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INFLUENCE OF CORE PROPERTIES ON THE FAILURE OF COMPOSITE SANDWICH BEAMS

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The initiation of failure in composite sandwich beams is heavily dependent on properties of the core material. Several core materials, including PVC foams and balsa wood were characterized. The various failure modes occurring in composite sandwich beams are described and their relationship to the relevant core properties is explained and discussed. Under flexural loading of sandwich beams, plastic yielding or cracking of the core occurs when the critical yield stress or strength (usually shear) of the core is reached. Indentation under localized loading depends principally on the square root of the core yield stress. The critical stress for facesheet wrinkling is related to the core Young's and shear moduli in the thickness direction. Experimental mechanics methods were used to illustrate the failure modes and verify analytical predictions.

1. Introduction

The overall performance of sandwich structures depends in general on the properties of the facesheets, the core, the adhesive bonding the core to the skins, and geometric dimensions. Sandwich beams under general bending, shear and in-plane loading display various failure modes. Their initiation, propagation, and interaction depend on the constituent material properties, geometry, and type of loading. Failure modes and their initiation can be predicted by conducting a thorough stress analysis and applying appropriate failure criteria in the critical regions of the beam. This analysis is difficult because of the nonlinear and inelastic behavior of the constituent materials and the complex interactions of failure modes. Possible failure modes include tensile or compressive failure of the facesheets, debonding at the core/facesheet interface, indentation failure under localized loading, core failure, wrinkling of the compression facesheet, and global buckling. Following initiation of a particular failure mode, this mode may trigger and interact with other modes and final failure may follow a different failure path. A general review of failure modes in composite sandwich beams was given in [Daniel et al. 2002]. Individual failure modes in sandwich columns and beams are discussed in [Abot et al. 2002; Gdoutos et al. 2002b; 2003]. Of all the factors influencing failure initiation and mode, the properties of the core material are the most predominant.

Commonly used materials for facesheets are composite laminates and metals, while cores are made of metallic and nonmetallic honeycombs, cellular foams, balsa wood, or truss.

The facesheets carry almost all of the bending and in-plane loads while the core helps to stabilize the facesheets and defines the flexural stiffness and out-of-plane shear and compressive behavior. A number of core materials, including aluminum honeycomb, various types of closed-cell PVC foams,

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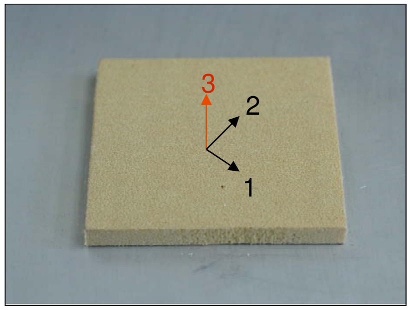

Figure 1. Material coordinate system for sandwich cores.

a polyurethane foam, foam-filled honeycomb and balsa wood, were characterized under uniaxial and biaxial states of stress.

In the present work, failure modes were investigated experimentally in axially loaded composite sandwich columns and sandwich beams under bending. Failure modes observed and studied include indentation failure, core failures, and facesheet wrinkling. The transition from one failure mode to another for varying loading or state of stress and beam dimensions was discussed. Experimental results were compared with analytical predictions.

2. Characterization of core materials

The core materials characterized were four types of a closed-cell PVC foam (Divinycell H80, H100, H160 and H250, with densities of 80, 100, 160 and 250kg/m^3, respectively), an aluminum honeycomb (PAMG 8.1-3/16 001-P-5052, Plascore Co.), a polyurethane foam, a foam-filled honeycomb, and balsa wood. Of these, the low density foam cores are quasi-isotropic, while the high density foam cores, the honeycombs, and balsa wood are orthotropic with the 1-2 plane parallel to the facesheets being a plane of isotropy and the through-thickness direction (3-direction) a principal axis of higher stiffness, as shown in Figure 1. All core materials were characterized in uniaxial tension, compression, and shear along the in-plane and through-thickness directions. Typical stress-strain curves are shown in Figures 2 and 3. Some

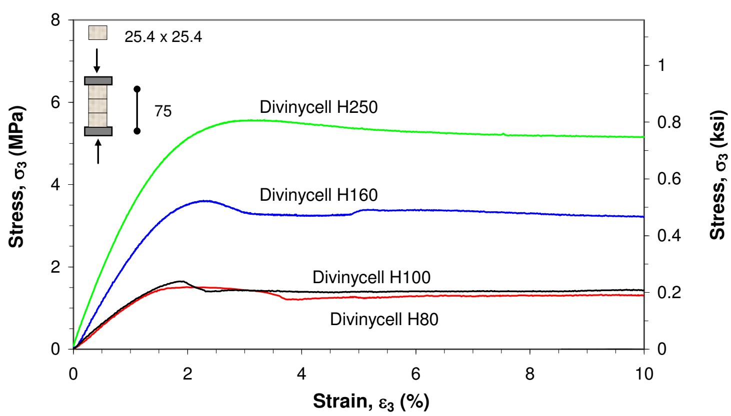

Figure 2. Stress-strain curves of PVC foam cores under compression in the through-thickness direction.

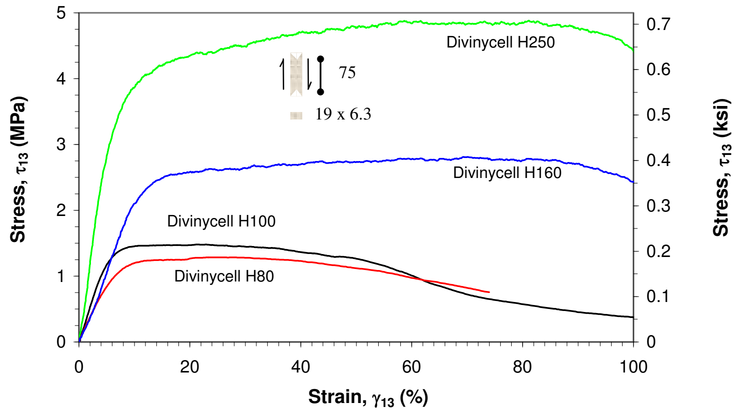

Figure 3. Shear stress-strain curves of PVC foam cores under through-thickness shear.

of their characteristic properties are tabulated in Table 1. The core materials (honeycomb or foam) were provided in the form of 25.4 mm thick plates. The honeycomb core was bonded to the top and bottom facesheets with FM73 M film adhesive and the assembly was cured under pressure in an oven following the recommended curing cycle for the adhesive. The foam cores were bonded to the facesheets using a commercially available epoxy adhesive (Hysol EA 9430) [Daniel and Abot 2000]. Beam specimens 25.4 mm wide and of various lengths were cut from the sandwich plates.

Two core materials, Divinycell H100 and H250 were fully characterized under multiaxial stress conditions [Gdoutos et al. 2002a]. A series of biaxial tests were conducted including constrained strip specimens in tension and compression with the strip axis along the through-thickness and in-plane directions; constrained thin-wall ring specimens in compression and torsion; thin-wall tube specimens in tension and torsion; and thin-wall tube specimens under axial tension, torsion and internal pressure. From these tests and uniaxial results in tension, compression, and shear, failure envelopes were constructed. It

Sandwich core material	ρ	E_1	E_2	E_3	G_{13}	F_{1c}	F_{1t}	F_{2c}	F_{3c}	F_5
Divinycell H80	80	77	77	110	18	1.0	2.3	1.0	1.4	1.1
Divinycell H100	100	95	95	117	25	1.4	2.7	1.4	1.6	1.4
Divinycell H160	160	140	140	250	26	2.5	3.7	2.5	3.6	2.8
Divinycell H250	250	255	245	360	73	4.5	7.2	4.5	5.6	4.9
Balsa Wood CK57	150	110	110	4600	60	0.8	1.2	0.8	9.7	3.7
Aluminum Honeycomb PAMG 5052	130	8.3	6.0	2200	580	0.2	1.2	0.2	11.8	3.5
Foam Filled Honeycomb Style 20	128	25	7.6	240	8.7	0.4	0.5	0.3	1.4	0.75
Polyurethane FR-3708	128	38	38	110	10	1.2	1.1	1.1	1.8	1.4

Table 1. Properties of sandwich core materials: the density, ρ (in units of kg/m^3); and the in-plane moduli, E_1 and E_2, the out of plane modulus, E_3, the transverse shear modulus, G_{13}, the in-plane compressive strength, F_{1c}, the in-plane tensile strength, F_{1t}, the in-plane compressive strength, F_{2c}, the out of plane compressive strength, F_{3c}, and the transverse shear strength, F_5 (all in units of MPa).

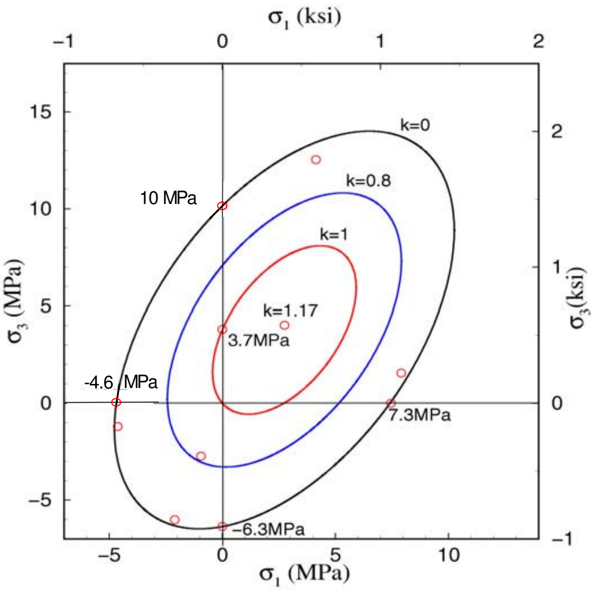

Figure 4. Failure envelopes predicted by the Tsai–Wu failure criterion for PVC foam (Divinycell H250) for $k = 0$, 0.8 and 1, and experimental results ($k = \tau_{13}/F_{13} = \tau_5/F_5$).

was shown that the failure envelopes were described well by the Tsai–Wu criterion [1971], as shown in Figure 4.

The Tsai–Wu criterion for a general two-dimensional state of stress on the 1-3 plane is expressed as

$$f_1\sigma_1 + f_3\sigma_3 + f_{11}\sigma_1^2 + f_{33}\sigma_3^2 + 2f_{13}\sigma_1\sigma_3 + f_{55}\tau_5^2 = 1, \tag{1}$$

where

$$f_1 = \frac{1}{F_{1t}} - \frac{1}{F_{1c}}, \qquad f_3 = \frac{1}{F_{3t}} - \frac{1}{F_{3c}}, \qquad f_{11} = \frac{1}{F_{1t}F_{1c}},$$

$$f_{33} = \frac{1}{F_{3t}F_{3c}}, \qquad f_{13} = -\frac{1}{2}(f_{11}f_{33})^{1/2}, \qquad f_{55} = \frac{1}{F_5^2}.$$

Here F_{1t}, F_{1c}, F_{3t}, and F_{3c} are the tensile and compressive strengths in the in-plane (1, 2) and out-of-plane (3) directions, and F_5 is the shear strength on the 1-3 plane.

Setting $\tau_5 = kF_5$, we can rewrite (1) as

$$f_1\sigma_1 + f_3\sigma_3 + f_{11}\sigma_1^2 + f_{33}\sigma_3^2 + 2f_{13}\sigma_1\sigma_3 = 1 - k^2. \tag{2}$$

It was assumed that the failure behavior of all core materials can be described by the Tsai–Wu criterion. Failure envelopes of all core materials constructed from the values of F_{1t}, F_{1c} and F_5 are shown in Figure 5. Note that the failure envelopes of all Divinycell foams are elongated along the σ_1-axis, which indicates that these materials are stronger under normal longitudinal stress than in-plane shear stress. Aluminum honeycomb and balsa wood show the opposite behavior. For all materials, the most critical combinations of shear and normal stress fall in the second and third quadrants (the failure envelopes are symmetrical with respect to the σ_1-axis).

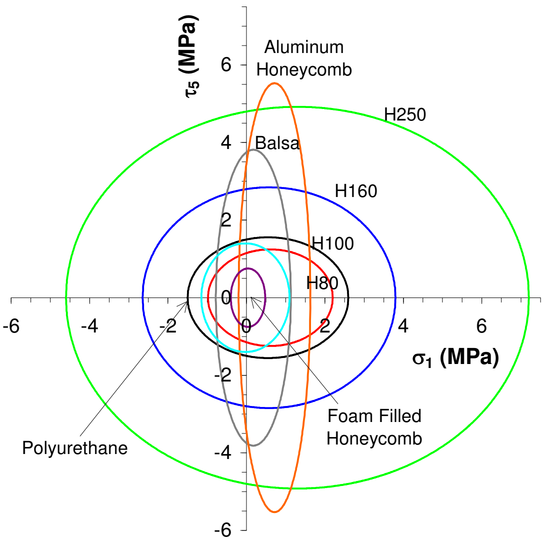

Figure 5. Failure envelopes for various core materials based on the Tsai–Wu failure criterion for interaction of normal and shear stresses.

3. Core failures

The core is primarily selected to carry the shear loading. Core failure by shear is a common failure mode in sandwich construction [Allen 1969; Hall and Robson 1984; Zenkert and Vikström 1992; Zenkert 1995; Daniel et al. 2001a; 2001b; Sha et al. 2006]. In short beams under three-point bending the core is mainly subjected to shear, and failure occurs when the maximum shear stress reaches the critical value (shear strength) of the core material. In long-span beams the normal stresses become of the same order of magnitude as, or even higher than the shear stresses. In this case, the core in the beam is subjected to a biaxial state of stress and fails according to an appropriate failure criterion. It was shown earlier that failure of the PVC foam core Divinycell H250 can be described by the Tsai–Wu failure criterion [Gdoutos et al. 2002a; Bezazi et al. 2007].

For a sandwich beam of rectangular cross section, with facesheets and core materials displaying linear elastic behavior, subjected to a bending moment, M, and shear force, V, the in-plane maximum normal stress, σ, and shear stress, τ, in the core, for a low stiffness core and thin facesheets are given by [Daniel et al. 2001a]

$$\sigma = \frac{PL}{C_1 bd^2}\left(\frac{E_c}{E_f}\right)\frac{h_c}{h_f}, \qquad \tau = \frac{P}{C_2 bh_c}, \tag{3}$$

where

$$M = \frac{PL}{C_1}, \qquad V = \frac{P}{C_2}, \tag{4}$$

P being the applied concentrated load, L the length of beam, E_f and E_c the Young's moduli of the facesheet and core material, h_f and h_c the thicknesses of the facesheets and core, d the distance between the centroids of the facesheets, b the beam width, and C_1 and C_2 constants depending on the loading configuration ($C_1 = 4$, $C_2 = 2$ for three-point bending; $C_1 = C_2 = 1$ for a cantilever beam).

The maximum normal stress, σ, for a beam under three-point bending occurs under the load, while for a cantilever beam under end loading it occurs at the support. The shear stress, τ, is constant along the beam span and through the core thickness, as verified experimentally [Daniel and Abot 2000; Daniel et al. 2002].

When the normal stress in the core is small relative to the shear stress, it can be assumed that core failure occurs when the shear stress reaches a critical value. Furthermore, failure in the facesheets occurs when the normal stress reaches its critical value, usually the facesheet compressive strength. Under such circumstances we obtain from (3) that failure mode transition from shear core failure to compressive facesheet failure occurs when

$$\frac{L}{h_f} = C\frac{F_f}{F_{cs}}, \tag{5}$$

where F_f is the facesheet strength in compression or tension, F_{cs} is the core shear strength, and C is a constant ($C = 2$ for a beam under three-point bending; $C = 1$ for a cantilever beam under an end load).

When the left-hand term of (5) is smaller than the right hand term, failure occurs by core shear, whereas in the reverse case failure occurs by facesheet tension or compression.

The deformation and failure mechanisms in the core of sandwich beams have been studied experimentally by means of moiré gratings and photoelastic coatings [Daniel and Abot 2000; Daniel et al. 2001a; 2001b; Gdoutos et al. 2001; 2002b; Abot and Daniel 2003]. Figure 6 shows moiré fringe patterns in the core of a sandwich beam under three-point bending for an applied load that produces stresses in the core within the linear elastic range. The moiré fringe patterns corresponding to the u (horizontal) and w (through-the-thickness) displacements away from the applied load consist of nearly parallel and equidistant fringes from which it follows that

$$\varepsilon_x = \frac{\partial u}{\partial x} \cong 0, \qquad \varepsilon_z = \frac{\partial w}{\partial z} \cong 0, \qquad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \text{constant}. \tag{6}$$

Thus, the core is under nearly uniform shear stress. This is true only in the linear range, as will be illustrated below.

Figure 7 shows photoelastic coating fringe patterns for a beam under three-point bending. The fringe pattern for a low applied load (2.3 kN) is nearly uniform, indicating that the shear strain (stress) in the

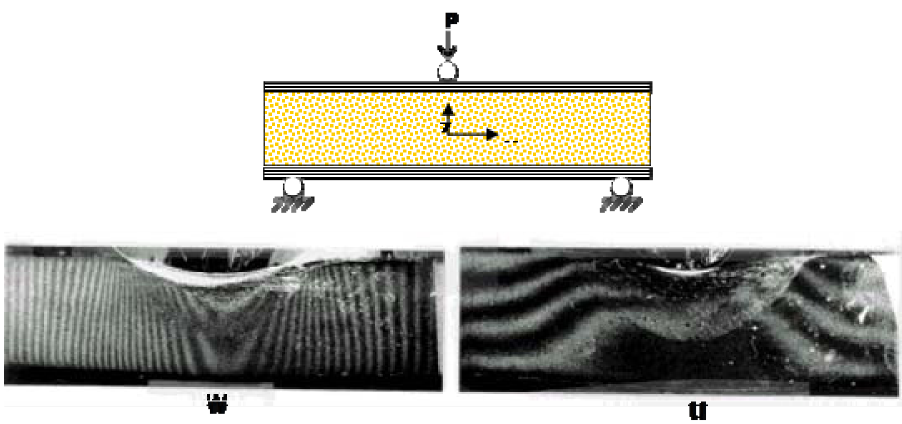

Figure 6. Moiré fringe patterns corresponding to horizontal and vertical displacements in sandwich beam under three-point bending (12 lines/mm, Divinycell H250 core).

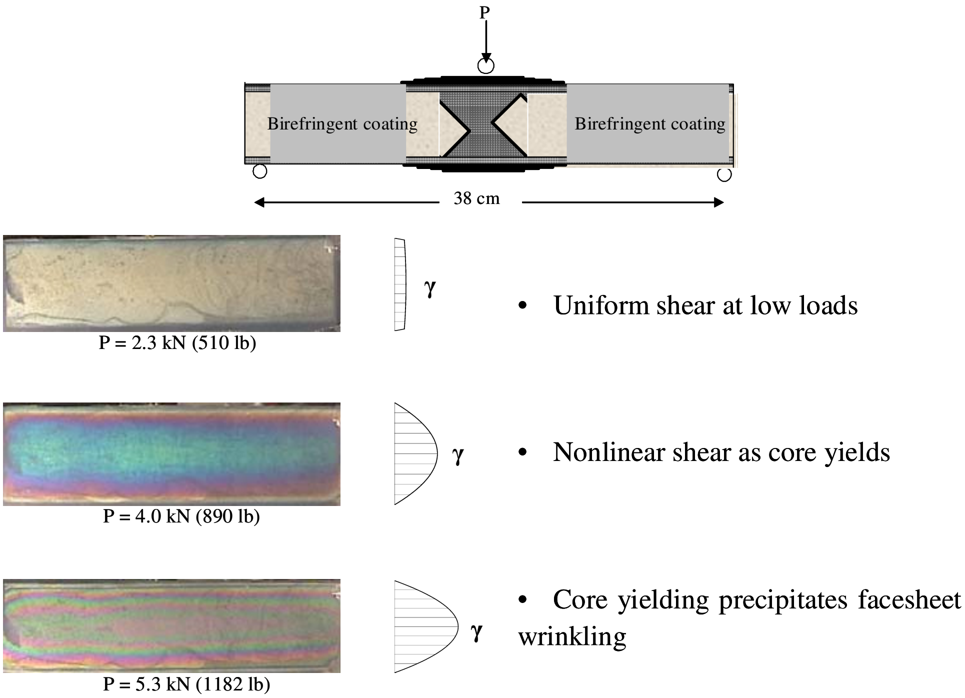

Figure 7. Isochromatic fringe patterns in birefringent coating of sandwich beam under three-point bending (Divinycell H250 core).

core is constant. This pattern remains uniform up to an applied load of 3.3 kN which corresponds to an average shear stress in the core of 2.55 MPa. This is close to the proportional limit of the shear stress-strain curve of the core material (Figure 3). For higher loads, the core begins to yield and the shear strain becomes highly nonuniform peaking at the center and causing plastic flow. The onset of core failure in beams is directly related to the core yield stress in the thickness direction. A critical condition for the core occurs at points where shear stress is combined with compressive stress.

The deformation and failure of the core is obviously dependent on its properties and especially its anisotropy. Honeycomb and balsa wood cores are highly anisotropic with much higher stiffness and strength in the thickness direction, a desirable property. Figure 8 shows isochromatic fringe patterns in the photoelastic coating and the corresponding load deflection curve for a composite sandwich beam under three-point bending. The beam consists of glass/vinylester facesheets and balsa wood core. The fringe patterns indicate that the shear deformation in the core is initially nearly uniform, but it becomes nonuniform and concentrated in a region between the support and the load at a distance of approximately one beam depth from the support. The pattern at the highest load shown is indicative of a vertical crack along the cells of the balsa wood core. The loads corresponding to the fringe patterns are marked on the load deflection curve. It is seen that the onset of nonlinear behavior corresponds to the beginning of fringe concentration and failure initiation in the critical region of the core.

Figure 9 shows the damaged region of the beam. Although the fringe patterns did not show that, it appears that a crack was initiated near the upper facesheet/core interface and propagated parallel to it.

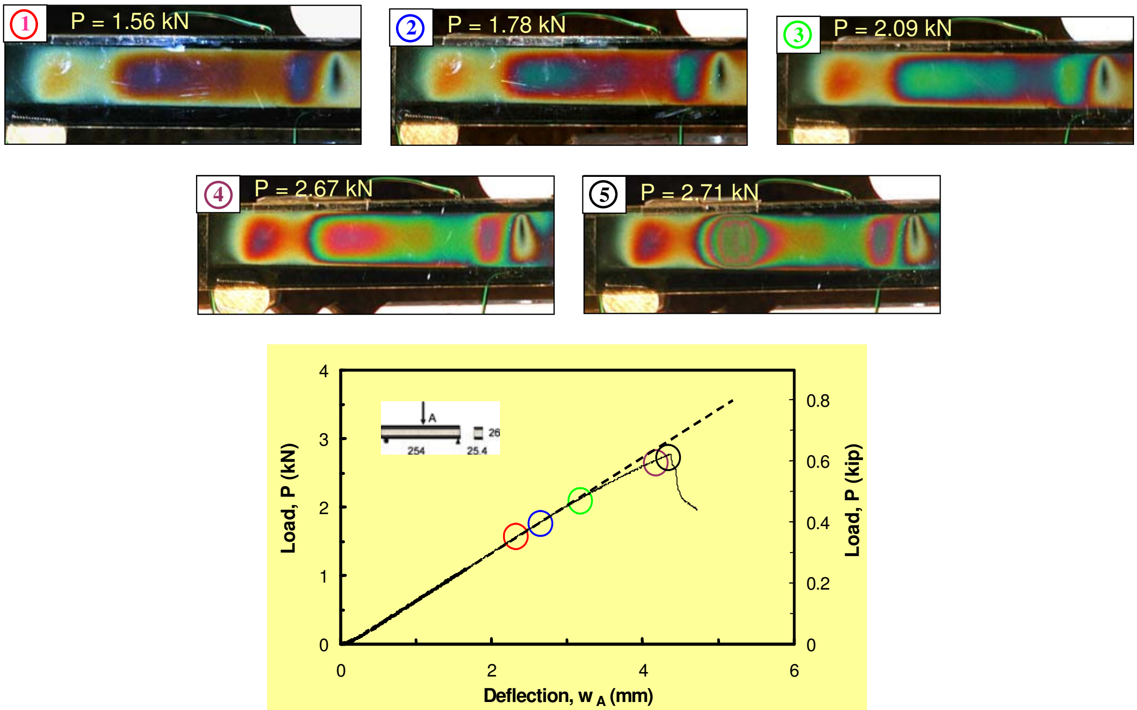

Figure 8. Isochromatic fringe patterns in photoelastic coating and load deflection curve of a composite sandwich beam under three-point bending (glass/vinylester facesheets; balsa wood core).

Figure 9. Cracking in balsa wood core of sandwich beam under three-point bending near support.

The crack traveled for some distance and then turned downwards along the cell walls of the core until it approached the lower interface. It then traveled parallel to the interface towards the support point.

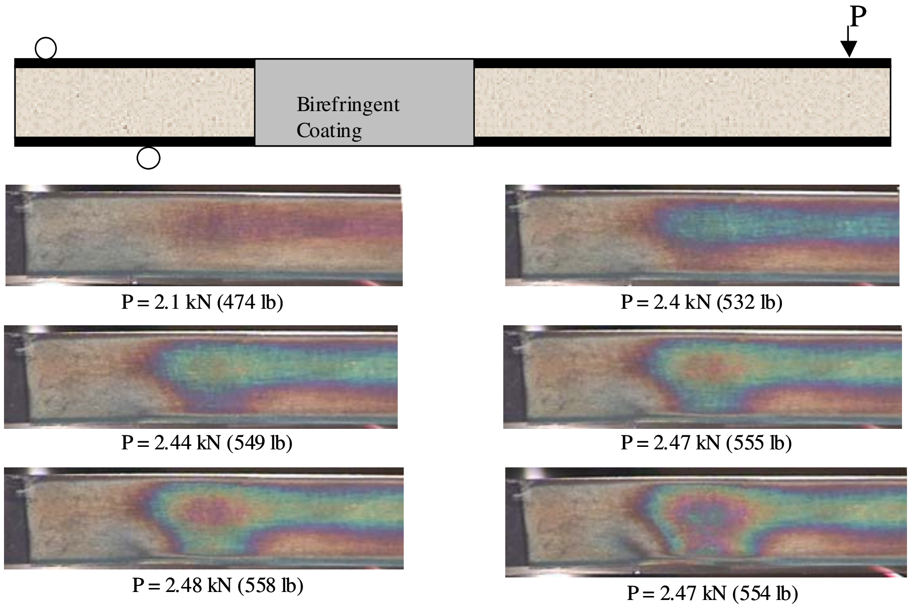

Figure 10. Isochromatic fringe patterns in birefringent coating of cantilever sandwich beam under end loading.

Core failure is accelerated when compressive and shear stresses are combined. This critical combination is evident from the failure envelope of Figure 4. The criticality of the compression/shear stress biaxiality was tested with a cantilever sandwich beam loaded at the free end. The isochromatic fringe patterns of the birefringent coating in Figure 10 show how the peak birefringence moves towards the fixed end of the beam at the bottom where the compressive strain is the highest and superimposed on the shear strain. Plastic deformation of the core, whether due to shear alone or a combination of compression and shear, degrades the supporting role of the core and precipitates other more catastrophic failure modes, such as facesheet wrinkling.

4. Indentation failure

Indentation failure in composite sandwich beams occurs under concentrated loads, especially in the case of soft cores. Under such conditions, significant local deformation takes place of the loaded facesheet into the core, causing high local stress concentrations. The indentation response of sandwich panels was first modeled by [Meyer-Piening 1989] who assumed linear elastic bending of the loaded facesheet resting on a Winkler foundation (core). Soden [1996] modeled the core as a rigid-perfectly plastic foundation, leading to a simple expression for the indentation failure load. Shuaeib and Soden [1997] predicted indentation failure loads for sandwich beams with glass-fiber-reinforced plastic facesheets and thermoplastic foam cores. The problem was modeled as an elastic beam, representing the facesheet, resting on an elastic-plastic foundation representing the core. Thomsen and Frostig [1997] studied the local bending effects in sandwich beams experimentally and analytically. The indentation failure of composite sandwich beams was also studied by [Anderson and Madenci 2000; Petras and Sutcliffe 2000; Gdoutos et al. 2002b].

For linear elastic behavior, the core is modeled as a layer of linear tension/compression springs. The stress at the core/facesheet interface is proportional to the local deflection w, $\sigma = kw$, where the foundation modulus k is given by

$$k = 0.64\frac{E_c}{h_f}\sqrt[3]{\frac{E_c}{E_f}}, \tag{7}$$

and where E_f and E_c are the facesheet and core moduli, respectively, and h_f is the facesheet thickness. Initiation of indentation failure occurs when the core under the load starts yielding. The load at core yielding was calculated as

$$P_{cy} = 1.70\sigma_{cy}bh_f\sqrt[3]{\frac{E_f}{E_c}}, \tag{8}$$

where σ_{cy} is the yield stress of the core, and b is the beam width.

Core yielding causes local bending of the facesheet which, combined with global bending of the beam, results in compression failure of the facesheet. The compressive failure stress in the facesheet is related to the critical beam loading P_{cr} by

$$\sigma_f = F_{fc} = \frac{9P_{\mathrm{cr}}^2}{16b^2h_f^2F_{cc}} + \frac{P_{\mathrm{cr}}L}{4bh_f(h_f+h_c)}, \tag{9}$$

where h_c is the core thickness, L the span length, b the beam width, and F_{cc}, F_{fc} the compressive strengths of the core (in the thickness direction) and facesheet materials, respectively. In the above equation, the first term on the right hand side is due to local bending following core yielding and indentation and the second term is due to global bending.

The onset and progression of indentation failure is illustrated by the moiré pattern for a sandwich beam under three-point bending (Figure 11).

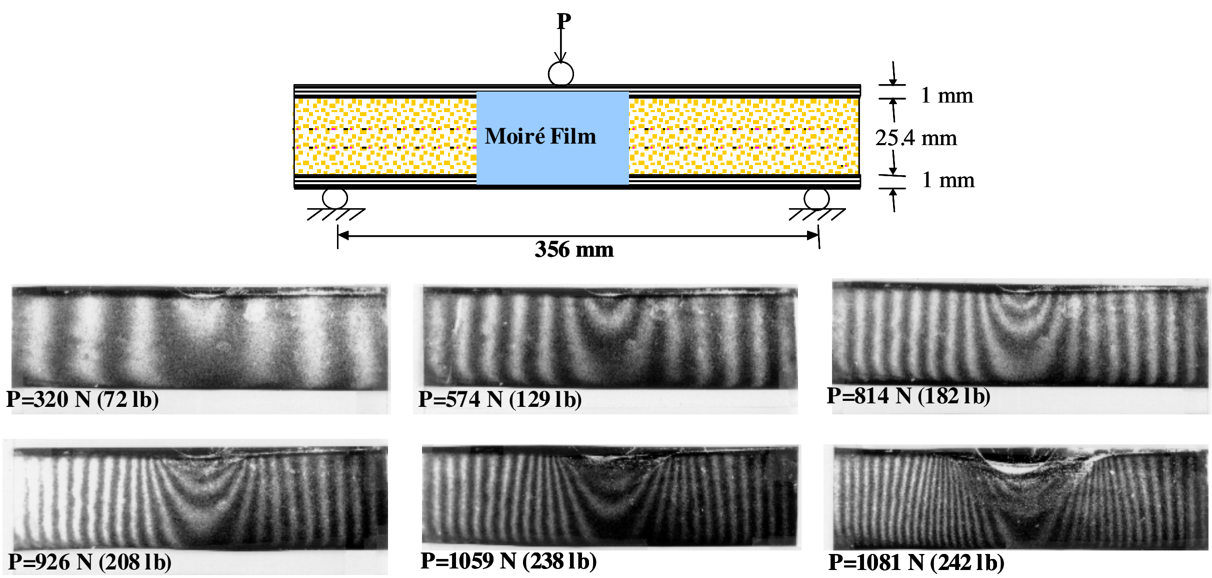

Figure 11. Moiré fringe patterns in sandwich beam with foam core corresponding to vertical displacements at various applied loads (11.8 lines/mm grating; carbon/epoxy facesheets; Divinycell H100 core).

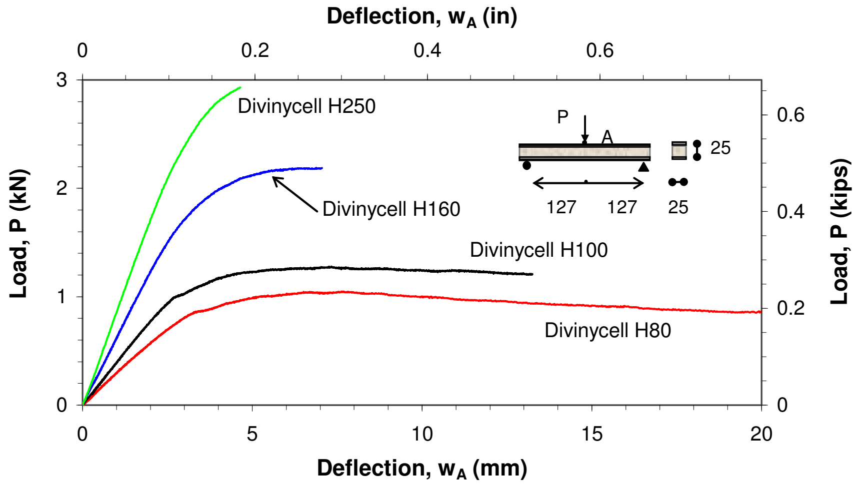

Figure 12. Load versus deflection under load of sandwich beam under three-point bending (carbon/epoxy facesheets, Divinycell cores).

Figure 12 shows load displacement curves for beams of the same dimensions but different cores. The displacement in these curves represents the sum of the global beam deflection and the more dominant local indentation. Therefore, the proportional limit of the load-displacement curves is a good indication of initiation of indentation.

The measured critical indentation loads in Figure 12 were compared with predicted values using (9), which can be approximated as [Soden 1996]

$$P_{\mathrm{cr}} \cong \tfrac{4}{3} b h_f \sqrt{F_{fc}\sigma_{cy}}. \tag{10}$$

Thus, the critical indentation load is proportional to the square root of the core material yield stress. The results obtained are compared in Table 2. The approximate theory with the assumption of rigid-perfectly plastic behavior overestimates the indentation failure load for soft cores, but it underestimates it for stiff cores.

5. Facesheet wrinkling failure

Wrinkling of sandwich beams subjected to compression or bending is defined as a localized short-wave length buckling of the compression facesheet. Wrinkling may be viewed as buckling of the compression facesheet supported on an elastic or elastoplastic continuum [Gdoutos et al. 2003]. It is a common failure mode leading to loss of the beam stiffness. The wrinkling phenomenon is characterized by the interaction

Indentation Load (N)	H80	H100	H160	H250
Measured	1050	1250	2150	2900
Calculated	1370	1500	2000	2380

Table 2. Critical indentation loads for sandwich beams with different cores under three-point bending.

between the core and the facesheet of the sandwich panel. Thus, the critical wrinkling load is a function of the stiffnesses of the core and facesheet, the geometry of the structure, and the applied loading.

A large number of theoretical and experimental investigations has been reported on wrinkling of sandwich structures. Some of the early works were presented and compiled in [Plantema 1966; Allen 1969]. Hoff and Mautner [1945] tested sandwich panels in compression and gave an approximate formula for the wrinkling stress, which depends only on the elastic moduli of the core and facesheet materials. Heath [1960] extended the theory for end loaded plates and proposed a simple expression for facesheet wrinkling in sandwich plates with isotropic components. The theory does not account for shear interaction between the facesheets and the core and thus is more applicable to compressively loaded sandwich columns and to beams under pure bending. Benson and Mayers [1967] developed a unified theory for the study of both general instability and facesheet wrinkling simultaneously for sandwich plates with isotropic facesheets and orthotropic cores. This theory was extended in [Hadi and Matthews 2000] to solve the problem of wrinkling of anisotropic sandwich panels. More studies on the wrinkling of sandwich plates are found in [Vonach and Rammerstorfer 2000; Fagerberg 2004; Birman and Bert 2004; Meyer-Piening 2006; Lopatin and Morozov 2008]. The critical wrinkling stress given in [Hoff and Mautner 1945] is

$$\sigma_{\mathrm{cr}} \cong c\sqrt[3]{E_{f1}E_{c3}G_{c13}}, \tag{11}$$

where E_{f1} and E_{c3} are the Young's moduli of facesheet and core, in the axial and through-thickness directions, respectively, G_{c13} is the shear modulus of the core on the 1-3 plane, and c is a coefficient, usually varying in the range of 0.5–0.9.

In the relation above, the core moduli are the initial ones while the material is in the linear range. After the core yields and its stiffnesses degrade (E'_c, G'_c), it does not provide adequate support for the facesheet, thereby precipitating facesheet wrinkling. The reduced critical stress after core degradation is

$$\sigma_{\mathrm{cr}} \cong c\sqrt[3]{E_f E'_c G'_c}. \tag{12}$$

Heath's original expression was modified here for a one-dimensional beam and by considering only the facesheet modulus along the axis of the beam and the core modulus in the through-thickness direction. The critical wrinkling stress can then be obtained by

$$\sigma_{\mathrm{cr}} = \left[\frac{2}{3}\frac{h_f}{h_c}E_{c3}E_f\right]^{1/2}. \tag{13}$$

Sandwich columns were subjected to end compression and strains were measured on both faces. The stress-strain curves for three columns with aluminum honeycomb, Divinycell H100 and Divinycell H250 cores are shown in Figure 13. Photographs of these columns after failure are shown in Figure 14. The wrinkling stress is defined as the stress at which the strain on the convex side of the panel reaches a maximum value. Note that the column with the honeycomb core failed by facesheet compression and not by wrinkling. The measured failure stress of 1,550 MPa is much lower than the critical wrinkling stresses of 2,850 MPa and 2,899 MPa predicted by (11) and (13), the former for $c = 0.5$. The columns with Divinycell H100 and H250 foam cores failed by facesheet wrinkling, as seen in the stress-strain curves of Figure 13. The measured wrinkling stresses at maximum strain for the Divinycell H100 and H250 cores were 627 MPa and 1,034 MPa, respectively, and are close to the values of 667 MPa and

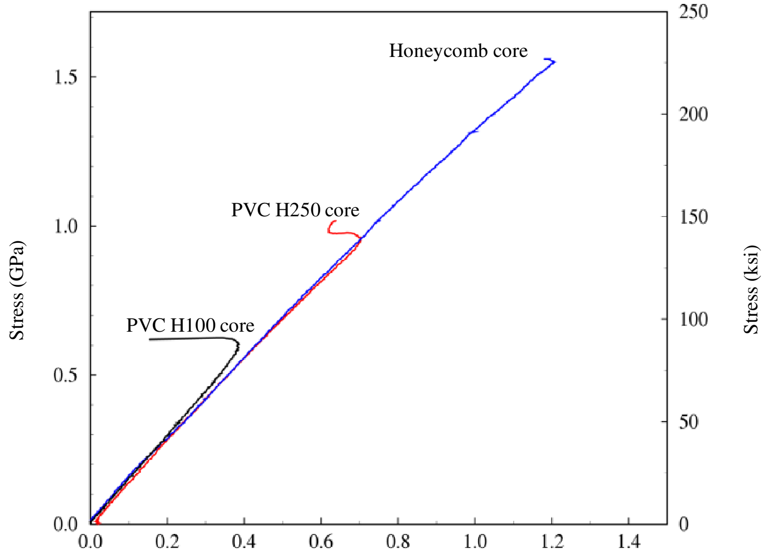

Figure 13. Compressive stress-strain curves for sandwich columns with different cores.

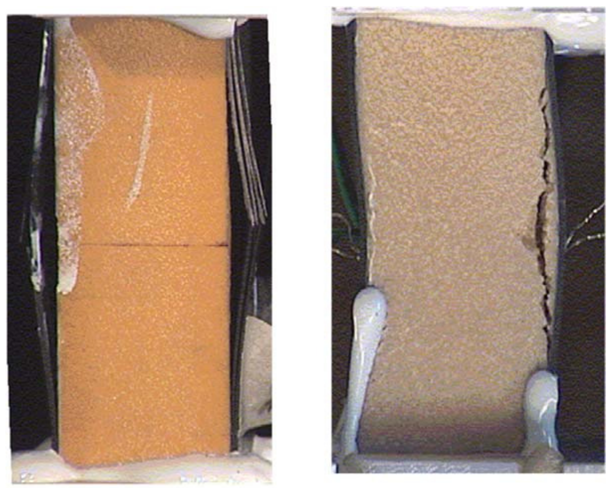

Figure 14. Failure of sandwich columns with two different cores.

1170 MPa predicted by (13). Agreement with the [Hoff and Mautner 1945] prediction would require coefficient values of $c = 0.834$ and $c = 0.662$ in (11).

Figure 15 shows moment versus strain results for two different tests of sandwich beams with Divinycell H100 foam cores under four-point bending. Evidence of wrinkling is shown by the sharp change in recorded strain on the compression facesheet, indicating inward and outward wrinkling in the two tests. In both cases the critical wrinkling stress was $\sigma_{cr} = 673$ MPa. Heath's relation (13) [Heath 1960] was selected because of the lack of shear interaction due to the pure bending loading. The predicted critical wrinkling stress of 667 MPa is very close to the experimental value.

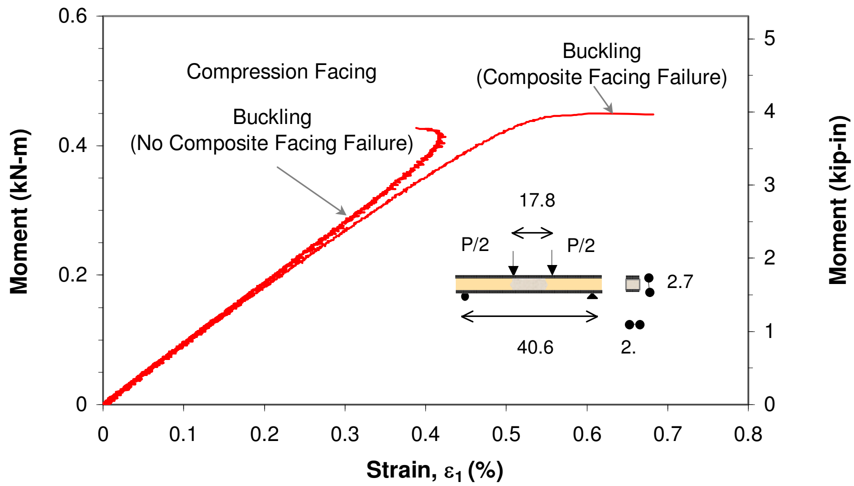

Figure 15. Facesheet wrinkling in sandwich beam under four-point bending (Divinycell H100 foam core; dimensions are in cm).

Sandwich beams were also tested in three-point bending and as cantilever beams. The moment-strain curves shown in Figure 16 illustrate the onset of facesheet wrinkling. Critical stresses obtained from the figure for the maximum moment for specimens 1 and 2 are $\sigma_{cr} = 860$ MPa and 947 MPa, respectively. The predicted value by (11) would agree with the average of the two measurements, 903 MPa, for $c = 0.578$. In the case of the short beam (specimen 3), core failure preceded wrinkling. The measured wrinkling stress was 517 MPa. The core shear stresses at wrinkling for specimens 2 and 3 are 3.2 MPa and 4.55 MPa, respectively. Thus, the core material for specimen 2 is in the linear elastic region, whereas for specimen 3 it is close to the yield point. Equation (14) predicts the measured wrinkling stress with a reduced core shear modulus of $G'_{c13} = 21.2$ MPa for $c = 0.5$.

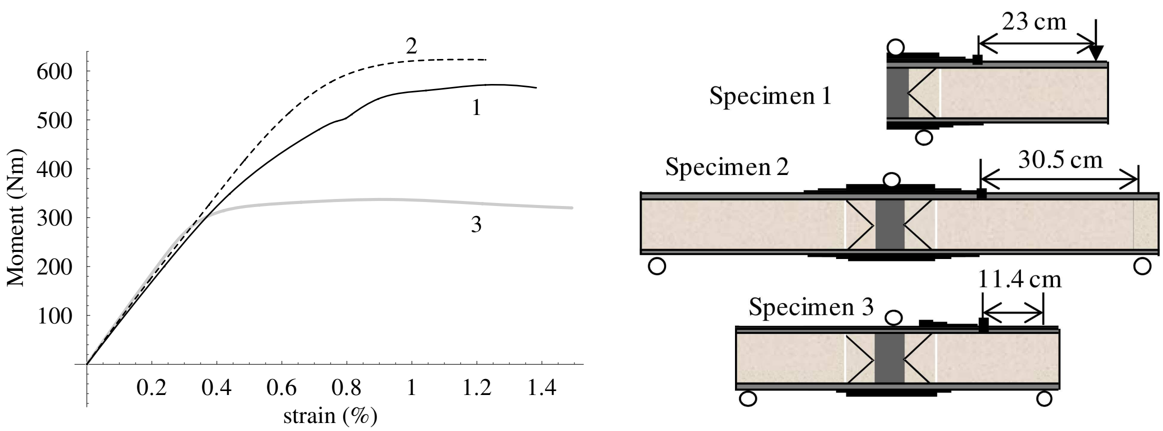

Figure 16. Facesheet wrinkling failure in sandwich beams with Divinycell H250 cores. Curve numbers correspond to specimen numbers on the right.

6. Conclusions

The initiation of failure in composite sandwich beams is heavily dependent on properties of the core material. Plastic yielding or cracking of the core occurs when the critical yield stress or strength (usually shear) of the core is reached. Indentation under localized loading depends principally on the square root of the core yield stress. Available theory predicts indentation failure approximately, overestimating it for soft cores and underestimating it for stiffer ones. The critical facesheet wrinkling stress is predicted fairly closely by Heath's formula for cases not involving shear interaction between the facesheets and the core, such as compressively loaded columns and beams under pure bending. In the case of cantilever beams or beams under three-point bending, entailing shear interaction between the facesheets and core, the Hoff and Mautner formula predicts a value for the critical wrinkling stress which is proportional to the cubic root of the product of the core Young's and shear moduli in the thickness direction. The ideal core should be highly anisotropic with high stiffness and strength in the thickness direction.

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