mechanical (stress) field occurring in the medium, besides the common one coming from the type II superconducting material itself. That field near the lower critical magnetic intensity limit  $H_{C1}$  is also of an elastic character. However, if the intensity of the supercurrent is above its critical value, the temperature is sufficiently high, and/or the value of the applied magnetic field tends to its upper critical limit  $H_{C2}$ , a flow (creep or diffusion) of the vortices occurs. The vortex array then loses its configuration and behaves as a fluid.

It has been observed that the vortex motion is accompanied by an energy dissipation. That motion is damped by a force proportional to the velocity of the vortex field point. Hence, except for the elastic properties, the vortex field is also of a viscous character. The resistivity in area of the vortex motion is the same as the resistivity of a current which would flow inside the vortex core where the material is

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in the normal state and where Ohm's law holds true. The result of the above superconducting material properties is a temperature gradient along the vortices and a heat flux that occurs in the vortex field.

In superconductors the vortex lattice mostly consists of a parallel straight vortex line set whose cross section forms the previously mentioned symmetries [Orlando and Delin 1991; Cyrot and Pavuna 1992]. However, recent research shows that the vortex lines can be curved or even tangled along the material [Blatter et al. 1994; Brant 1995]. Moreover, since the vortices form, among others, sets called twisted triplets, twisted quadruplets, single loops or pairs [Schönenberger et al. 1997], the vortex field can be considered even in three dimensions. Because of the fact that each vortex line has a sign (has the definite vorticity), lines of the opposite signs annihilate.

In the paper we focus solely on the kinetic part of interactions occurring in the vortex field of the type II superconductor. The subject of our considerations are the reciprocal links between normal current, supercurrent, heat flux and vortex diffusion flux in the vortex array. Those links are the source of kinetic laws, both linear (Fourier's, Fick's, Ohm's, London's, Soret's, Dufour's, Seebeck's, Peltier's, etc.) and nonlinear (Righi-Leduc's, Ettingshausen-Nernst's, Hall's, etc.), all of which describe thermogalvanoelectromagnetic effects extended on interactions with the supercurrent [Maruszewski 1984; 1988; Sirotnin and Šaskolskaya 1979; Freimuth 2002]. All these laws have a purely kinetic character but from the thermodynamical model presented in the paper laws of relaxation-kinetic nature like the generalized Maxwell-Cattaneo equation, the generalized Fick-Nonnemacher equation, and the generalized first London's equation result as well [Kluitenberg 1981; Restuccia and Kluitenberg 1987].

## 2. The unconventional thermodynamical model

Let us consider the elastic vortex array that exists in the type II superconductor placed in an external magnetic field. For the sake of simplicity we deal solely with soft (depinned) vortices to avoid direct material connections between the superconducting medium and the vortex medium (to ensure our description is related only to the vortex array).

Following the above properties the unconventional (extended-like) thermodynamical model for the viscoelastic field of vortices in the type II superconductor is presented below. We have assumed that the mass density  $\rho$  of the vortex field concerns the density of the material in the normal state as the counterpart in the mixed type II superconductor [Kopnin 2002] (that is, the mass of the normal part of the body related to the total volume of the material), and the energy dissipation occurs only because of the Ohmic-like resistivity (normal-state resistivity) inside the vortex core [Blatter et al. 1994]. Hence the general form of the state vector (the set of independent variables) reads [Maruszewski and Restuccia 1999; Schönenberger et al. 1997]

$$C = \{\varepsilon_{ij}, \varphi, A_i, T, T_{,i}, c, c_{,i}, \psi, \psi^*, \psi_{,i}, \psi_{,i}^*, q_i, j_i^c, j_i^S\}, \quad (1)$$

where  $\varepsilon_{ij}$  denotes the strain tensor,  $\varphi$  and  $A_i$  are the scalar and vector potentials, respectively,  $T$  is the absolute temperature,  $\psi$  is the order parameter (the wave function of a Cooper pair) and  $\psi^*$  is its complex conjugate,  $j_i^S$  is the supercurrent density,  $j_i^c$  is the diffusion flux of vortices and  $q_i$  is the heat flux in the vortex field.  $c$  denotes the concentration of vortices defined as  $c = \frac{\rho}{\rho_{tot}}$ , where  $\rho_{tot}$  is the density of the superconducting material.

The fundamental laws, which govern the set (1), are the *balances*

$$\rho \dot{c} + j_{k,k}^c = 0, \quad \rho \dot{v}_k - \sigma_{jk,j} - \epsilon_{kij} j_i B_j - f_k = 0, \quad \epsilon_{ijk} \sigma_{jk} = 0, \quad \rho \dot{U} - \sigma_{ji} v_{i,j} + q_{k,k} - j_i \mathcal{E}_i - \rho r = 0, \quad (2)$$

the *evolution equations*

$$\dot{q}_k^* - Q_k(C) = 0, \quad \dot{j}_k^c - j_k^c(C) = 0, \quad \dot{j}_k^S - j_k^S(C) = 0, \quad \dot{\psi} - \Psi(C) = 0, \quad \dot{\psi}^* - \Psi^*(C) = 0, \quad (3)$$

where the superimposed asterisk denotes the Zaremba–Jaumann time derivative, *Maxwell's equations*

$$\epsilon_{ijk} E_{k,j} + \frac{\partial B_i}{\partial t} = 0, \quad \epsilon_{ijk} H_{k,j} - j_i = 0, \quad D_{k,k} = 0, \quad B_{k,k} = 0, \quad (4)$$

where  $j_i = j_i^N + j_i^S$ , and the *balance of superelectrons* [van de Ven 1991]

$$\frac{\partial n^S}{\partial t} + j_{k,k}^S = N^S(C), \quad j_{k,k}^S - N^S(C) = (\psi^* \psi_{,k} + \psi \psi_{,k}^*) - [\psi^* \Psi(C) + \psi \Psi^*(C)]. \quad (5)$$

Here  $v_k$  denotes the velocity of the vortex field point,  $\sigma_{ik}$  is the viscoelastic stress tensor,  $j_i^N$  is the normal current,  $B_j$  is the magnetic induction and  $H_j$  is the magnetic field strength,  $f_k$  is the body force,  $U$  is the internal energy density,  $\mathcal{E}_i$  is the electromotive intensity in a moving frame and  $E_i$  is the electromotive intensity in a resting frame,  $r$  is the heat source distribution, and  $n^S$  is the number density of superelectrons (Cooper pairs). The sets (2), (3), (4), and (5) consist of the equation whose form ensures conservation of the vortex mass in the sense indicated above, the momentum balance in the vortex field where elastic interactions are due to the Lorentz force, the equation determining the symmetry of the stress tensor, the internal energy balance of the vortex field where the dissipation term comes only from the Joule-like heat produced by the total current, the first law of thermodynamics, the evolution equation for heat flux, the evolution equation for diffusion flux, the evolution equation for supercurrent, the evolution equations for Cooper pairs wave function as the order parameter (internal variable) evolution equations, the electromagnetic field evolution equations, and the balance equations for superelectrons. Such equations form the structure of an unconventional thermodynamical model based on extended thermodynamics with internal variables [Maruszewski 1990]. The extended-like thermodynamical description has been chosen here since all the interactions run within low temperatures. Moreover, for the electromagnetic field quantities the following relations hold

$$D_k = \epsilon E_k, \quad B_k = \mu_0 H_k, \quad E_k = -\varphi_{,k} - \frac{\partial A_k}{\partial t}, \quad B_k = \epsilon_{ijk} A_{j,i}, \quad \mathcal{E}_i = E_i + \epsilon_{ijk} v_j B_k.$$

In the sequel we follow the assumption that  $\varphi$  vanishes because of gauging [Orlando and Delin 1991; Yeh and Chen 1993].

The use of the second law of thermodynamics in the form of entropy inequality is to ensure solutions of the set (2)–(5) to be related to description of real physical processes.

The *entropy inequality* is taken in its classical form

$$\rho \dot{S} + \Phi_{k,k} - \frac{\rho r}{T} \geq 0, \quad (6)$$

where  $S$  is the entropy density and  $\Phi_k$  denotes the entropy flux.

Now, the inequality (6) gives us a possibility of determining all the constitutive functions which in our case form the set of dependent variables

$$Z = \{\sigma_{ij}, \mu^c, U, Q_k, \Psi, \Psi^*, J_k^c, J_k^S, N^S, S, \Phi_k\}, \quad Z = Z(C). \tag{7}$$

We omit from now on investigations and analysis of the above thermodynamical structure for laws concerning states of the vortex field [Maruszewski 2007; Maruszewski et al. 2007]. Our attention is focused only on laws dealing with processes running in the vortex array, that is, kinetic relations.

A detailed analysis of the entropy inequality and the introduction of the free energy density

$$F = U - TS, \quad F = F^N(\varepsilon_{ij}, T, c, q_i, j_i^c, j_i^S) + F^S(\varepsilon_{ij}, T, c, A_i, \psi, \psi^*, \psi_{,k}, \psi_{,k}^*)$$

[van de Ven 1991; Maruszewski 1998; Maugin 1992] lead us to the residual inequality

$$-\frac{1}{T}q_k T_{,k} - h_c j_k^c c_{,k} + j_i^N \mathcal{E}_i - \rho \frac{\partial F}{\partial q_i} \dot{q}_i - \rho \frac{\partial F}{\partial j_i^c} \dot{j}_i^c - \rho \frac{\partial F}{\partial j_i^S} \dot{j}_i^S - \left[ \rho \frac{\partial F}{\partial \psi} - \left( \rho \frac{\partial F}{\partial \psi_{,k}} \right)_{,k} \right] \frac{\partial \psi}{\partial t} - \left[ \rho \frac{\partial F}{\partial \psi^*} - \left( \rho \frac{\partial F}{\partial \psi_{,k}^*} \right)_{,k} \right] \frac{\partial \psi^*}{\partial t} \geq 0, \tag{8}$$

which stands for the kinetic part of the modelled and described interaction among the elastic, thermal, diffusion, and electromagnetic fields in the vortex array. Here  $h_c = \partial \mu^c / \partial c$  [Maruszewski 1997], where  $\mu^c$  is the vortex chemical potential,

As we see, the residual inequality has a bilinear form and can be presented as follows:

$$J_\alpha X_\alpha \geq 0, \tag{9}$$

where  $J_\alpha$  are the generalized fluxes and  $X_\alpha$  denote generalized forces. Based on the irreversible thermodynamical model, the relation between generalized fluxes and forces is linear

$$J_\alpha = \ell^{\alpha\beta} X_\beta, \tag{10}$$

where the phenomenological coefficients  $\ell^{\alpha\beta}$  satisfy Onsager–Casimir’s reciprocity relations

$$\ell^{\alpha\beta} = \ell^{\beta\alpha}. \tag{11}$$

The use of (9), (10), and (11) in (8) allows us to determine matrices of generalized fluxes, forces and phenomenological coefficients, as follows:

$$J_\alpha = \begin{pmatrix} q_k \\ j_k^c \\ j_k^N \\ \dot{q}_k \\ j_k^c \\ j_k^S \\ j_k \\ \dot{\psi} \\ \dot{\psi}^* \end{pmatrix}, \quad X_\beta = \begin{pmatrix} -(1/T)T_{,k} \\ -h_c c_{,k} \\ \mathcal{E}_i \\ -\rho \partial F / \partial q_i \\ -\rho \partial F / \partial j_i^c \\ -\rho \partial F / \partial j_i^S \\ -[\rho \partial F / \partial \psi - (\rho \partial F / \partial \psi_{,k})_{,k}] \\ -[\rho \partial F / \partial \psi^* - (\rho \partial F / \partial \psi_{,k}^*)_{,k}] \end{pmatrix}, \tag{12}$$

$$\ell^{\alpha\beta} = \begin{pmatrix} \ell^{11} & \ell^{12} & \ell^{13} & 0 & 0 & 0 & 0 & 0 \\ \ell^{21} & \ell^{22} & \ell^{23} & 0 & 0 & 0 & 0 & 0 \\ \ell^{31} & \ell^{32} & \ell^{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ell^{44} & \ell^{45} & \ell^{46} & 0 & 0 \\ 0 & 0 & 0 & \ell^{54} & \ell^{55} & \ell^{56} & 0 & 0 \\ 0 & 0 & 0 & \ell^{64} & \ell^{65} & \ell^{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ell^{77} & \ell^{78} \\ 0 & 0 & 0 & 0 & 0 & 0 & \ell^{87} & \ell^{88} \end{pmatrix}. \tag{13}$$

The basic thermogalvanomagnetic effects and effects which include relaxation features of the considered processes can be described, in the first approximation, if the phenomenological coefficients are assumed in the following form [Maruszewski 1984; Sirotin and Šaskolskaya 1979]  $\ell_{ij}^{\alpha\beta}(H_k) = \ell_{ij}^{\alpha\beta(0)} + \ell_{ijk}^{\alpha\beta(1)} H_k$ . After laborious but routine calculations, the final forms of the expected kinetic relations both without and with relaxation properties (for the sake of simplicity and easy interpretation we present them in the isotropic form assuming that  $\ell_{kj}^{\alpha\beta(0)} = \ell^{\alpha\beta(0)} \delta_{kj}$ ,  $\ell_{kjl}^{\alpha\beta(1)} = \ell^{\alpha\beta(1)} \in_{kjl}$ , [Orlando and Delin 1991; Cyrot and Pavuna 1992; Maruszewski 1984; 1988; 1990; 1997; Sirotin and Šaskolskaya 1979; Freimuth 2002; Kluitenberg 1981; Restuccia and Kluitenberg 1987]) become the *generalized Fourier law*

$$\mathbf{q} = -\kappa \nabla T + \frac{1}{T} \ell \nabla T \times \mathbf{H} - h_c \kappa^c \nabla c + h_c K^c \nabla c \times \mathbf{H} + \kappa^e \mathcal{E} + N \mathcal{E} \times \mathbf{H}, \tag{14}$$

the *generalized Fick law*

$$\mathbf{j}^c = -\frac{1}{T} \kappa^c \nabla T + \frac{1}{T} K^c \nabla T \times \mathbf{H} - \rho D \nabla c + M h_c \nabla c \times \mathbf{H} + \Sigma^c \mathcal{E} + \Gamma^c \mathcal{E} \times \mathbf{H}, \tag{15}$$

the *generalized Ohm law*

$$\mathbf{j}^N = -\frac{1}{T} \kappa^e \nabla T + \frac{1}{T} N \nabla T \times \mathbf{H} - h_c \Sigma^c \nabla c + \rho D \nabla c + h_c \Gamma^c \nabla c \times \mathbf{H} + \sigma \mathcal{E} + R \mathcal{E} \times \mathbf{H}, \tag{16}$$

the *generalized Maxwell–Cattaneo law*

$$\tau^q \dot{\mathbf{q}} = \kappa \nabla T - \frac{1}{T} \ell \nabla T \times \mathbf{H} + h_c \kappa^c \nabla c - h_c K^c \nabla c \times \mathbf{H} + \kappa^e \mathcal{E} - N \mathcal{E} \times \mathbf{H} - \mathbf{q} - D^c \mathbf{j}^c - D^S \mathbf{j}^S, \tag{17}$$

the *generalized Fick–Nonnenmacher law*

$$\tau^c \dot{\mathbf{j}}^c = \frac{1}{T} \kappa^c \nabla T - \frac{1}{T} K^c \nabla T \times \mathbf{H} + \rho D \nabla c - M h_c \nabla c \times \mathbf{H} + \Sigma^c \mathcal{E} - \Gamma^c \mathcal{E} \times \mathbf{H} - D^q \mathbf{q} - \mathbf{j}^c - D^{Sq} \mathbf{j}^S, \tag{18}$$

and the *generalized first London equation*

$$\tau^S \dot{\mathbf{j}}^S = P^T T \nabla T - \frac{1}{T} R^T \nabla T \times \mathbf{H} + P^c \nabla c - h_c R^c \nabla c \times \mathbf{H} + \frac{1}{\mu_0 \lambda_0^2} \mathcal{E} - R^e \mathcal{E} \times \mathbf{H} - D^q \mathbf{q} - \mathbf{j}^S - D^{cS} \mathbf{j}^c. \tag{19}$$

In Eqs. (14), (15), (16), (17), (18), and (19) we recognize the following phenomena and effects described by definite coefficients:

$\kappa$	heat conductivity	$D^c$	thermodiffusive constant
$\ell$	Righi–Leduc effect coefficient	$D^S$	thermosupercurrent constant

$h^c$	diffusion constant [Sirotin and Šaskolskaya 1979]	$\tau^c$	diffusive relaxation time
$\kappa^c$	Dufour–Soret effect coefficient	$D^q$	diffusive-thermal constant
$K^c$	magnetothermodiffusive kinetic coefficient	$D^{Sq}$	diffusive-supercurrent constant
$\kappa^e$	Peltier effect coefficient	$\tau^S$	supercurrent relaxation time
$N$	Ettingshausen–Nernst effect coefficient	$P^T$	superthermal constant
$D$	diffusion coefficient	$R^T$	supermagnetothermal constant
$M$	magnetodiffusive kinetic coefficient	$P^c$	superdiffusive constant
$\Sigma^c$	electrodiffusive kinetic coefficient	$R^c$	supermagnetodiffusive constant
$\Gamma^c$	electromagnetodiffusive kinetic coefficient	$R^e$	superelectromagnetic constant
$\sigma$	electric conductivity	$D^{qS}$	superthermal kinetic constant
$R$	Hall constant	$D^{cS}$	superdiffusive kinetic constant
$\tau^q$	thermal relaxation time		

In addition, (3), (12), and (13) still yield the generalized Ginzburg–Landau kinetic equation as well [Orlando and Delin 1991; Maruszewski 1998]. Since we have, however, decided that the gauge can be chosen such that the scalar electric potential vanishes [Yeh and Chen 1993], then we use the experimental observations that the supercurrent exists reasonably long in time and we assume that the local density of Cooper pairs to be constant (this approach is true in many practical situations where the local fluctuations of the density of superelectrons in steady state are of such length and time scales that they are too small to be of engineering interest [Orlando and Delin 1991]). That fact leads to the conclusion that  $X_7 = X_8 = 0$  in (12). Hence, we assume that the generalized Ginzburg–Landau equation in such a situation (within the model of interactions presented in the paper) can be neglected.

### 3. Conclusions

The paper has proved, in the opinion of the author, that the dynamics of the vortex field in a type II superconductor is very rich in interesting phenomena. The kinetic part of interactions and processes running in that array show that reciprocal links among heat transfer, diffusion of vortices and normal electron conduction (the Ohmic-like current) with relaxation of heat flux, diffusion flux, and supercurrent result in known and unknown linear and nonlinear kinetic effects. Those effects, particularly nonlinear ones, demand detailed physical analysis and interpretation. Finally, experimentation should verify and answer the fundamental question: do all the effects presented in (14)–(19) really exist?

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