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The dimensions of pyramidal lattice truss core sandwich beams loaded in 3-point bending are designed to achieve the most weight-efficient structure. The bending stiffness and strength of the sandwich beam is predicted theoretically, and the identified failure modes include core member buckling, core member crushing, face wrinkling, and face crushing. To predict the relevant limit loads of the four failure modes, a force analysis is carried out based on structural mechanics. The inclined angle of the core member is determined by maximizing the bending stiffness-to-weight ratio. The length and radius of the core member, the thickness of the face, and the length of the sandwich beam are determined by maximizing the bending strength-to-weight ratio for the given height of the core. The effectiveness of the design method is validated by experiments, and the specimen with optimal geometric parameters possesses the largest weight efficiency. The useful results are extended to give an available design tool for sandwich beam manufacturers.

1. Introduction

Sandwich structures are widely used in weight-sensitive structures where high stiffness-to-weight and strength-to-weight ratios are required [Vinson 2001]. The cores of sandwich structures are conventionally made of foams and honeycombs [Bitzer 1997; Gibson and Ashby 1997; He and Hu 2008]. Recent research has shown the higher weight efficiency of lattice structures compared to that of foams [Evans et al. 2001]. Numerous studies on metallic and polymer foams have shown that the strength of the foams is governed by cell wall bending (that is, bending-dominated materials) for all loading conditions and scales as $\bar{\rho}^{1.5}$, where $\bar{\rho}$ is the relative density of the foam [Chen et al. 1999; Ashby et al. 2000]. On the other hand, the strength of a structure that deforms by cell wall stretching (that is, strength-dominated materials) scales with $\bar{\rho}$. Deshpande et al. [2001] have analyzed the topological criteria for lattice structures to be stretching-dominated. They considered a periodic assembly of pin-jointed trusses with similarly situated nodes, and determined the degree of connectivity of bars per node in order to ensure stretching domination. For the case of a sandwich panel comprised of solid faces and a pin-jointed truss core, at least three core trusses per node are required for collapse to occur by truss stretching. Based on this criterion, numerous lattice topology structures have been proposed including those with tetrahedral [Deshpande et al. 2001; Deshpande and Fleck 2001; Wallach and Gibson 2001; Wicks and Hutchinson 2001; Chiras et al. 2002], pyramidal [Zok et al. 2004; Cote et al. 2007; Queheillalt and Wadley 2009], 3D-Kagome [Hyun et al. 2003; Wang et al. 2003; Lim and Kang 2006; Fan et al. 2007] structures, and so on. Besides the superior mechanical properties, truss core sandwich structures have additional potential by virtue of their open structure for multifunctional applications [Evans et al. 1998;

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Lu et al. 2005; Wadley 2006]. For example, sandwich panels with solid faces and truss cores can serve as a heat transfer element simultaneously carrying loads. The cavity between the faces could be used for storage of a liquid in other applications.

For truss core sandwich structures, optimal design is an important problem that has attracted the attention of many researchers. The goal is to find the optimal geometric parameters that minimize the weight subject to a given load. Wicks and Hutchinson [2001] have studied the structural optimization of sandwich panels with octet truss cores and truss faces or solid faces in combinations of bending and transverse shear loads. Cote et al. [2007] have studied the minimum weight designs of pyramidal truss core sandwich columns in in-plane compression. It appears that only a few researchers have dealt with the optimal design of pyramidal truss core sandwich structures subject to 3-point bending loads. Zok et al. [2004], for example, investigated optimal design of pyramidal truss core sandwich panels in combinations of bending and transverse shear loads.

In the above-mentioned works, however, we cannot determine the thickness of the core, the thickness of the face, the radius of the core member, and the length of the sandwich structure simultaneously, and there is always a family of relationships among these variables. Thus, it is inconvenient for a structural designer. A simple and direct mathematical relation is preferable for a structural designer. Furthermore, for the lattice truss core sandwich beams, the common failure modes include face yielding, face buckling, core member yielding, and core member buckling [Wicks and Hutchinson 2001; Zok et al. 2004]. However, the four failure modes often can't converge at one point in the failure mechanism maps, and the optimal design is based on three failure modes: face yielding, face buckling, and core buckling (or face yielding, face buckling, and core yielding). Theoretically, the optimal design should be based on all four failure modes simultaneously. In the present paper, we consider a pyramidal truss core sandwich beam comprised of a pyramidal lattice truss core and two solid faces. When the sandwich beam is loaded in 3-point bending, the structural parameters are designed on the basis of optimal bending stiffness as well as bending strength, which is not according to the failure mechanism maps. Thus, all failure modes are active simultaneously. The present optimal design method offers a principle foundation for designing pyramidal truss core sandwich beams.

In this study, three assumptions are given, and then the bending stiffness and strength of a pyramidal truss core sandwich beam loaded in 3-point bending are predicted theoretically. To better understand the failure modes, force analysis of the pyramidal truss core sandwich beam is conducted. Based on the principle of minimizing the weight, the inclined angle, and the radius of the core member, the thickness of the face and the length of the sandwich beam are calculated. Experiments are carried out to validate the effectiveness of the structural design method on the specimen with optimal geometric parameters that possesses the largest weight efficiency. Several conclusions are given in the last section.

2. Predictions of the stiffness and strength of pyramidal truss core sandwich beams in 3-point bending

Consider a 3D sandwich beam with pyramidal truss cores and solid faces loaded in 3-point bending and with the geometry as shown in Figure 1. The loading is imposed on the center line of the top face. The sandwich beam is comprised of faces of thickness t_f and a square base pyramidal truss core with circular section of radius r and length l, and inclined at an angle ω with respect to the face. Thus, the height, H_c,

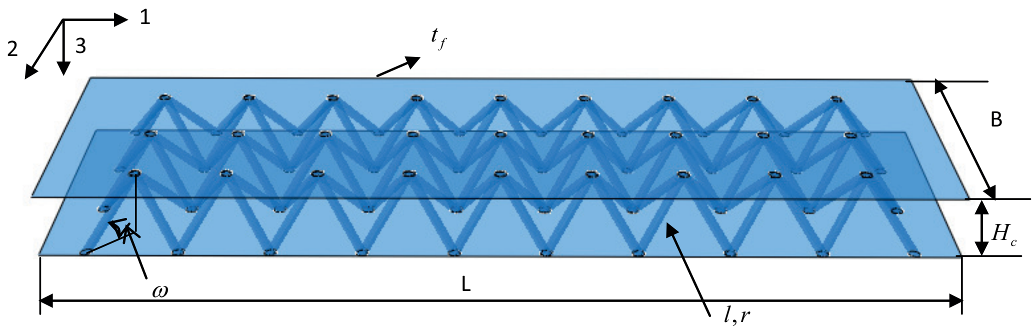

Figure 1. Geometry of the pyramidal truss core sandwich beam.

of the core is given by

$$H_c = l \sin\omega. \tag{1}$$

The span between the supports and the width of the sandwich beam are denoted by L and B, respectively. In the present paper, the span between the supports of the beam is almost equal to the length of the beam. Thus, L also represents the length of the beam in the following content. The relative density $\bar{\rho}$ of the core is given by

$$\bar{\rho} = \frac{2\pi r^2}{\sin\omega (l\cos\omega)^2}. \tag{2}$$

The underlying assumptions considered in the analytical predictions are:

- The faces are very thin compared with the total thickness of the sandwich beam.
- The macroscopic mechanical performance of the pyramidal truss core is too weak to provide a significant contribution to the bending rigidity of the sandwich beam.
- The nodes between the truss core and the faces are pin-joints offering no rotational restriction, and will not rupture in the loading process.

2.1. ***Analytical predictions of the bending stiffness.*** Referring to [Allen 1969, pp. 15–21], the deflection of a sandwich beam loaded in 3-point bending is the sum of the bending and shear deflections:

$$\delta = \frac{PL^3}{48(EI)_{\mathrm{eq}}} + \frac{PL}{4(AG)_{\mathrm{eq}}}, \tag{3}$$

where δ is the deflection at the beam's center due to P. The equivalent bending and shear stiffness are, respectively, given by

$$(EI)_{\mathrm{eq}} \approx \tfrac{1}{2} E_f B t_f H_c^2, \qquad (AG)_{\mathrm{eq}} = G_{13} B H_c. \tag{4}$$

E_f is the elastic modulus of the faces and G_{13} is the transverse shear modulus of the pyramidal truss core. According to assumptions (1) and (2), the core gives a negligible contribution to the overall bending stiffness and the faces give a negligible contribution to the overall transverse shear stiffness. The transverse shear modulus G_{13} is given by [Deshpande and Fleck 2001]

$$G_{13} = \tfrac{1}{8}\bar{\rho} E_c \sin^2(2\omega), \tag{5}$$

where E_c is the elastic modulus in the axial direction of the core member.

2.2. ***Analytical predictions of the bending strength.*** When a pyramidal truss core sandwich beam is loaded in 3-point bending, four main failure modes have been identified [Wicks and Hutchinson 2001]: core member buckling, core member crushing, face wrinkling, and face crushing. (If the material comprising the sandwich beam is elastic-brittle, such as a fiber reinforced composite, it will crush in a compression stress state; if the material is elastic-plastic, such as a metal, it will yield. We take crushing as the failure mode). To better understand the failure modes, it is essential to analyze the force distribution of a pyramidal truss core sandwich beam loaded in 3-point bending.

Force analysis based on structural mechanics. For simplicity, we initially consider a 2D sandwich structure with truss cores and truss faces subjected to a 3-point bending load, as sketched in Figure 2. The sandwich structure possesses m unit cells along the length direction, where the unit cell is represented by the gray area. The height of the core is also denoted as H_c and the inclined angle is denoted as θ. According to the foregoing pin-joint idealization of assumption (3), all members in the truss core sandwich beam are two-force rods which can only bear axial force. Based on the theory of structural mechanics, the forces in the face members and core members are shown in Figure 2. The maximum force in the face members is beneath the loading roller, which is $(m/2)P\cot\theta$. The span L of the sandwich is given by

$$L = 2mH_c\cot\theta, \tag{6}$$

and therefore the maximum force F_f in the face members is given by

$$F_f = \frac{m}{2}P\cot\theta = \frac{PL}{4H_c}. \tag{7}$$

The forces in all of the core members are identical, and are given by

$$F_c = \frac{P}{2\sin\theta}. \tag{8}$$

Subsequently, consider a 3D sandwich beam with a pyramidal truss core and solid faces as shown in Figure 1. The sandwich beam is subjected to a 3-point bending load P along the whole width, so the load carried by a unit cell along the width direction is $P\sqrt{2}l\cos\omega/B$. The schematic illustration of the

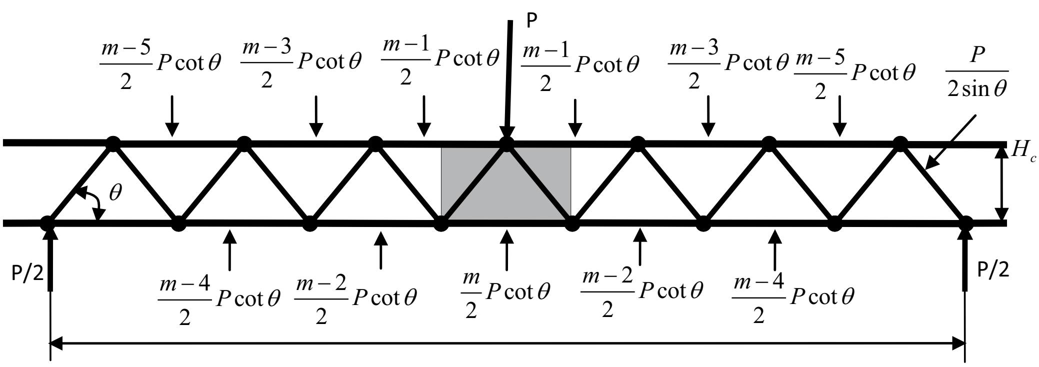

Figure 2. Force distribution of 2D sandwich beam with truss cores and truss faces.

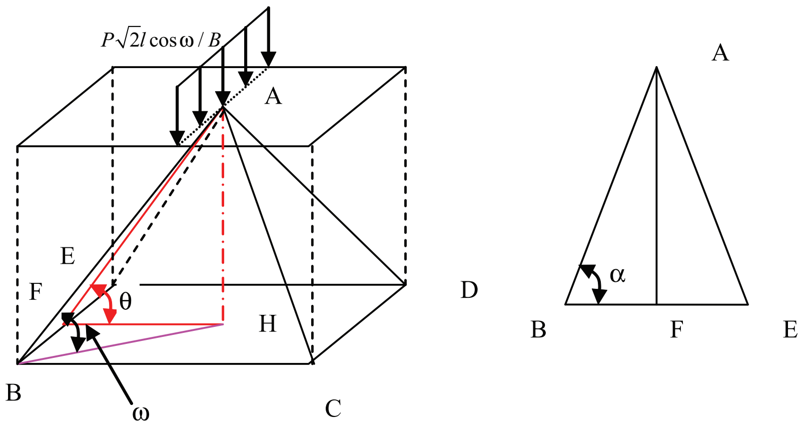

Figure 3. Force analysis and geometrical relations of a pyramidal unit cell.

force distribution of a unit cell is shown in Figure 3. From the above analysis, we have

$$F_{AF} = 2F_{AB} \sin\alpha = \frac{P\sqrt{2}l \cos\omega}{2B \sin\theta}. \tag{9}$$

According to the geometric relations, $\sin\alpha = AF/AB$, $\sin\theta = AH/AF$, and $\sin\omega = AH/AB$, we obtain

$$F_{AB} = \frac{P\sqrt{2}l \cos\omega}{4B \sin\alpha \sin\theta} = \frac{P\sqrt{2}l \cos\omega}{4B \sin\omega}. \tag{10}$$

So, the forces in the core members are

$$F_c = F_{AB} = \frac{P\sqrt{2}l \cos\omega}{4B \sin\omega}. \tag{11}$$

The maximum force in the faces of the 3D sandwich beam is also given by (7).

Failure modes: limit load calculations. According to (7) and (11), limit loads corresponding to the four failure modes are given by

$$\begin{aligned} P_{cb} &= \frac{4B \sin\omega}{\sqrt{2}l \cos\omega}\sigma_{cb}\pi r^2 \quad \text{(core member buckling)}, &\quad P_{fw} &= \frac{4H_c}{L}\sigma_{fw}Bt_f \quad \text{(face wrinkling)}, \\ P_{cc} &= \frac{4B \sin\omega}{\sqrt{2}l \cos\omega}\sigma_{cc}\pi r^2 \quad \text{(core member crushing)}, &\quad P_{fc} &= \frac{4H_c}{L}\sigma_{fc}Bt_f \quad \text{(face crushing)}, \end{aligned} \tag{12}$$

where σ_{cb} and σ_{cc} are the buckling and crushing strength of the core member, respectively, and σ_{fw} and σ_{fc} are the wrinkling and crushing strength of the faces, respectively. Core member buckling occurs between the two end joints of the truss loaded in a compressive stress state, and the buckling strength of the core member is given by

$$\sigma_{cb} = k_1^2\pi^2 E_c\left(\frac{r}{2l}\right)^2, \tag{13}$$

where $k_1 = 1$ for a core member with simply supported ends, consistent with the pin-joint assumption. The upper face is subjected to compressive stress, and may fail by elastic buckling or crushing at the location on the face with maximum force (beneath the central roller). Face wrinkling occurs between the adjacent nodes of attachment, which is known as intercellular buckling. The wrinkling strength of the faces is given by

$$\sigma_{fw} = \frac{k_2^2 \pi^2 E_f t_f^2}{12(\sqrt{2}l \cos\omega)^2}. \tag{14}$$

The rotational restraining effect of the truss core on the faces at the nodes of attachment is neglected, so $k_2 = 1$. The crushing strength of the core member σ_{cc} and the crushing strength of the face σ_{fc} can be measured experimentally.

3. Structural design of the pyramidal truss core sandwich beam

3.1. ***Design of the inclined angle of the core member.*** From (3), the overall bending stiffness-to-weight ratio will increase with the increase of the shear-modulus-to-weight ratio. The shear-modulus-to-weight ratio is given as

$$\frac{G_{13}}{\rho\bar{\rho}} = \frac{1}{8\rho} E_c \sin^2(2\omega), \tag{15}$$

where ρ is the density of the parent material. The shear-modulus-to-weight ratio reaches its maximum when the inclined angle, ω, equals 45°; the overall bending stiffness-to-weight ratio also reaches its maximum here, while other geometric parameters remain constant. Thus, the inclined angle of the core member should be fixed at 45°.

3.2. ***Design of the radius of the core member.*** For a core member loaded in axial compression loading, its weight efficiency will achieve its maximum value when core member crushing and buckling occur simultaneously. Based on this principle, we can obtain the optimal radius of the core member upon equating P_{cb} to P_{cc}, as follows:

$$r_{\mathrm{cr}} = H_c \frac{2}{k_1 \pi \sin\omega} \sqrt{\frac{\sigma_{cc}}{E_c}}. \tag{16}$$

Then, take $\sigma_{c,\mathrm{cr}}$ as the failure strength of the core member, which is given by $\sigma_{c,\mathrm{cr}} = \sigma_{cc} = \sigma_{cb}$.

3.3. ***Design of the thickness of the faces.*** Based on the same principle used in Section 3.2, the weight efficiency of the face will achieve its maximum value when the face crushing and wrinkling occur simultaneously. The optimal thickness, $t_{f,\mathrm{cr}}$, of the faces can be obtained upon equating P_{fc} to P_{fw}:

$$t_{f,\mathrm{cr}} = H_c \frac{\cos\omega}{k_2 \pi \sin\omega} \sqrt{\frac{24\sigma_{fc}}{E_f}}. \tag{17}$$

Then, take $\sigma_{f,\mathrm{cr}}$ as the failure strength of the faces, which is given by $\sigma_{f,\mathrm{cr}} = \sigma_{fc} = \sigma_{fw}$.

Design of the length of the sandwich beam. Using (12), the failure loads of the core member and faces are given as

$$P_{c,\mathrm{cr}} = \frac{4B\sin\omega}{\sqrt{2}l\cos\omega}\sigma_{c,\mathrm{cr}}\pi(r_{\mathrm{cr}})^2, \qquad P_{f,\mathrm{cr}} = \frac{4H_c}{L}\sigma_{f,\mathrm{cr}} B t_{f,\mathrm{cr}}. \tag{18}$$

The weight efficiency of the whole sandwich structure achieves its maximum when every part fails simultaneously. So the faces and core members of the sandwich beam should fail simultaneously, which means the failure load $P_{f,\mathrm{cr}}$ of the faces equates to the failure load $P_{c,\mathrm{cr}}$ of the core member, given as

$$\frac{4H_c}{L}\sigma_{f,\mathrm{cr}}Bt_{f,\mathrm{cr}} = \frac{4B\sin\omega}{\sqrt{2}l\cos\omega}\sigma_{c,\mathrm{cr}}\pi(r_{\mathrm{cr}})^2. \tag{19}$$

Substituting (1), (16), and (17) into (19), we obtain

$$L = H_c\frac{k_1^2}{k_2}\frac{(\cos\omega)^2}{\sin\omega}\frac{\sigma_{f,\mathrm{cr}}E_c}{\sigma_{c,\mathrm{cr}}^2}\sqrt{\frac{3\sigma_{f,\mathrm{cr}}}{E_f}}. \tag{20}$$

Hitherto, the design procedure can be summarized as follows:

- From (14), the inclined angle of the core member should be fixed at 45°.
- Given H_c, one can determine the length of the core member, the radius of the core member, the thickness of the faces and the length of the sandwich beam from (1), (16), (17), and (20), respectively.

4. Experiments and discussion

In order to validate the feasibility and effectiveness of the proposed structural design method, experiments are conducted.

4.1. ***Fabrication of specimen.*** The sandwich beam specimen is made of a carbon fiber reinforced composite T700/epoxy with a hot compression molding method. A set of molds with special structures are designed to fabricate the pyramidal truss core sandwich beam. The faces are comprised of a laminate of prepreg with the stack sequence $(0/90)_n$, where the number n can vary to gain a different face thickness. The core member is rolled into a circular cross section from two plies of prepreg with the stack sequence (0/90). The preformed sandwich beam is cured at 125° C in an autoclave under a pressure of 0.5 MPa for an hour, and cools down to room temperature naturally. Then the specimen is taken out from the autoclave and removed from the molds. Because the sandwich beam specimen is fabricated with the same composite material and stack sequence, the equivalent elastic modulus and crushing strength of the face and core member are equal. Thus, $E_f = E_c = 60\,\mathrm{GPa}$ and $\sigma_{cc} = \sigma_{fc} = 329\,\mathrm{MPa}$. The density of the T700/epoxy composite is $\rho = 1500\,\mathrm{kg\,m^{-3}}$.

4.2. ***Test protocol.*** Ideally, the specimen geometries are designed to probe the different failure modes of face crushing, face wrinkling, core member crushing, and core member buckling. However, in the current study, it was not feasible to make a range of pyramidal truss core sandwich beams with different truss dimensions. From the above analysis, the optimal inclined angle of the core member is 45°, and thus the inclined angle of the specimen is fixed at 45° and the height of the core is 15 mm. The optimal radius of the core member is 1 mm. For the sandwich beam specimen, the radius of the core member is chosen as 1.25 mm, which is slightly lager than the optimal radius computed from (16). The reasons are as follows. First, it is difficult for fabrication if the truss radius is too small. Second, the material properties of the composite possess discreteness, so, we take a conservative value of the truss radius to prevent core member crushing from occurring prematurely. The optimal length of the sandwich beam and the thickness of the face are 248 mm and 1.73 mm, respectively. In order to find the largest weight efficiency, three types of specimens with face thicknesses 0.84 mm, 1.73 mm, and 2.6 mm are designed and fabricated. The width of the specimen is 100 mm, and the span of the sandwich beam is taken as 248 mm, which is equal to the optimal value.

The weight of the sandwich beam is

$$W = \rho L B(\bar{\rho} H_c + 2t_f). \tag{21}$$

The weight efficiency η is defined as

$$\eta = \frac{P_{\mathrm{cr}}}{W}, \tag{22}$$

where P_{cr} is the failure load of the pyramidal lattice truss core sandwich beam loaded in 3-point bending.

The specimen is tested under 3-point bending using an Instron 5569 machine, applying loading through a 10 mm diameter cylindrical roller in accordance with ASTM standard C393 [ASTM 2006]. The load is applied at a rate of 1 mm min^{-1} and recorded with the load cell of the testing machine. A laser extensometer is used to measure the deflection, δ, of the specimen.

4.3. ***Results and discussion.*** The failure loads and weight efficiencies of the three types of specimens are listed in Table 1, and the corresponding bending responses are shown in Figure 4. From Table 1, good agreement is found between the measured and predicted failure loads, and the measured failure loads are somewhat lower than the predicted ones. These discrepancies are attributed to imperfections in the manufactured specimens, while the analytical predictions pertain to the idealized geometry. The predicted and measured weight efficiencies of the specimen with face thickness 1.73 mm are larger than those of the specimens with face thicknesses 0.84 mm and 2.6 mm, which validates the effectiveness of the structural design method.

The corresponding failure mode photographs are shown in Figures 5, 6, and 7. From Figure 5, it is found that the upper face of the specimen with face thickness 0.84 mm wrinkles like a sine wave in the compressive stress. For the specimen with face thickness 1.73 mm, it is found, from Figure 6, that the upper face and the core member crush simultaneously. From Figure 7, it is found that only core member

Predicted				
Thickness of face-sheet (mm)	Failure mode	Failure load (kN)	Weight (g)	Weight efficiency
0.84	FW	1.57	74.5	2107
1.73	FCC	13.73	151.9	9038
2.6	CC	13.78	227.6	6054
Measured				
Thickness of face-sheet (mm)	Failure mode	Failure load (kN)	Weight (g)	Weight efficiency
0.84	FW	1.55	85	1823
1.73	FCC	11.63	156	7455
2.6	CC	12.87	235	5476

Table 1. The predicted and measured failure loads and weight efficiencies. FW indicates face wrinkling, FCC face and core member crushing simultaneously, and CC core member crushing; for simplicity, the acceleration of gravity is taken as 10 N kg^{-1} in the computation of weight efficiency.

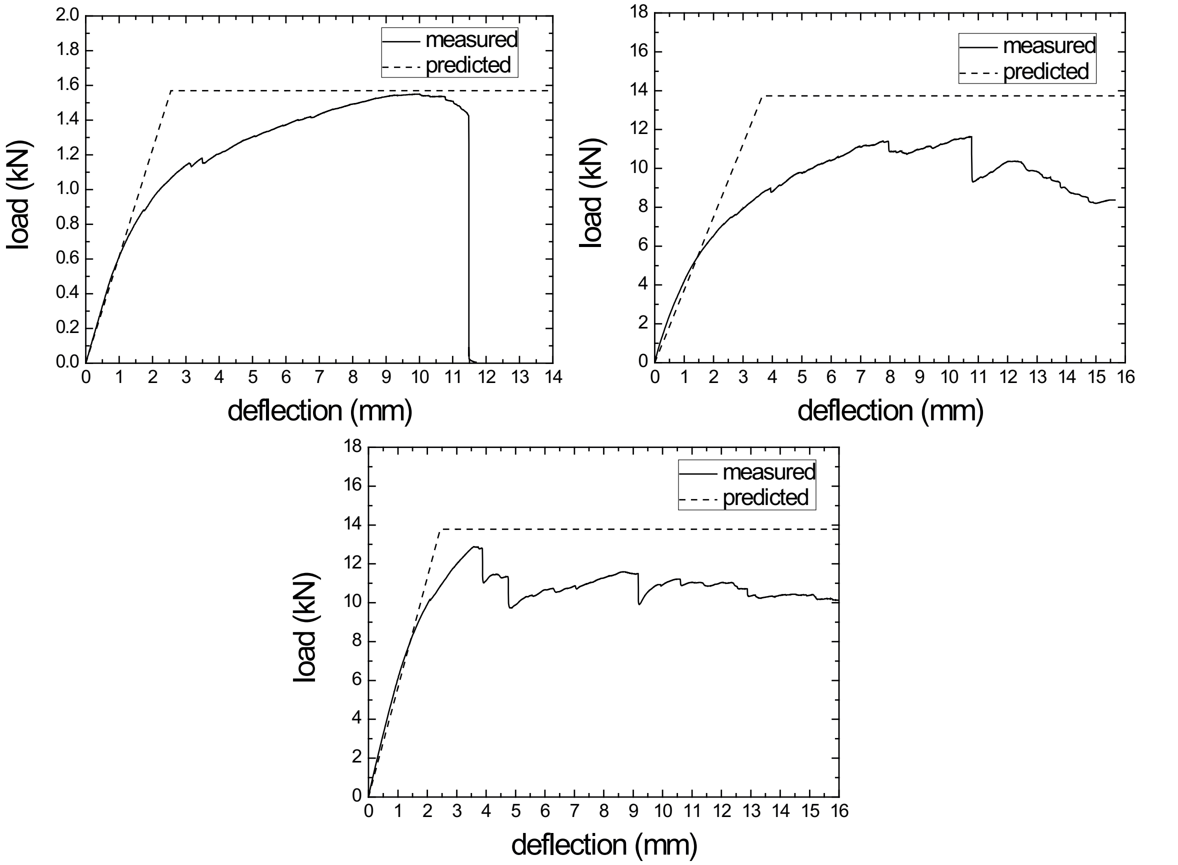

Figure 4. Bending response of sandwich structures with different face-sheet thicknesses: 0.84 mm (upper left), 1.73 mm (upper right), and 2.6 mm (lower).

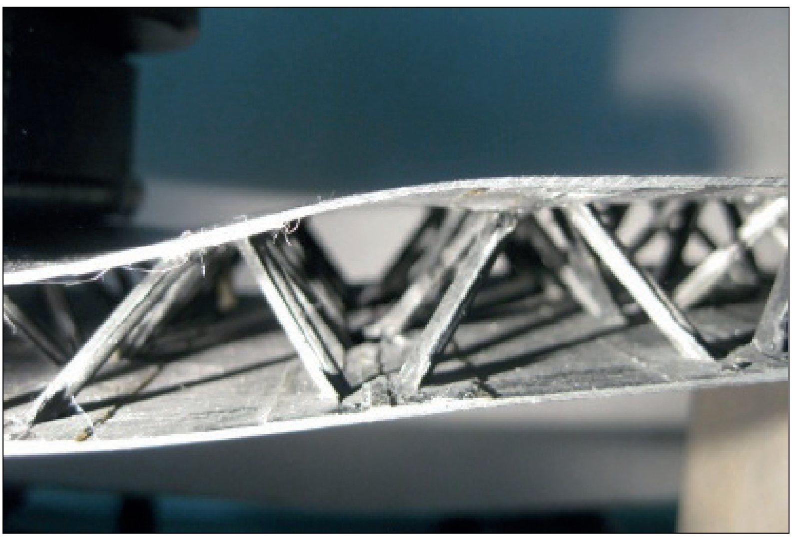

Figure 5. Face-sheet wrinkling of the specimen with face-sheet thickness 0.84 mm.

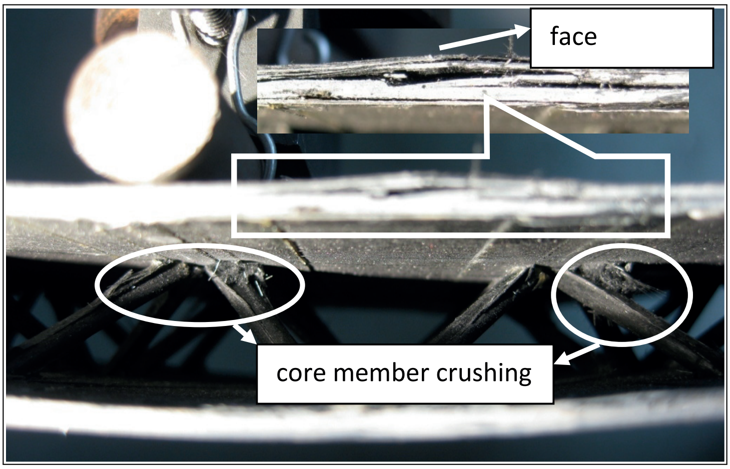

Figure 6. Face-sheet and core member crushing occur simultaneously for the specimen with face-sheet thickness 1.73 mm.

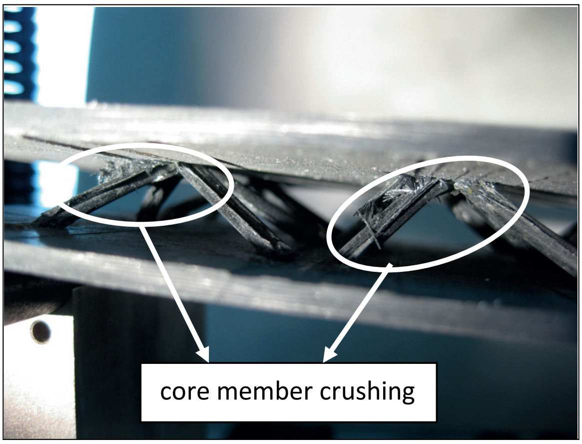

Figure 7. Core member crushing of the specimen with face-sheet thickness 2.6 mm.

crushing occurs for the specimen with face thickness 2.6 mm. All the failure modes observed above agree well with the predicted ones.

From Table 1, it is found that the ratio of the core thickness to the face thickness is about 9 at the condition of optimal geometric parameters. According to the quantitative definition of thin and thick face-sheets of sandwich structures given by [Allen 1969, pp. 15–21], the face can be treated as thin when the core thickness is lager than 5.77 times the face thickness. So, assumption (1) is reasonable. The length to the radius of the core member is about 20, which causes the transverse force of the core member to be rather smaller than the axial force, so the pin-joint idealization of assumption (2) is reasonable.

Generally, in actual loading conditions, the length of the beam is given. For example, for a bridge, the total span is directly given. Thus, in the geometric parameter optimization of a sandwich beam, the thickness of the core is considered as a design variable and the length of the sandwich beam is considered to be fixed. The thickness of the core can be determined from (20). Subsequently, the length of the core member, the radius of the core member and the thickness of the face can be determined with (1), (16), and (17), respectively.

5. Conclusions

- Based on the pin-joint assumption, force analysis is implemented in a pyramidal truss core sandwich beam loaded in 3-point bending, and the limit load calculations are then conducted.
- The inclined angle of the core member should be fixed at 45° to maximize the overall bending stiffness-to-weight ratio.
- The length of the core member, the radius of the core member, the thickness of the face, and the length of the sandwich beam are all deduced with the condition of maximizing the bending strength-to-weight ratio.
- Experiments validate the feasibility and effectiveness of the proposed structural design method, and the specimen with optimal geometric parameters possesses the largest weight efficiency.
- In the present study, a simple and direct design procedure is developed, which is easy to implement for manufacturers.

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