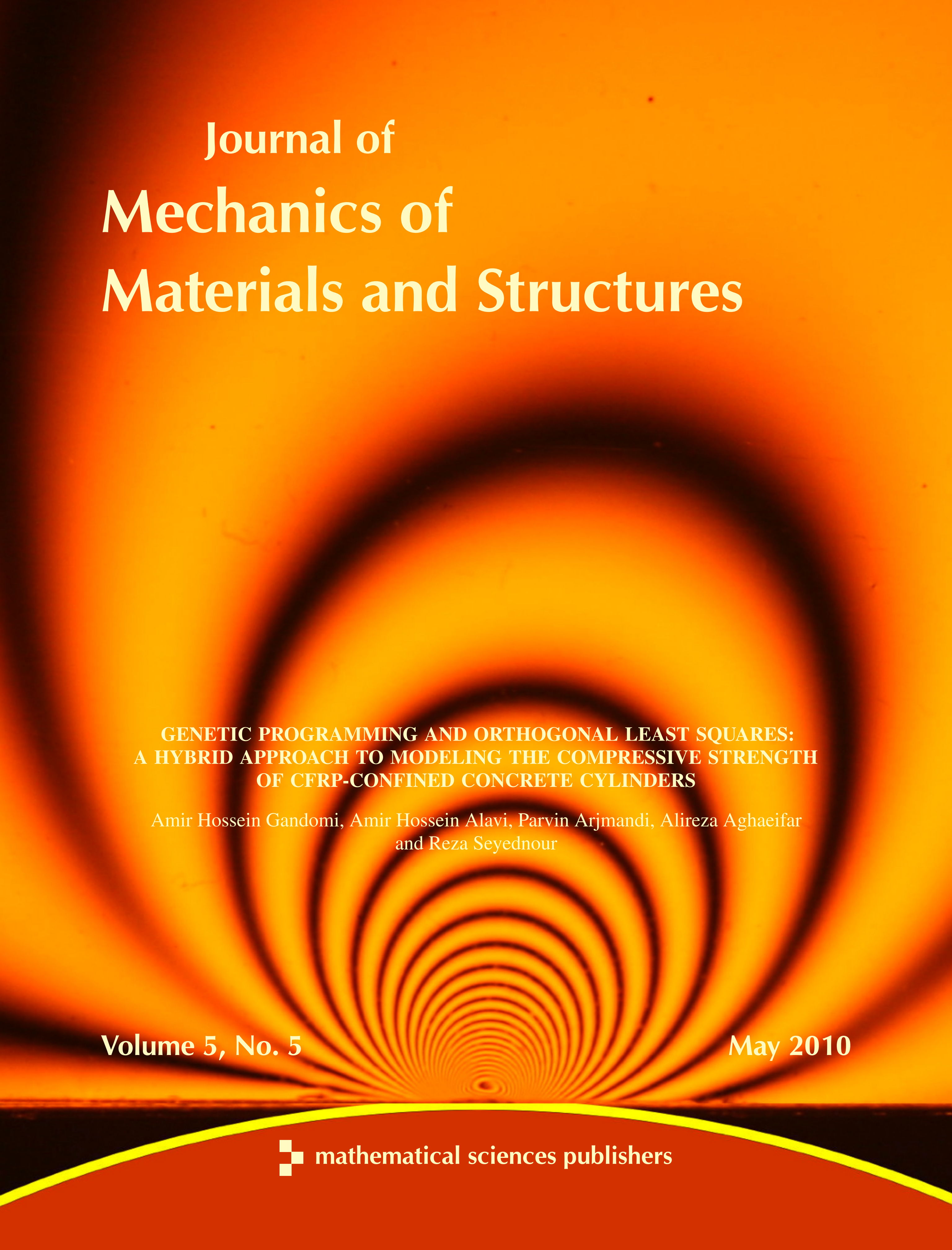

Journal of
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GENETIC PROGRAMMING AND ORTHOGONAL LEAST SQUARES: A HYBRID APPROACH TO MODELING THE COMPRESSIVE STRENGTH OF CFRP-CONFINED CONCRETE CYLINDERS

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Volume 5, No. 5 May 2010

mathematical sciences publishers

JOURNAL OF MECHANICS OF MATERIALS AND STRUCTURES
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The main objective of this paper is to apply genetic programming (GP) with an orthogonal least squares (OLS) algorithm to derive a predictive model for the compressive strength of carbon fiber-reinforced plastic (CFRP) confined concrete cylinders. The GP/OLS model was developed based on experimental results obtained from the literature. Traditional GP-based and least squares regression analyses were performed using the same variables and data sets to benchmark the GP/OLS model. A subsequent parametric analysis was carried out and the trends of the results were confirmed via previous laboratory studies. The results indicate that the proposed formula can predict the ultimate compressive strength of concrete cylinders with an acceptable level of accuracy. The GP/OLS results are more accurate than those obtained using GP, regression, or several CFRP confinement models found in the literature. The GP/OLS-based formula is simple and straightforward, and provides a valuable tool for analysis.

1. Introduction

Concrete is a frictional material with considerable sensitivity to hydrostatic pressure. Lateral stress has advantageous effects on concrete strength and deformation. When concrete is uniaxially loaded and cannot dilate laterally, it exhibits increased strength and axial deformation capacity, indicated as confinement. Concrete confinement can generally be provided through transverse reinforcement in the form of spirals, circular hoops, or rectangular ties or by encasing the concrete columns in steel tubes that act as permanent formwork [de Lorenzis 2001]. Fiber-reinforced polymers (FRPs) are also used for the confinement of concrete columns. Compared to steel [Fardis and Khalili 1982], FRPs present several advantages, such as ease and speed of application, continuous confining action to the entire cross-section, lack of change in the shape and size of the strengthened elements, and corrosion resistance [de Lorenzis 2001]. A typical response of FRP-confined concrete is shown in Figure 1, where normalized axial stress is plotted against axial, lateral, and volumetric strains. The stress is normalized with respect to the unconfined strength of the concrete core. The figure shows that both axial and lateral responses are bilinear with a transition zone at or near the peak strength of the unconfined concrete core. The volumetric response shows a similar transition toward volume expansion. However, as soon as the jacket takes over, the volumetric response undergoes another transition which reverses the dilation trend and results in volume compaction. This behavior is shown to be remarkably different from plain concrete and steel-confined concrete [Mirmiran et al. 2000].

Keywords: genetic programming, orthogonal least squares, CFRP confinement, concrete compressive strength, formulation.

Carbon fiber-reinforced plastic (CFRP) is one of the main types of FRP composites. The advantages of CFRP include anticorrosion, easy cutting and construction, as well as high strength-to-weight ratio and high elastic modulus. These features caused widely usage of CFRP in the retrofitting and strengthening of reinforced concrete structures for over 50 years. A typical CFRP-confined concrete cylinder is illustrated in Figure 2.

Several studies have been conducted to analyze the effect of CFRP confinement on the strength and deformation capacity of concrete columns. On the basis of this research, a number of empirical and theoretical models have been developed [de Lorenzis 2001]. In spite of the extensive research in this field, the existing models have significant limitations, such as specific loading systems and conditions,

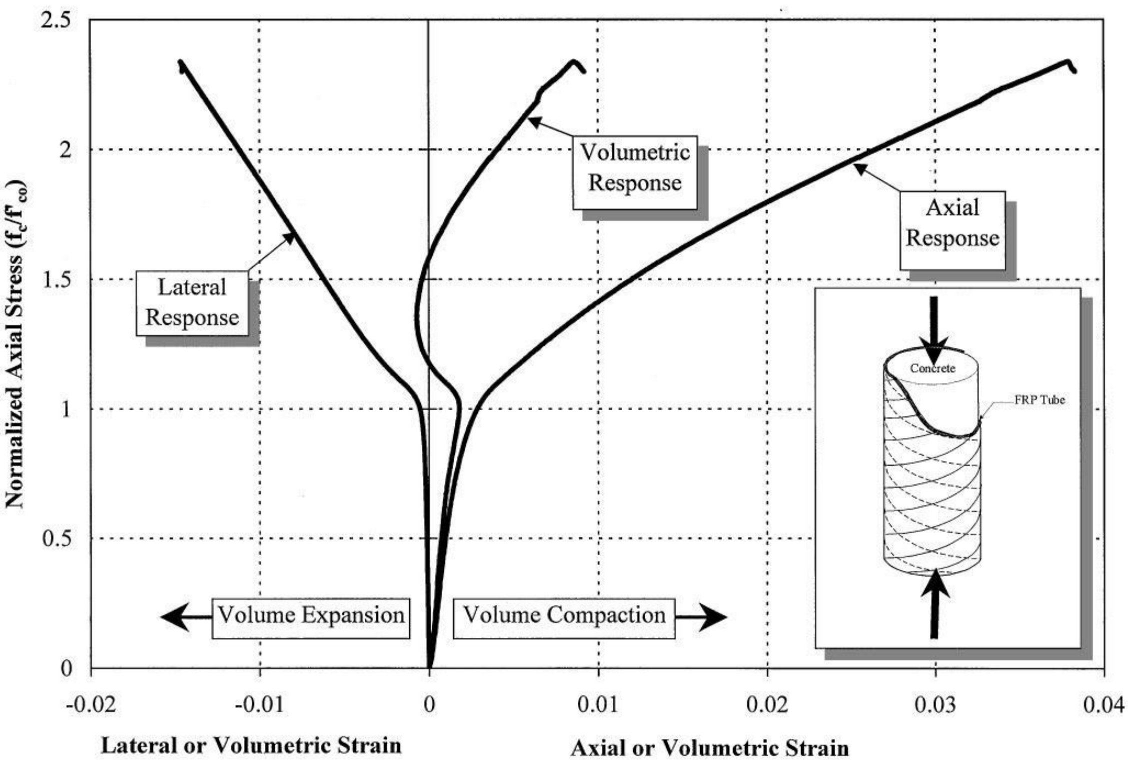


Figure 1. Typical response of FRP-confined concrete [Mirmiran et al. 2000].

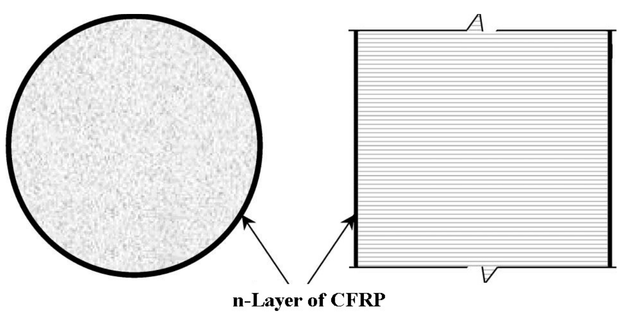


Figure 2. A typical CFRP-confined concrete cylinder.

and the need for calibration of parameters. These limitations suggest the necessity of developing more comprehensive mathematical models for assessing the behavior of CFRP-confined concrete columns.

Genetic programming (GP) [Koza 1992; Banzhaf et al. 1998] is a developing subarea of evolutionary algorithms, where programs are represented as tree structures (see Section 3). GP and its variants have successfully been applied to various kinds of civil engineering problems [Alavi et al. 2010; Gandomi et al. 2009; 2010; Alavi and Gandomi $\geq$ 2010].

The orthogonal least squares (OLS) algorithm [Billings et al. 1988; Chen et al. 1989] is an effective algorithm for determining which terms are significant in a linear-in-parameters model. The OLS algorithm introduces the error reduction ratio, which is a measure of the decrease in the variance of output by a given term. [Madár et al. 2005b; 2005c] combined GP and OLS to make a hybrid algorithm with better efficiency, showing that introducing OLS to the GP process results in more robust and interpretable models. GP/OLS uses the data alone to determine the structure and parameters of the model. This technique has rarely been applied to civil engineering problems [Gandomi and Alavi 2010]. The GP/OLS approach can be substantially useful in deriving empirical models for characterizing the compressive strength behavior of CFRP-confined concrete cylinders by directly extracting the knowledge contained in the experimental data.

The main purpose of this paper is to utilize GP/OLS to generate a linear-in-parameters predictive model of the compressive strength of CFRP concrete cylinders represented by tree structures. The predictor variables included in the analysis were unconfined concrete strength and ultimate confinement pressure. Traditional GP and least squares regression models were developed to benchmark the derived model. A reliable database of previously published test results was utilized to develop the models.

2. Previous research on behavior of CFRP-confined concrete

The characteristic response of confined concrete includes three distinct regions of uncracked elastic deformation, crack formation and propagation, and plastic deformation. It is generally assumed that concrete behaves like an elastic-perfectly plastic material after reaching its maximum strength capacity. The failure surface is considered to be fixed in stress space. Constitutive models for concrete should be concerned with pressure sensitivity, path dependence, stiffness degradation, and cyclic response. Existing plasticity models include nonlinear elasticity, endochronic plasticity, classical plasticity, multilaminate or micro-plane plasticity, and bounding surface plasticity. Many of these models, however, are only suitable in specific applications and loading systems for which they are devised and may give unrealistic results in other cases. Also, some of these models require several parameters to be calibrated based on experimental results [Mirmiran et al. 2000]. Considerable experimental research has been performed on the behavior of CFRP-confined concrete columns [Miyauchi et al. 1997; Kono et al. 1998; Matthys et al. 1999; Rochette and Labossière 2000; Shahawy et al. 2000; Micelli et al. 2001; Rousakis 2001]. Numerous studies have concentrated on assessing the strength enhancement of CFRP-wrapped concrete cylinders in the literature. Some of the most important models in this field are shown in Table 1.

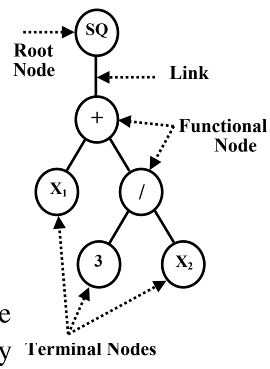
By extending developments in computational software and hardware, several alternative computer-aided data mining approaches have been developed. Thus, Cevik and Guzelbey [2008] presented an application of neural networks (NN) to the modeling of the compressive strength of a CFRP-confined concrete cylinder. They also obtained the explicit formulation of the compressive strength using NN.

ID	Authors	Expression for f'_{cc}/f'_{co}
1	[Fardis and Khalili 1981]	$1+3.7(p_u/f'_{co})^{0.85}$
2	[Mander et al. 1988]	$2.254\sqrt{1+7.94P_u/f'_{co}}-2P_u/f'_{co}-1.254$
3	[Miyauchi et al. 1997]	$1+3.485(p_u/f'_{co})$
4	[Xiao and Wu 2000]	$1.1+(4.1-0.75{f'_{co}}^2/E_l)p_u/f'_{co}$
5	[Samaan et al. 1998]	$1+0.6p_u^{0.7}$
6	[Lam and Teng 2001]	$1+2(p_u/f'_{co})$
7	[Toutanji 1999]	$1+3.5(p_u/f'_{co})^{0.85}$
8	[Saafi et al. 1999]	$1+2.2(p_u/f'_{co})^{0.84}$
9	[Spoelstra and Monti 1999]	$0.2+3\sqrt{p_u/f'_{co}}$
10	[Karbhari and Gao 1997]	$1+2.1(p_u/f'_{co})^{0.87}$
11	[Richart et al. 1928]	$1+4.1(p_u/f'_{co})$
12	[Berthet et al. 2006]	$1+K_1\dfrac{p_u}{f'_{co}},\quad K_1=\begin{cases}3.45 & \text{if } 20\le f'_{co}/\text{MPa}\le 50\\ 0.95(f'_{co})^{-1/4} & \text{if } 50\le f'_{co}/\text{MPa}\le 200\end{cases}$
13	[Li et al. 2003] (L-L Model)	$1+\tan\left(45^\circ-\tfrac{1}{2}\phi\right)(P_u/f'_{co}),\quad \phi=36^\circ+1^\circ(f'_{co}/35)\le 45^\circ$
14	[Vintzileou and Panagiotidou 2008]	$1+2.8(P_u/f'_{co})$

Table 1. Different models for the strength enhancement of FRP confined concrete cylinders. f'_{co} is the compressive strength of the unconfined concrete cylinder, f'_{cc} the ultimate compressive strength of the confined concrete cylinder, P_u the ultimate confinement pressure ($P_u = E_l \cdot \varepsilon_f = 2t \cdot f'_{com}/D$), E_l the lateral modulus, ε_f the ultimate tensile strain of the FRP laminate, f'_{com} the ultimate tensile strength of the FRP layer, t the thickness of the FRP layer, and D the diameter of concrete cylinder.

3. Genetic programming

GP is a symbolic optimization technique that creates computer programs to solve problems using the principle of natural selection [Koza 1992]. GP may generally be defined as a supervised machine learning technique that searches a program space instead of a data space [Banzhaf et al. 1998]. The symbolic optimization algorithms present the potential solutions by structural ordering of several symbols. In GP, a random population of individuals (trees) is created to achieve high diversity. A population member in GP is a hierarchically structured tree comprising functions and terminals. The functions and terminals are selected from appropriate sets. For example, the function set F can contain the basic arithmetic operations (+, −, ×, /, et cetera), Boolean logic functions (AND, OR, NOT, et cetera), or any other mathematical functions. The terminal set T contains the arguments for the functions and can consist of numerical constants, logical constants, variables, et cetera. The functions and terminals are chosen at random and combined to form a computer model in a tree-like structure with a root point with branches extending from each function and ending in a terminal. For example, the tree shown on the right represents the GP model $(X_1 + 3/X_2)^2$.



The creation of the initial population is a blind random search for solutions in the large space of possible solutions. Once a population of models has been randomly

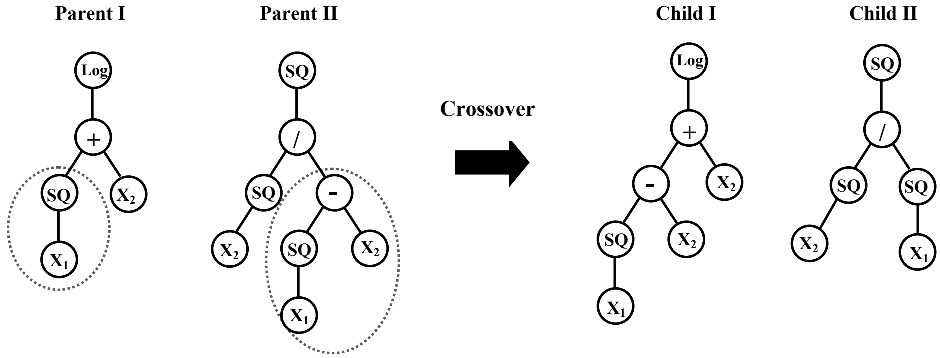


Figure 3. Typical crossover operation in GP.

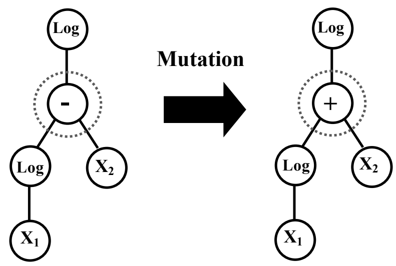


Figure 4. Typical mutation operation in GP.

created, the GP algorithm evaluates the individuals, selects individuals for reproduction, generates new individuals by mutation, crossover, and direct reproduction, and finally creates the new generation in all iterations [Koza 1992].

During the crossover procedure, a point on a branch of each solution (program) is randomly selected and the set of terminals and/or functions from each program is then swapped to create two new programs, as can be seen in Figure 3. The evolutionary process continues by evaluating the fitness of the new population and starting a new round of reproduction and crossover. During this process, the GP algorithm occasionally selects a function or terminal at random from a model and mutates it (see Figure 4).

3.1. ***GP for linear-in-parameters models.*** In general, GP creates not only nonlinear models but also linear-in-parameters models. In order to avoid parameter models, the parameters must be removed from the set of terminals. That is, it contains only variables: $T = \{x_0(k), \dots, x_i(k)\}$, where $x_i(k)$ denotes the i-th repressor variable. Hence, a population member represents only F_i nonlinear functions [Pearson 2003]. The parameters are assigned to the model after extracting the F_i function terms from the tree, and determined using a least square (LS) algorithm [Reeves 1997]. A simple technique for the decomposition of the tree into function terms can be used. The subtrees, representing the F_i function terms, are determined by decomposing the tree starting from the root and going as far as nonlinear nodes (nodes distinct from "+" or "−"). As can be seen in Figure 5, the root node is a + operator; therefore, it is possible to decompose the tree into two subtrees A and B. The root node of the A tree is again a linear operator; therefore, it can be decomposed into C and D trees. As the root node of the B tree is a nonlinear node (/), it cannot be decomposed. The root nodes of the C and D trees are

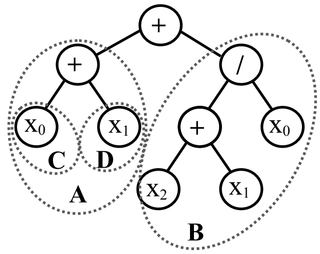


Figure 5. Decomposition of a tree to function terms [Madár et al. 2005a].

also nonlinear. Consequently, the final decomposition procedure results in three subtrees: B, C, and D. According to the results of the decomposition, it is possible to assign parameters to the functional terms represented by the obtained subtrees. The resulting linear-in-parameters model for this example is $y : p_0 + p_1(x_2 + x_1)/x_0 + p_2 x_0 + p_3 x_1$.

GP can be used for selecting from special model classes, such as polynomial models. To achieve this, the set of operators must be restricted and some simple syntactic rules must be introduced. For instance, if the set of operators is defined as $F = \{\times, +\}$ and there is a syntactic rule that exchanges the internal nodes that are below $\times$-type internal nodes to $\times$-type nodes, GP will generate only polynomial models [Koza 1992; Madár et al. 2005a].

3.2. ***OLS algorithm.*** The great advantage of using linear-in-parameter models is that the LS method can be used for identifying the model parameters. This is much less computationally demanding than other nonlinear optimization algorithms, because the optimal $p = [p_1, \dots, p_m]T$ parameter vector can analytically be calculated:

$$p = (U^{-1}U)^T U_y, \tag{1}$$

where $y = (y(1), \dots, y(N))T$ is the measured output vector and the U regression matrix is

$$U = \begin{pmatrix} U_1(x(1)) & \cdots & U_M(x(1)) \\ \vdots & \ddots & \vdots \\ U_1(x(N)) & \cdots & U_M(x(N)) \end{pmatrix}. \tag{2}$$

The OLS algorithm [Billings et al. 1988; Chen et al. 1989] is an effective algorithm for determining which terms are significant in a linear-in-parameters model. The OLS technique introduces the error reduction ratio (err), which is a measure of the decrease in the variance of the output by a given term. The matrix form corresponding to the linear-in-parameters model is

$$y = U_p + e, \tag{3}$$

where U is the regression matrix, p is the parameter vector, and e is the error vector. The OLS method transforms the columns of the U matrix into a set of orthogonal basis vectors to inspect the individual contributions of each term [Cao et al. 1999]. It is assumed in the OLS algorithm that the regression matrix U can be orthogonally decomposed as $U = WA$, where A is a M by M upper triangular matrix (that is, $A_{ij} = 0$ if $i > j$) and W is a N by M matrix with orthogonal columns in the sense that $WTW = D$ is a diagonal matrix (N is the length of the y vector and M is the number of repressors). After this decomposition, the OLS auxiliary parameter vector g can be calculated as

$$g = D^{-1} W^T y, \tag{4}$$

where g_i represents the corresponding element of the OLS solution vector. The output variance $(y^T y)/N$ can be described as

$$y^T y = \sum_{i=1}^{M} g_i^2 w_i^T w_i + e^T e. \tag{5}$$

Therefore, the error reduction ratio $[\text{err}]_i$ of the U_i term can be expressed as

$$[\text{err}]_i = \frac{g_i^2 w_i^T w}{y^T y}. \tag{6}$$

This ratio offers a simple mean for order and selects the model terms of a linear-in-parameters model on the basis of their contribution to the performance of the model.

3.3. *Hybrid GP/OLS algorithm.* Application of OLS to the GP algorithm leads to significant improvements in the performance of GP. The main feature of this hybrid approach is to transform the trees into simpler trees which are more transparent, but with accuracy close to that of the original trees. In this coupled algorithm, GP generates a lot of potential solutions in the form of a tree structure during the GP operation. These trees may have better and worse terms (subtrees) that contribute more or less to the accuracy of the model represented by the tree. OLS is used to estimate the contribution of the branches of the tree to the accuracy of the model; using the OLS, one can select the less significant terms in a linear regression problem. According to this strategy, terms (subtrees) having the smallest error reduction ratio are eliminated from the tree [Pearson 2003]. This "tree pruning" approach is realized in every fitness evaluation before the calculation of the fitness values of the trees. Since GP works with the tree structure, the further goal is to preserve the original structure of the trees as far as possible. The GP/OLS method always guarantees that the elimination of one or more function terms of the model can be done by pruning the corresponding subtrees, so there is no need for structural rearrangement of the tree after this operation. The way the GP/OLS method works on its basis is simply demonstrated in Figure 6. Assume that the function which must be identified is $y(x) = 0.8u(x-1)^2 + 1.2y(x-1) - 0.9y(x-2) - 0.2$. As can be seen in Figure 6, the GP algorithm finds a solution with four terms: $u(x-1)^2$, $y(x-1)$, $y(x-2)$, and $u(x-1) \times u(x-2)$. Based on the OLS algorithm, the subtree with the least error reduction ratio ($F_4 = u(x-1) \times u(x-2)$) is eliminated from the tree. Subsequently, the error reduction ratios and mean

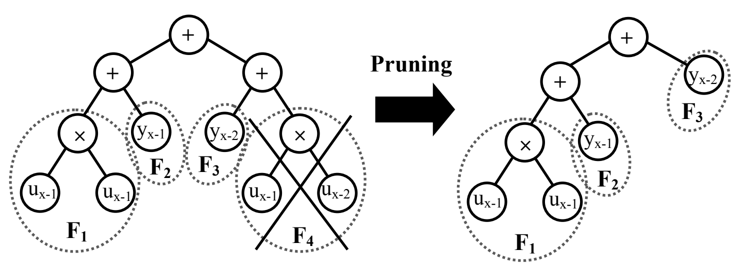


Figure 6. Pruning of a tree with OLS.

square error values (and model parameters) are calculated again. The new model (after pruning) may have a higher mean square error but it obviously has a more adequate structure.

4. Experimental database

A comprehensive experimental database was obtained for the compressive strength of CFRP-wrapped concrete cylinders from the literature [Cevik and Guzelbey 2008]. The database contains 101 samples from seven separate studies. The ranges of different input and output parameters used for the model development are given in Table 2. To visualize the sample distribution, the data are presented in Figure 7.

Parameter	min.	max.	range	SD	skewness	kurtosis	mean
Unconfined ultimate concrete strength	19.40	82.13	62.73	17.35	0.781	−0.175	45.11
Ultimate confinement pressure	3.44	38.38	34.94	8.69	1.483	1.803	13.51
Confined ultimate concrete strength	33.8	137.9	104.1	23.03	0.389	−0.566	78.32

Table 2. Ranges of parameters in database (SD is the standard deviation); values in MPa.

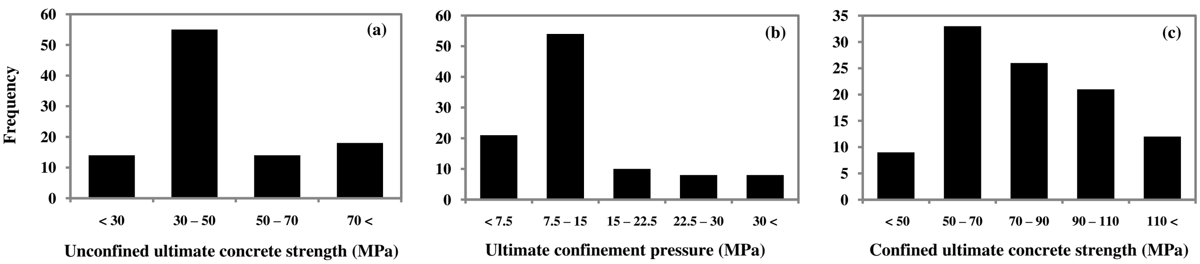


Figure 7. Histograms of the variables used in the model development.

5. Building a GP/OLS predictive model for compressive strength

Thus, the main goal of this study is to derive an explicit formulation for the compressive strength of CFRP-confined concrete cylinders (f'_{cc}) as follows:

$$f'_{cc} = f(f'_{co}, P_u) \tag{7}$$

in which f'_{co} is the unconfined ultimate concrete strength and P_u is the ultimate confinement pressure.

In the FRP confinement models developed by other researchers, f'_{co} and P_u are the most widely used parameters. As indicated in Table 1, P_u is a function of the diameter of the concrete cylinder (D), the thickness of the CFRP layer (t), and the ultimate tensile strength of the CFRP layer (f'_{com}) [Spoelstra and Monti 1999]. Therefore, the effects of D, t, and (f'_{com}) were implicitly incorporated into the model development.

For the analysis, the data sets were randomly divided into training and testing subsets (75 data sets were used as training and the rest as testing). In order to obtain a consistent data division, several combinations of the training and testing sets were considered. The selection was such that the maximum, minimum, mean, and standard deviation of the parameters were consistent in the training and testing data sets. The

GP/OLS approach was implemented using MATLAB. The best GP/OLS model was chosen on the basis of a multiobjective strategy as follows:

- The total number of inputs involved in each model.
- The best model fitness value on the training set of data.

During the evolutionary process, different participating parameters were gradually picked up in order to form the equations representing the input-output relationship. After checking several normalization methods [Swingler 1996; Rafiq et al. 2001], the following method was used for normalizing the data. The inputs and output of the GP/OLS model were normalized between 0 and 0.91 using the rule

$$X_n = \frac{X_i}{1.1 X_{i,\max}}, \tag{8}$$

where $X_{i,\max}$ are the maximum values of X_i and X_n is the normalized value. Various parameters are involved in the GP/OLS algorithm. The parameter selection will affect the generalization capability of GP/OLS. The GP/OLS parameters were selected based on some previously suggested values [Madár et al. 2005c] and after a trial and error approach. The parameter values are shown in Table 3. The correlation coefficient (R), the mean absolute percent error (MAPE), and the root mean squared error (RMSE) were used as the target error parameters to evaluate the performance of the models.

The GP/OLS-based formulation of the compressive strength f'_{cc} in terms of f'_{co} and P_{u} is

$$f'_{\mathrm{cc}} = \left(\frac{P_{\mathrm{u}}}{25} + \frac{2}{3}\right) f'_{\mathrm{co}} + 25. \tag{9}$$

Figure 8 shows the expression tree of the best GP model formulation. A comparison of the GP/OLS predicted and experimental compressive strengths of the CFRP-wrapped concrete cylinder is shown in Figure 9.

Parameter	Values
Function set	$+, -, \times, /$
Population size	1000
Maximum tree depth	3-8
Maximum number of evaluated individuals	2500
Generation	100
Type of selection	roulette-wheel
Type of mutation	point-mutation
Type of crossover	one-point (2 parents)
Type of replacement	elitist
Probability of crossover	0.5
Probability of mutation	0.5
Probability of interchanging terminal and nonterminal nodes during mutation	0.25

Table 3. Parameter values for GP/OLS.

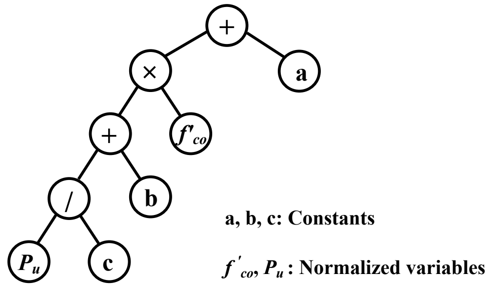


Figure 8. Expression tree of the best GP/OLS model.

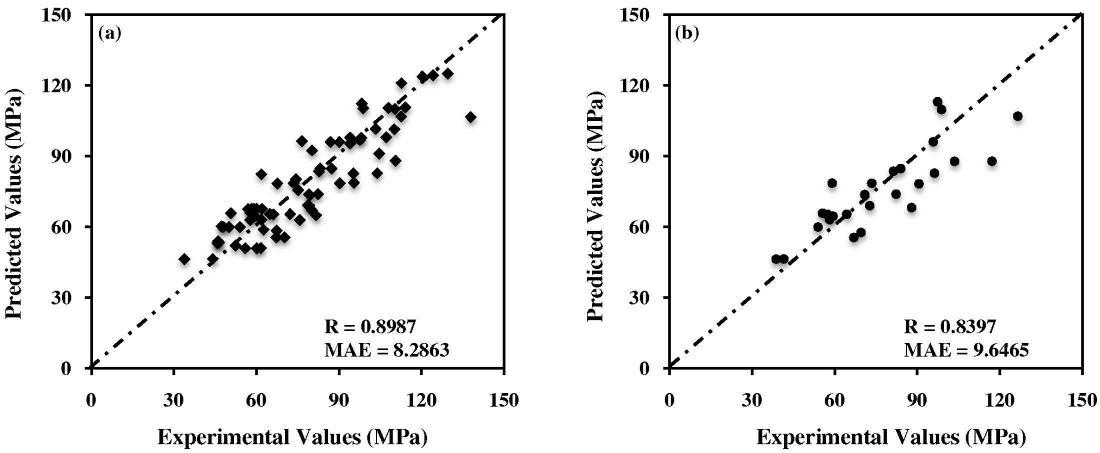


Figure 9. Predicted versus experimental compressive strengths using the GP/OLS model: (a) training data and (b) testing data.

6. Building models for benchmarking the GP/OLS model

6.1. ***Traditional GP predictive model for compressive strength.*** A tree-based GP analysis was performed to compare the GP/OLS technique with a traditional GP approach. The general parameter settings for the tree-based GP model are similar to those of GP/OLS. The tree-based GP software GPLAB [Silva 2007] was used, in conjunction with subroutines coded in MATLAB.

Similarly to the use in the GP/OLS model, out of the 101 data sets, 75 were used as the training data and 26 were used for the testing of the GP model. The formulation of f'_{cc} in terms of f'_{co} and P_u, for the best results from the GP, is

$$f'_{cc} = f'_{co} - \frac{P_u((f'_{co} - 8) - (f'_{co} - 5))}{8} + P_u - \left(\frac{f'_{co}}{P_u} - 2\right) + 4 - \left(\frac{P_u}{f'_{co}}\right) + \frac{f'_{co}}{4}. \tag{10}$$

The GP-based equation was obtained by converting the related expression tree into a mathematical form. A comparison of the GP predicted and experimental compressive strengths of the CFRP-wrapped concrete cylinder is shown in Figure 10.

6.2. ***LSR predictive model for compressive strength.*** A multivariable LSR analysis was performed to assess the predictive power of the GP/OLS technique, in comparison with a classical statistical approach. The LSR method is extensively used in regression analysis primarily because of its interesting nature. Under certain assumptions, LSR has some attractive statistical properties that have made it one of the most powerful and popular methods of regression analysis. The major task is to determine the multivariable LSR-based equation connecting the input variables to the output variable:

$$f'_{\mathrm{cc}} = \alpha_1 f'_{\mathrm{co}} + \alpha_2 P_{\mathrm{u}} + \alpha_3, \tag{11}$$

where f'_{cc} is the compressive strength of the CFRP-confined concrete cylinders, f'_{co} is the unconfined ultimate concrete strength, P_{u} is the ultimate confinement pressure, and α denotes the coefficient vector. The software package EViews [Maravall and Gomez 2004] was used to perform the regression analysis.

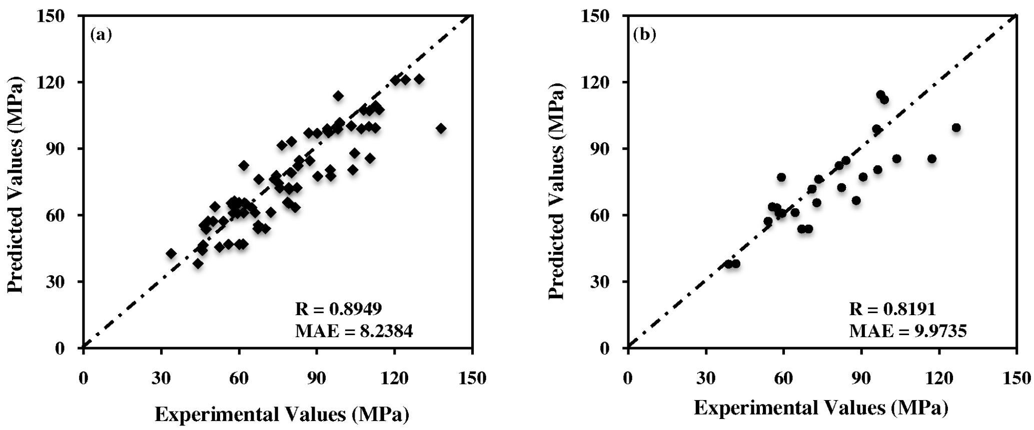


Figure 10. Predicted versus experimental compressive strengths using the GP model: (a) training data and (b) testing data.

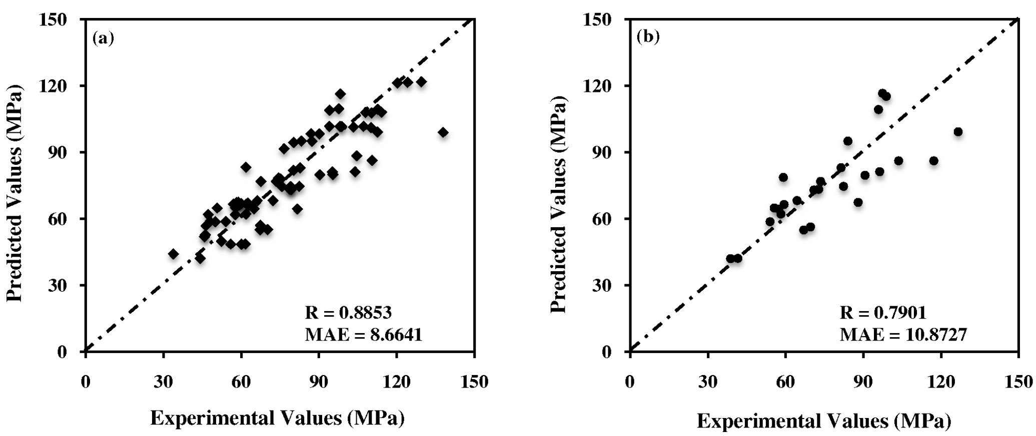


Figure 11. Predicted versus experimental compressive strengths using the LSR model: (a) training data and (b) testing data.

The formulation of f'_{cc} in terms of f'_{co} and P_u, for the best result from the LSR, is

$$f'_{cc} = 1.118 f'_{co} + 1.479 P_u + 7.903. \tag{12}$$

A comparison of the LSR predicted and experimental compressive strengths of the CFRP-wrapped concrete cylinder is shown in Figure 11. The resulting Fisher value (F) of the performed regression analysis is equal to 130.4.

7. Comparison of the CFRP confinement models

A GP/OLS-based formula was obtained for the compressive strength of CFRP-wrapped concrete cylinders. A comparison of the ratios between the predicted compressive strength values from the GP/OLS, GP, and LSR models, as well as those found in the literature, and the experimental values is shown in Figure 12. Some other models in the literature, such as the second formula of [Karbhari and Gao 1997], require additional details that are not available in the experimental database. Thus, they were not included in the comparative study.

The performance statistics of the formulas obtained by the different methods on the whole of the data are summarized in Table 4. As can be seen in Figures 9–12 and Table 4, the GP/OLS-based formula has provided the best performance on the training, testing, and whole data sets compared with the GP, LSR, and existing FRP confinement models.

	correlation of exper. and predicted f'_{cc}			exper./predicted f'_{cc}	
Model ID	R	MAPE	RMSE	mean	SD
1	0.752	24.04	31.46	1.33	0.28
2	0.871	18.19	22.15	1.08	0.22
3	0.704	16.91	25.56	1.2	0.29
4	0.23	29.43	38.25	1	0.46
5	0.847	10.12	12.46	1.06	0.17
6	0.833	11.09	14.18	0.94	0.16
7	0.769	21.62	8.6	1.29	0.27
8	0.851	10.11	12.45	1.03	0.16
9	0.812	11.81	13.86	1.01	0.19
10	0.702	16.28	24.94	1.19	0.28
11	0.659	23.16	34.2	1.31	0.35
12	0.854	9.8	12.14	1	0.16
13	0.791	26.5	30.02	0.67	0.12
14	0.763	12.86	17.39	1.25	0.2
LSR	0.863	9.23	11.59	1.01	0.15
GP	0.877	9.23	11.59	0.98	0.14
GP/OLS	0.885	8.64	10.69	1.01	0.16

Table 4. Performance statistics of the compressive strength predictive models. For the meaning of the columns, see page 743.

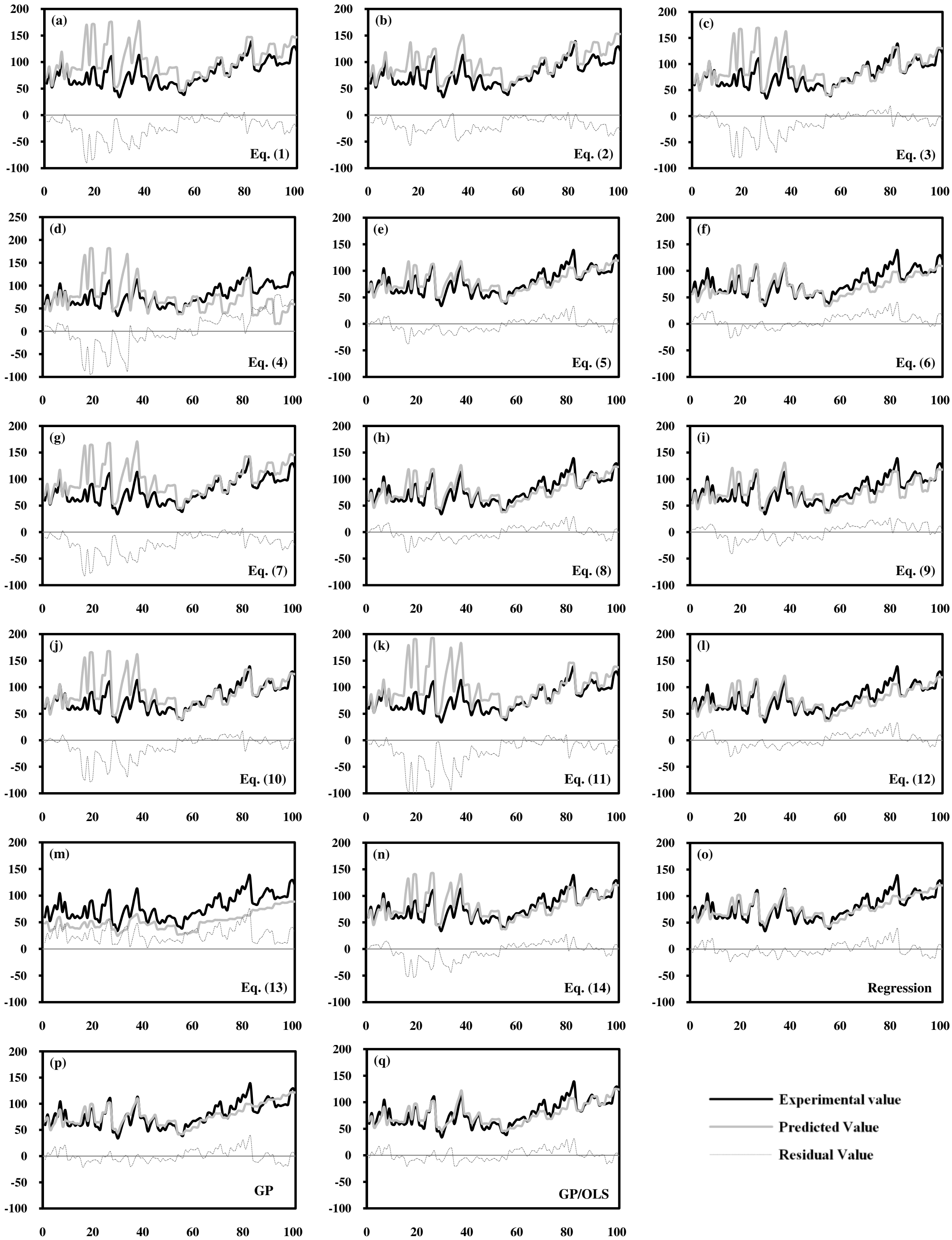


Figure 12. Comparison of the ratios between the predicted f'_{cc} values (in MPa, on the vertical axes) and experimental values (test number, on the horizontal axes) using different methods.

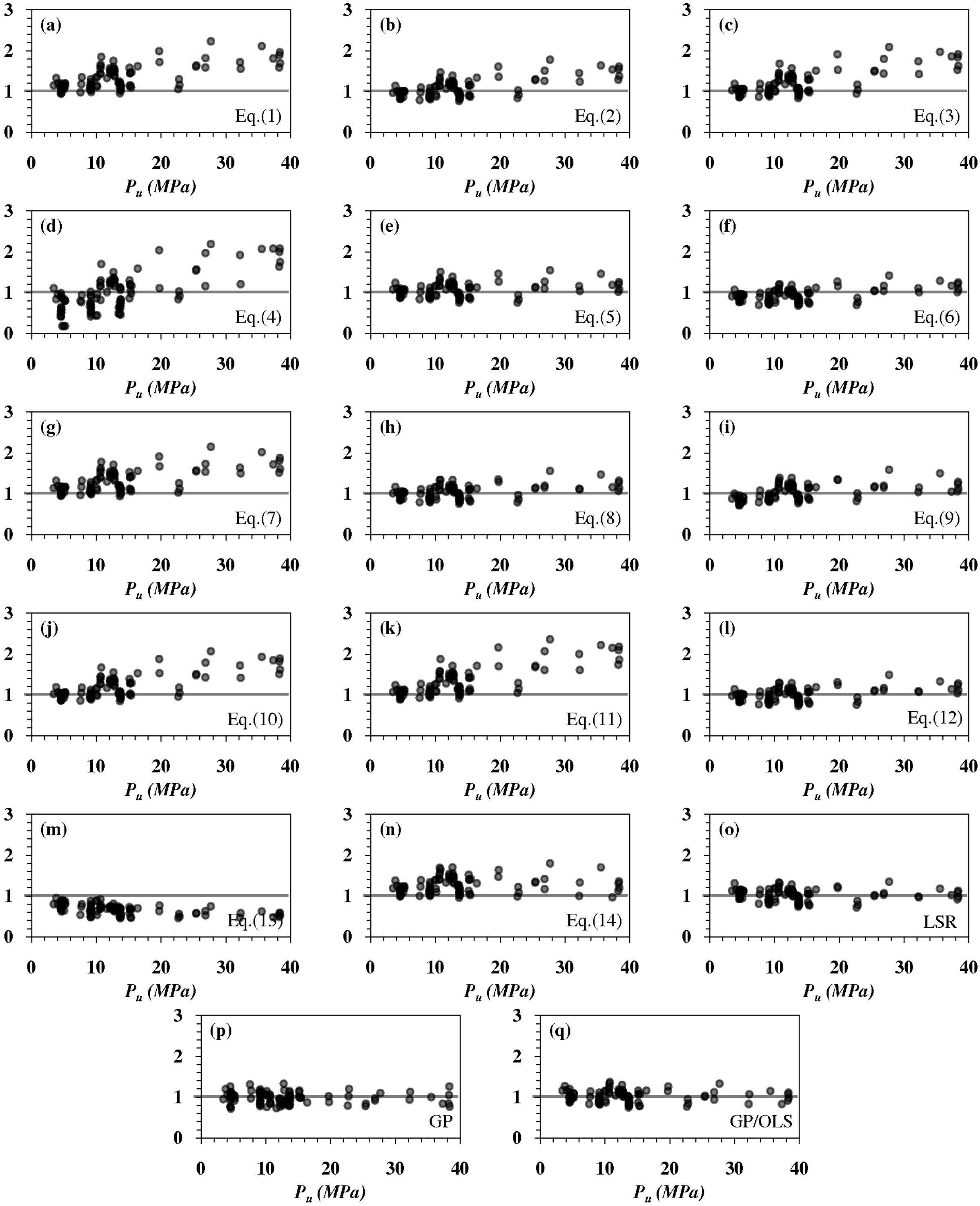


Figure 13. The ratios between the predicted and experimental compressive strength values with respect to P_{u}. Vertical axes in units of f'_{cc} predicted$/f'_{\mathrm{cc}}$ experimental.

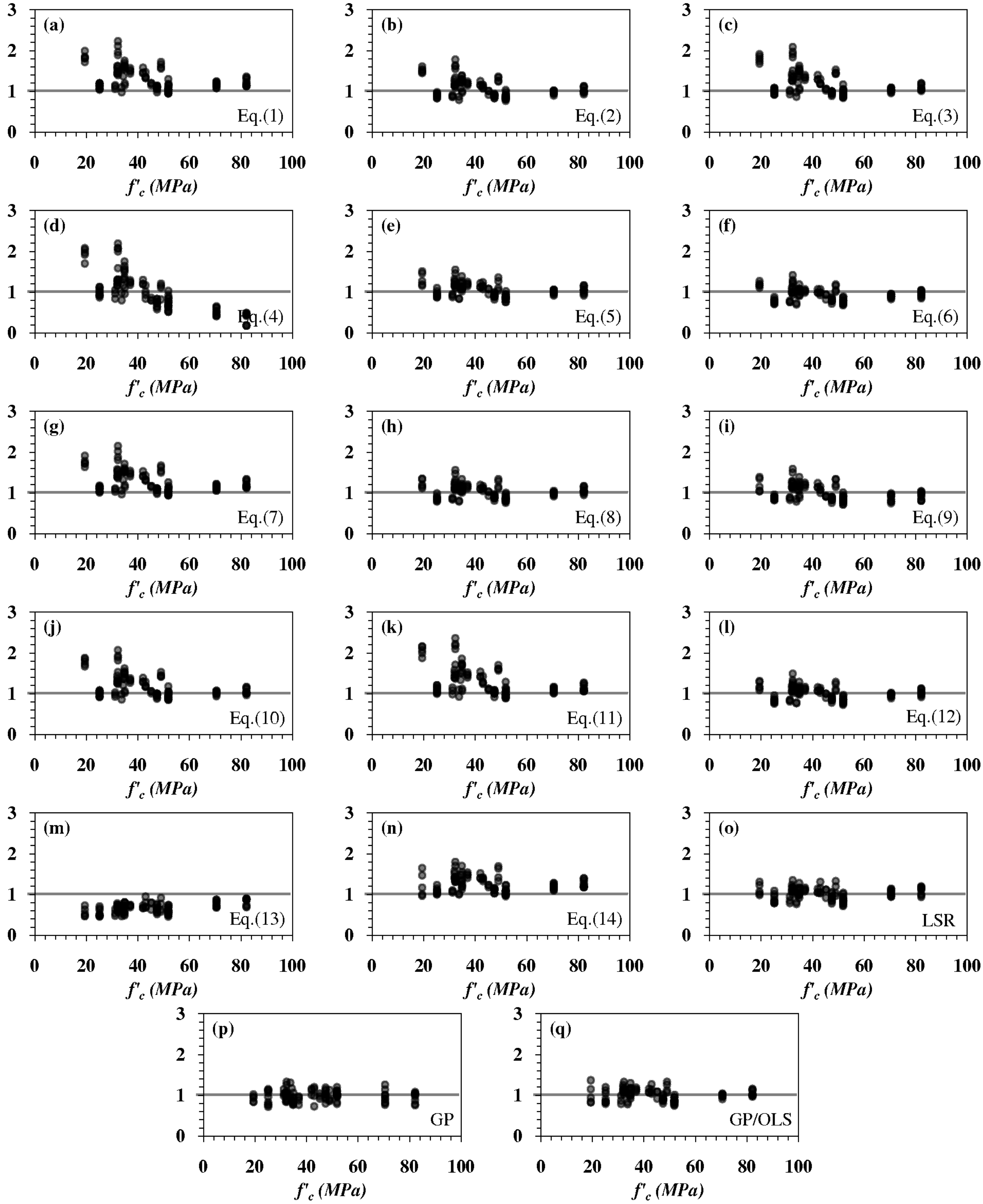


Figure 14. The ratios between the predicted and experimental compressive strength values with respect to f'_{co}. Vertical axes in units of f'_{cc} predicted/f'_{cc} experimental.

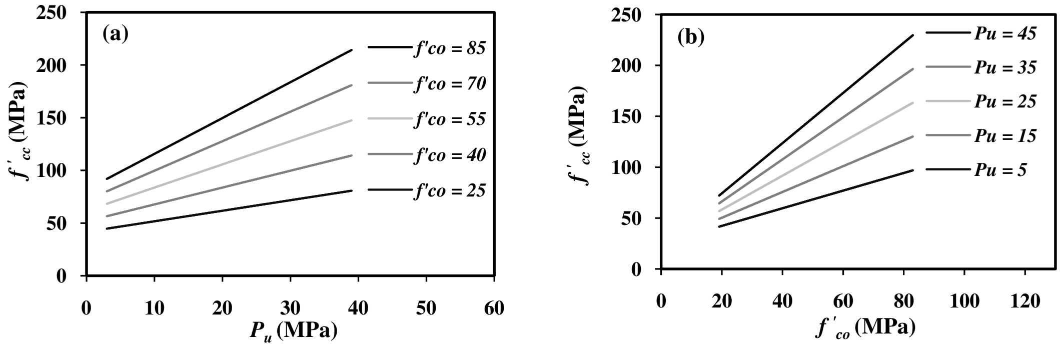


Figure 15. Parametric analysis of f'_{cc} in the GP/OLS model.

Because of the tree pruning process, the GP/OLS-based equation is very short and simple, especially in comparison with the traditional GP model. The GP/OLS predictive equation can reliably be used for routine design practice via hand calculations. However, the proposed GP/OLS-based formula is valid for the ranges of the database used for the training of the model. It can also be seen that the developed GP and LSR models perform better than most of the available FRP confinement models.

Although the proposed regression-based model yields good results for the current database, empirical modeling based on statistical regression techniques has significant limitations. Most commonly used regression analyses can have large uncertainties, which has major drawbacks for the idealization of complex processes, approximation, and averaging widely varying prototype conditions. In regression analyses, modeling of the nature of the corresponding problem is attempted by a predefined linear or nonlinear equation, which is not always true.

Equation (9), obtained by means of GP/OLS, can be expressed similarly to the form of the other formulas presented in Table 1:

$$\frac{f'_{\mathrm{cc}}}{f'_{\mathrm{co}}} = \frac{P_{\mathrm{u}}}{25} + \frac{25}{f'_{\mathrm{co}}} + \frac{2}{3}. \tag{13}$$

Figures 13 and 14 show the ratios of the compressive strength values predicted by different methods to the experimental values, with respect to P_{u} and f'_{co}. It can be observed from these figures that predictions by the models found in the literature, in most cases, are scattered with respect to both P_{u} and f'_{co}. The scattering decreases with increasing f'_{co} and increases as P_{u} increases. Figures 13 and 14 indicate that the predictions obtained by the proposed methods have good accuracy with no significant trends with P_{u} or f'_{co}. The predictions made by the GP/OLS model, with a mean value of 1.01, are slightly better compared with those obtained by the GP and LSR models.

8. Parametric analysis

For further verification of the GP/OLS model, a parametric analysis was performed. The main goal was to find the effect of each parameter on the values of compressive strength of the CFRP-wrapped concrete cylinders. The methodology was based on the change of only one input variable at a time while other input variables were kept constant at the average values of their entire data sets. Figure 15 presents the predicted strengths of concrete cylinders after CFRP confinement as a function of each parameter. The change in the predictions with variations in P_{u} and unconfined f'_{co} can be determined from these figures.

The results of the parametric study indicate that f'_{cc} increases continuously with increasing P_u and f'_{co}. The results obtained are in close agreement with those reported in [Karbhari and Gao 1997; Spoelstra and Monti 1999], for example.

9. Conclusions

A combined genetic programming and orthogonal least squares algorithm (GP/OLS) was employed to predict the complex behavior of carbon fiber-reinforced plastic (CFRP)-confined concrete columns. A simplified predictive equation was derived for the compressive strength by means of GP/OLS. A reliable database including previously published test results of the ultimate strength of concrete cylinders after CFRP confinement was used for developing the models. The GP/OLS model was benchmarked against the traditional GP, regression-based, and several CFRP confinement models found in the literature. The major findings obtained are as follows:

- The GP/OLS model is capable of predicting the ultimate strength of concrete cylinders with reasonable accuracy. The formula evolved by GP/OLS outperforms the GP, regression, and other models found in the literature.
- The GP/OLS-based predictive equation is very simple compared with the formula generated via traditional GP. This is mainly because of the important role of the tree pruning process in the GP/OLS algorithm.
- The proposed GP/OLS formula can be used for practical preplanning and design purposes in that it was developed upon on a comprehensive database with a wide range of properties.
- The results of the parametric analysis are in close agreement with the physical behavior of the CFRP-confined concrete cylinders. The results confirm that the proposed design equation is robust and can confidently be used.
- Using the GP/OLS approach, the compressive strength can accurately be estimated without carrying out destructive, sophisticated, and time-consuming laboratory tests.
- A major advantage of GP/OLS for determining the compressive strength lies in its powerful ability to model the mechanical behavior without any prior assumptions or simplifications.
- As more data becomes available, including that for other types of FRP, these models can be improved to make more accurate predictions for a wider range.

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Received 10 Mar 2009. Revised 17 Nov 2009. Accepted 31 Mar 2010.

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JOURNAL OF MECHANICS OF MATERIALS AND STRUCTURES

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Cover design: Alex Scorpan — Cover photo: Wikimedia Commons

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JoMMS (ISSN 1559-3959) is published in 10 issues a year. The subscription price for 2010 is US $500/year for the electronic version, and $660/year (+$60 shipping outside the US) for print and electronic. Subscriptions, requests for back issues, and changes of address should be sent to Mathematical Sciences Publishers, Department of Mathematics, University of California, Berkeley, CA 94720–3840.

JoMMS peer-review and production is managed by EditFlow™ from Mathematical Sciences Publishers.

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http://www.mathscipub.org
A NON-PROFIT CORPORATION
Typeset in LaTeX

Journal of Mechanics of Materials and Structures

Volume 5, No. 5 May 2010

1559-3959(2010)5:5;1-D