Journal of
Mechanics of
Materials and Structures

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msp

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DISPERSION OF GUIDED WAVES IN INITIALLY STRESSED LAYERED PLATES

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Based on the theory of the mechanics of incremental deformations, guided wave propagation in multi-layered plates under initial stresses in the thickness and wave-propagation directions is investigated. The Legendre polynomial series method is used to solve the coupled wave equations and its convergence is discussed along with numerical examples. The effects of initial stresses on Lamb-like waves and SH waves are analyzed, and numerical results show that they are quite distinct. The frequency variation is not linear with the increase of the initial stresses in both directions. The effects of the initial stress in the thickness direction are very different from those in the longitudinal direction.

1. Introduction

Because of the limitations of manufacturing and assembling technology, initial stresses in structural elements, especially in layered structures, are inevitable. The working loads on structures can be taken as the initial stresses. Thus, the problem of initially stressed structures is very common and it is very important to study it in both the practical and the theoretical sense.

Wave propagation in structures with initial stresses has received considerable attention. Chen and Wright [1966] derived the frequency equations for wave propagation in an initially stressed circular cylinder. Murdoch [1977] considered the effect of interfacial stress on the propagation of Stoneley waves. Ogden and Sotiropoulos [1995] studied interfacial waves in prestressed layered incompressible elastic solids. Gei [2008] investigated small-amplitude interfacial waves along a thin film between two pre-stressed, incompressible elastic media. Rayleigh waves in a magnetoelastic initially stressed conducting medium with a gravity field were investigated in [El-Naggar et al. 1994]. Abd-Alla and Ahmed [1999] investigated Love wave propagation in a nonhomogeneous orthotropic elastic layer with initial stress over-lying a semiinfinite medium. Abd-Alla [1999] presented the effect of initial stress on axisymmetric waves in an orthotropic hollow cylinder. Qian et al. [2004] studied the effect of initial stress on Love waves in an elastic substrate with a piezoelectric layer and a piezoelectric substrate with an elastic layer. Du et al. [2007] studied Love wave propagation in layered magnetoelectroelastic structures with initial stress. Garg [2007] considered the effect of initial stress on harmonic plane waves in viscoelastic anisotropic media. Gupta et al. [2012] investigated torsional surface waves in an initially stressed heterogeneous half-space covered by a homogeneous layer. Bigoni et al. [2008] studied the filtering and band gap characteristics of a prestressed, stiff layer on an elastic half-space.

The above investigations are all about surface waves, subsurface waves, and interfacial waves. Guided wave propagation in structures with initial stresses has also received attention. Akbarov and Guliev [2009] studied axisymmetric longitudinal waves in a prestrained compound circular cylinder. Akbarov et al. [2011b] studied the case of torsional waves. Akbarov and Guliev [2010] investigated axisymmetric

longitudinal waves in a prestrained circular cylinder embedded in a prestrained compressible infinite medium. These three papers are all about circular cylinders. As they are more common structures, initially stressed multilayered plates have received more attention. Kayestha et al. [2010] obtained dispersion curves for a perfectly bonded prestressed compressible elastic bilayered plate. Wijeyewickrema and Leungvichcharoen [2009] investigated the influence of imperfections in the contact conditions between the layers of a prestrained three-layered plate. These two investigations do not allow readers to draw any concrete conclusions about the influence of the initial strains on the wave propagation velocity or on wave dispersion in layered media. Akbarov et al. [2008; 2011a] and Zamanov and Agasiyev [2011] investigated these issues. Akbarov et al. [2008] and Zamanov and Agasiyev [2011] studied the influence of initial strains in the wave-propagation direction on the Lamb wave dispersion curves. Akbarov et al. [2011a] studied the influence of initial strains in the thickness direction on the Lamb wave dispersion curves. Furthermore, small-amplitude waves in prestressed [Rogerson and Sandiford 1996; Gei et al. 2004; Gei 2008] and large deformed [Shmuel et al. 2012] multilayered structures have been given attention.

This paper simultaneously investigates the influence of initial stresses in two directions, the thickness and wave-propagation directions. Their different effects are discussed. We also illustrate the case of SH waves. The influences on Lamb waves and on SH waves are very different.

2. Mathematics and formulation of the problem

Consider an orthotropic N-layered plate with initial stresses in two directions, $S_{xx} = -\sum_{n=1}^{N} P_n \pi_{h_{n-1},h_n}(z)$ and $S_{zz} = -Q$, as shown in Figure 1. The rectangular window function is

$$\pi_{h_{n-1},h_n}(z) = \begin{cases} 1, & h_{n-1} \leq z \leq h_n, \\ 0, & \text{elsewhere.} \end{cases}$$

The plate is infinite in the horizontal direction and has a total thickness h_N. We place the horizontal (x, y)-plane of a Cartesian coordinate system on the bottom surface and let the plate be in the positive z region, where the medium occupies the region $0 \leq z \leq h_N$.
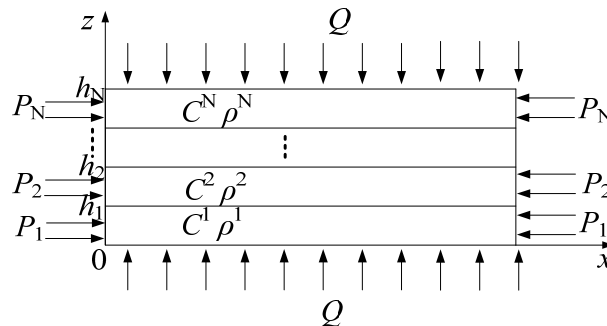

Figure 1. Schematic diagram of a layered plate under initial stresses.

According to [Biot 1965] (see also [Selim and Ahmed 2006]) the dynamic equations for the plate with initial stresses are governed by

$$\frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} - \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)\frac{\partial S_{xx}}{\partial z} + S_{xx}\frac{\partial \omega_z}{\partial y} + (S_{zz} - S_{xx})\frac{\partial \omega_y}{\partial z} = \rho\frac{\partial^2 u_x}{\partial t^2},$$

$$\frac{\partial T_{xy}}{\partial x} + \frac{\partial T_{yy}}{\partial y} + \frac{\partial T_{yz}}{\partial z} - S_{zz}\frac{\partial \omega_x}{\partial z} + S_{xx}\frac{\partial \omega_z}{\partial x} = \rho\frac{\partial^2 u_y}{\partial t^2}, \qquad (1)$$

$$\frac{\partial T_{xz}}{\partial x} + \frac{\partial T_{yz}}{\partial y} + \frac{\partial T_{zz}}{\partial z} + (S_{zz} - S_{xx})\frac{\partial \omega_y}{\partial x} - S_{zz}\frac{\partial \omega_x}{\partial y} = \rho\frac{\partial^2 u_z}{\partial t^2},$$

where

$$\omega_x = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right), \quad \omega_y = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right), \quad \omega_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right). \qquad (2)$$

The relationships between the strain and displacement components can be expressed as

$$\epsilon_{xx} = \frac{\partial u_x}{\partial x}, \qquad \epsilon_{yy} = \frac{\partial u_y}{\partial y}, \qquad \epsilon_{zz} = \frac{\partial u_z}{\partial z},$$

$$\epsilon_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right), \quad \epsilon_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right), \quad \epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right). \qquad (3)$$

Here u_i, T_{ij} and ϵ_{ij} are the elastic displacements, stresses, and strains, respectively, while ρ is the density of the plate.

By introducing the rectangular window function $\pi_{0,h_N}(z)$, the stress-free boundary ($T_{zz} = T_{xz} = T_{yz} = 0$ at $z = 0$, $z = h_N$) is automatically incorporated in the constitutive relations of the plate [Datta and Hunsinger 1978]

$$T_{xx} = C_{11}\epsilon_{xx} + C_{12}\epsilon_{yy} + C_{13}\epsilon_{zz}, \qquad T_{yy} = C_{12}\epsilon_{xx} + C_{22}\epsilon_{yy} + C_{23}\epsilon_{zz},$$

$$T_{zz} = (C_{13}\epsilon_{xx} + C_{23}\epsilon_{yy} + C_{33}\epsilon_{zz})\pi_{0,h_N}(z), \qquad T_{yz} = 2C_{44}\epsilon_{yz}\pi_{0,h_N}(z), \qquad (4)$$

$$T_{xz} = 2C_{55}\epsilon_{xz}\pi_{0,h_N}(z), \qquad T_{xy} = 2C_{66}\epsilon_{xy},$$

where C_{ij} are the elastic coefficients.

For the layered plate these coefficients are expressed as

$$C_{ij} = \sum_{n=1}^{N} C_{ij}^n \pi_{h_{n-1},h_n}(z), \qquad (5a)$$

where N is the number of the layers and C_{ij}^n is the elastic constant of the N-th material. Similarly, the mass density can be expressed as

$$\rho = \sum_{n=1}^{N} \rho^n \pi_{h_{n-1},h_n}(z). \qquad (5b)$$

For a free harmonic wave being propagated in the x direction in a plate, we assume the displacement components to be of the form

$$u_x(x, y, z, t) = \exp(ikx - i\omega t)U(z), \tag{6a}$$

$$u_y(x, y, z, t) = \exp(ikx - i\omega t)V(z), \tag{6b}$$

$$u_z(x, y, z, t) = \exp(ikx - i\omega t)W(z). \tag{6c}$$

$U(z)$, $V(z)$, and $W(z)$ represent the amplitude of vibration in the x, y, and z directions, respectively, while k is the magnitude of the wave vector in the propagation direction and ω is the angular frequency.

Substituting (3)–(6) into (1), the governing differential equations in terms of displacement components can be obtained:

$$
\begin{aligned}
U'' \sum_{n=1}^{N}(C_{55}^n + 0.5P_n - 0.5Q)\pi_{h_{n-1},h_n}(z) &+ U'\Big(\sum_{n=1}^{N}(C_{55}^n + 0.5P_n)\pi_{h_{n-1},h_n}(z)\Big)' \\
- k^2 U \sum_{n=1}^{N}(C_{11}^n + P_n)\pi_{h_{n-1},h_n}(z) &+ ikW' \sum_{n=1}^{N}(C_{13}^n + C_{55}^n + 0.5P_n + 0.5Q)\pi_{h_{n-1},h_n}(z) \\
+ ikW\Big(\sum_{n=1}^{N}(C_{55}^n + 0.5P_n)\pi_{h_{n-1},h_n}(z)\Big)' &= -\omega^2 U \sum_{n=1}^{N}\rho^n \pi_{h_{n-1},h_n}(z),
\end{aligned} \tag{7a}
$$

$$
\begin{aligned}
V'' \sum_{n=1}^{N}(C_{44}^n - 0.5Q)\pi_{h_{n-1},h_n}(z) + V'\Big(\sum_{n=1}^{N} C_{44}^n \pi_{h_{n-1},h_n}(z)\Big)' - k^2 V \sum_{n=1}^{N}(C_{66}^n - 0.5P_n)\pi_{h_{n-1},h_n}(z) \\
= -\omega^2 V \sum_{n=1}^{N}\rho^n \pi_{h_{n-1},h_n}(z),
\end{aligned} \tag{7b}
$$

$$
\begin{aligned}
W'' \sum_{n=1}^{N}(C_{33}^n + Q)\pi_{h_{n-1},h_n}(z) &+ ikU' \sum_{n=1}^{N}(C_{13}^n + C_{55}^n + 0.5P_n + 0.5Q)\pi_{h_{n-1},h_n}(z) \\
- k^2 W \sum_{n=1}^{N}(C_{55}^n - 0.5P_n + 0.5Q)\pi_{h_{n-1},h_n}(z) &+ W'\Big(\sum_{n=1}^{N}(C_{33}^n + Q)\pi_{h_{n-1},h_n}(z)\Big)' \\
+ ikU\Big(\sum_{n=1}^{N}(C_{13}^n + Q)\pi_{h_{n-1},h_n}(z)\Big)' &= -\omega^2 W \sum_{n=1}^{N}\rho^n \pi_{h_{n-1},h_n}(z)
\end{aligned} \tag{7c}
$$

where the superscript $(\ )'$ is the partial derivative for z. Obviously, (7b) is independent of the other two equations. It represents the antiplane SH waves. The other two, (7a) and (7c), control the in-plane Lamb-like waves.

To solve the coupled wave (7), we expand the field quantities of each layer into one specific Legendre orthogonal polynomial series:

• In the first layer

$$u_a^1 = \sum_{m=0}^{\infty} p_{m,1}^a Q_m^1(z) \exp(ikx), \tag{8a}$$

with

$$Q_m^1(z) = \sqrt{\frac{2m+1}{h_1}} L_m\left(\frac{2}{h_1}z - 1\right)$$

and

$$u_a^1(z = h_1) = u_a^{1,h_1} = \sum_{m=0}^{\infty} p_{m,1}^a Q_m^1(z = h_1) \exp(ikx).$$

- In the second layer

$$u_a^2 = u_a^{1,h_1} + (z - h_1) \sum_{m=0}^{\infty} p_{m,2}^a Q_m^2(z) \exp(ikx), \tag{8b}$$

with

$$Q_m^2(z) = \sqrt{\frac{2m+1}{h_2-h_1}} L_m\left(\frac{2}{h_2-h_1}z - \frac{h_2+h_1}{h_2-h_1}\right)$$

and

$$u_a^2(z = h_2) = u_a^{2,h_2} = u_a^{1,h_1} + (h_2 - h_1) \sum_{m=0}^{\infty} p_{m,2}^a Q_m^2(z = h_2) \exp(ikx).$$

- The subsequent layers follow as such.
- In the last layer

$$u_a^N = u_a^{N-1,h_{N-1}} + (z - h_{N-1}) \sum_{m=0}^{\infty} p_{m,N}^a Q_m^N(z) \exp(ikx), \tag{8c}$$

with

$$Q_m^N(z) = \sqrt{\frac{2m+1}{h_N-h_{N-1}}} L_m\left(\frac{2}{h_N-h_{N-1}}z - \frac{h_N+h_{N-1}}{h_N - h_{N-1}}\right)$$

and

$$u_a^N(z = h_N) = u_a^{N,h_N} = u_a^{N-1,h_{N-1}} + (h_N - h_{N-1}) \sum_{m=0}^{\infty} p_{m,N}^a Q_m^N(z = h_N) \exp(ikx)$$

and so on.

Here, L_m is the m-th order Legendre polynomial. Theoretically, m runs from 0 to ∞. In practice, the summation over the polynomials in (8) can be truncated at some finite value $m = M$, at which higher-order terms become negligible.

The forms of u_a ($a = 1, 2, 3$) and (u_x, u_y, u_z) are chosen in order to automatically incorporate in the calculation the continuity conditions at the interfaces relative to the components of the mechanical displacement.

Equation (7) is multiplied by $Q_j^{1*}(z)$, $Q_j^{2*}(z)$, ..., $Q_j^{N*}(z)$, with j running from 0 to M, so that integration over z from 0 to h_N yields the following $3N(M+1)$ equations:

$$ {}^nA_{11}^{j,m} p_{m,n}^1 + {}^nA_{12}^{j,m} p_{m,n}^2 + {}^nA_{13}^{j,m} p_{m,n}^3 = -\omega^2 \cdot {}^nM_m^j p_{m,n}^1, \tag{9a} $$

$$ {}^nA_{21}^{j,m} p_{m,n}^1 + {}^nA_{22}^{j,m} p_{m,n}^2 + {}^nA_{23}^{j,m} p_{m,n}^3 = -\omega^2 \cdot {}^nM_m^j p_{m,n}^1, \tag{9b} $$

$$ {}^nA_{31}^{j,m} p_{m,n}^1 + {}^nA_{32}^{j,m} p_{m,n}^2 + {}^nA_{33}^{j,m} p_{m,n}^3 = -\omega^2 \cdot {}^nM_m^j p_{m,n}^1. \tag{9c} $$

where ${}^nA_{\alpha\beta}^{j,m}$ ($\alpha, \beta = 1, 2, 3$) and ${}^nM_m^j$ are the elements of a nonsymmetric matrix. They can be easily obtained according to (7).

Equation (9) can be written as

$$ \begin{bmatrix} {}^nA_{11}^{j,m} & {}^nA_{12}^{j,m} & {}^nA_{13}^{j,m} \\ {}^nA_{21}^{j,m} & {}^nA_{22}^{j,m} & {}^nA_{22}^{j,m} \\ {}^nA_{31}^{j,m} & {}^nA_{32}^{j,m} & {}^nA_{33}^{j,m} \end{bmatrix} \begin{Bmatrix} p_{m,n}^1 \\ p_{m,n}^2 \\ p_{m,n}^3 \end{Bmatrix} = -\omega^2 \begin{bmatrix} {}^nM_m^j & 0 & 0 \\ 0 & {}^nM_m^j & 0 \\ 0 & 0 & {}^nM_m^j \end{bmatrix} \begin{Bmatrix} p_{m,n}^1 \\ p_{m,n}^2 \\ p_{m,n}^3 \end{Bmatrix}. \tag{10} $$

So, (10) yields a form of the eigenvalue problem. The eigenvalue ω^2 gives the angular frequency of the guided wave; eigenvectors $p_{m,n}^i$ ($i = 1, 2, 3$) allow the components of the particle displacement to be calculated. According to $Vph = \omega/k$ and $Vg = d\omega/dk$, the phase velocity and group velocity can be obtained. Equation (10) can be solved numerically making use of standard computer programs for the diagonalization of nonsymmetric square matrices. $3N(M+1)$ eigenmodes are generated from the order-M expansion. Acceptable solutions are those eigenmodes for which convergence is obtained as M is increased. We determine that the eigenvalues obtained are converged when a further increase in the matrix dimension does not result in a significant change in the eigenvalue. We consider 0.5% as a significant change.

3. Numerical results

Based on the foregoing formulations, computer programs have been written using Mathematica to calculate the dispersion curves. We take common ceramic-metal (silicon nitride (N) and steel (S)) layered plates as examples. Their material properties are listed in Table 1. Because SH waves and Lamb-like waves are independent, we will discuss them respectively.

3.1. Convergence of the method. In order to verify the convergence of the method, the Lamb-like wave and SH wave dispersion curves for a three-layered plate under an uniform initial stress $Q = 20\,\text{GPa}$ are calculated for $M = 6, 7, 8$, and 9, as shown in Figure 2. The stacking sequence and thickness (m) of the plate are N/S/N-1/2/1.

Material	C_{11}	C_{13}	C_{33}	C_{44}	C_{55}	C_{66}	ρ
Steel	282	113	282	84	84	84	7.932
Silicon nitride	380	120	380	130	130	130	2.37

Table 1. The material properties of the layered plate. C_{ij} values are in units of $10^9\,\text{N/m}^2$, ρ values in $10^3\,\text{kg/m}^3$.
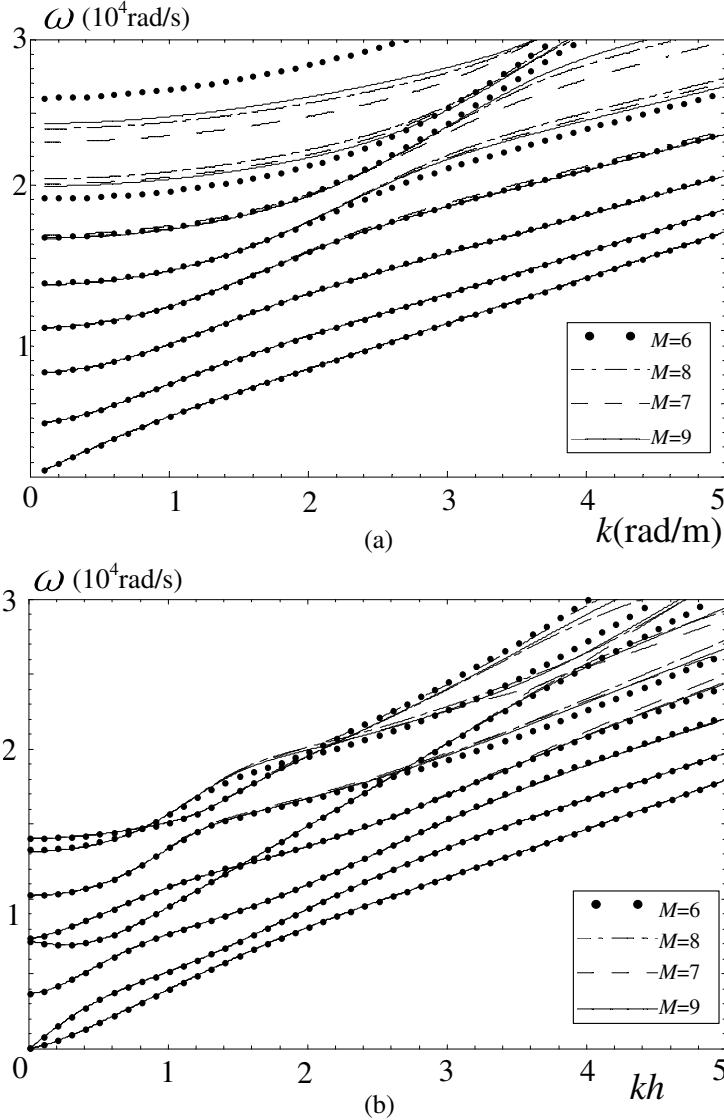
Figure 2. Dispersion curves for the N/S/N-1/2/1 plate under uniform initial stress $Q = 20\,\text{GPa}$ for various M values. (a) SH waves and (b) Lamb-like waves.

For SH waves, the first three modes are convergent when $M = 6$; the first four modes are convergent when $M = 7$ and 8. So, we think that at least the first $M/2$ SH modes are convergent. Similarly, for Lamb-like waves, the first three modes are convergent when $M = 6$; the first four modes are convergent when $M = 7$ and 8. So, we believe that at least the first $M/2$ Lamb-like wave modes are convergent.

For all the numerical calculations presented below, the series expansion is truncated at $M = 16$.

3.2. *Effects of the initial stresses on Lamb-like waves.* Figure 3 shows the phase velocity dispersion curves for the N/S/N-1/2/1 plate under different uniform initial stresses. Figure 3a is for the plate under initial stresses Q and Figure 3b is for the plate under uniform initial stresses P, that is, $P_1 = P_2 = P_3$.
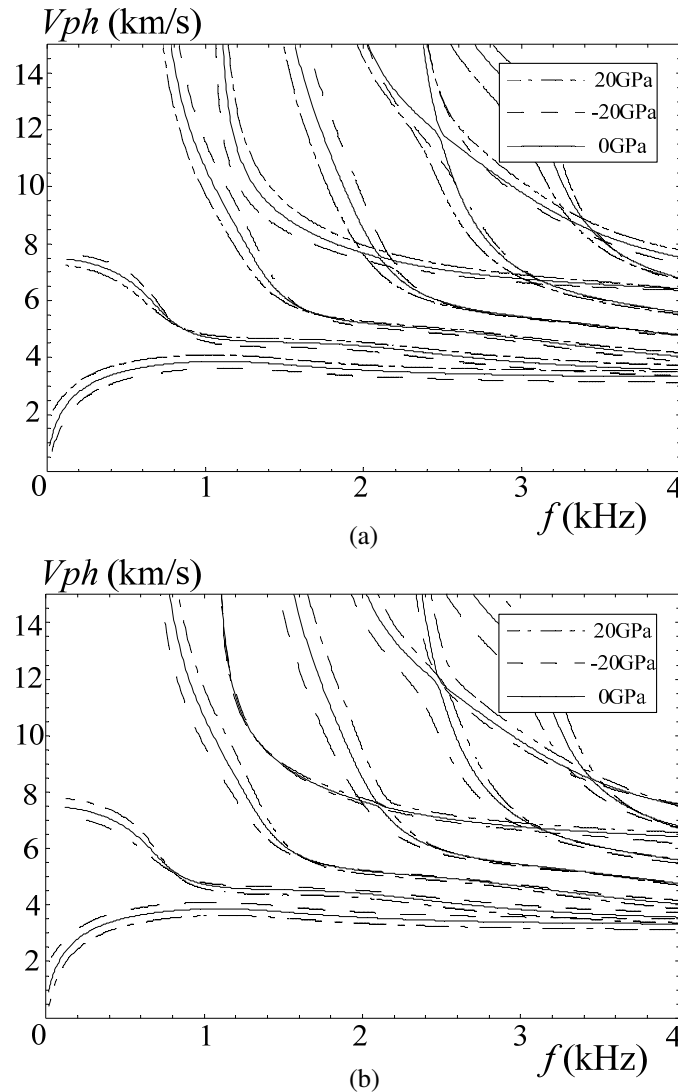
Figure 3. Lamb-like wave phase velocity spectra for the N/S/N-1/2/1 plate, (a) under
initial stresses Q and (b) under uniform initial stresses P.

According to Figure 3a, a tensile stress in the thickness direction usually makes the wave speed lower
at low frequencies (except for the first mode). As the frequency increases, a tensile stress makes the
wave speed higher. In many cases, the effect of the tensile stress is contrary to that of the compressive
stress. Comparing the two figures, we can see that the effect of the initial stress P is quite different from
that of the initial stress Q. In many cases, the effect of the initial stress P is almost contrary to that of
the initial stress Q.

Figure 3 reflects the relation between the effect of the initial stress and frequency. We observe the
relation between the frequency (or wave speed) and the magnitudes of initial stress Q and P, as shown
in Figures 4a and 4b, which show the frequency versus initial stress Q and P at $k = 1\,\text{rad/m}$. In fact,
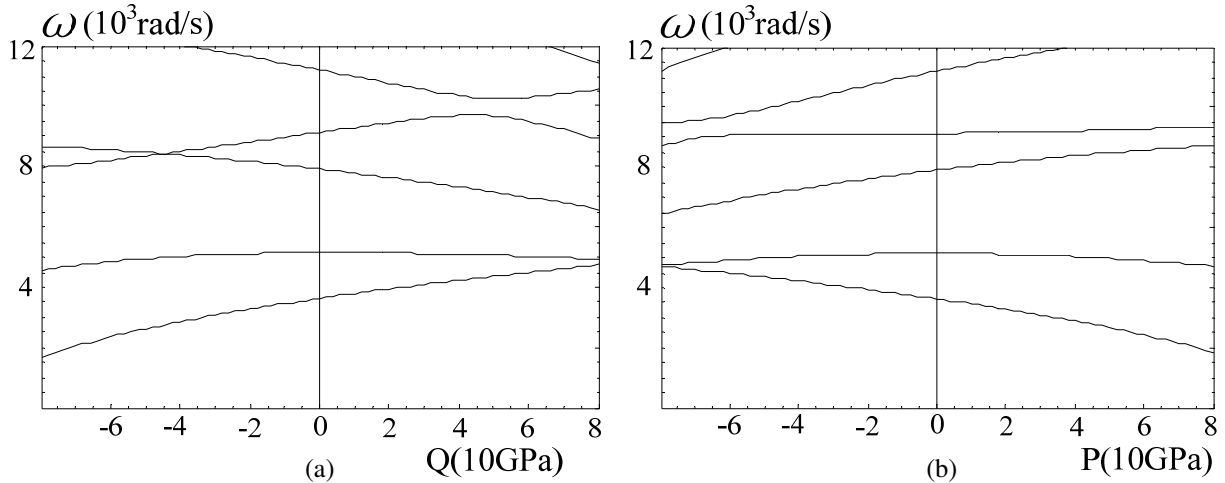
Figure 4. Lamb-like wave frequency curves versus initial stresses for the N/S/N-1/2/1 plate at $k = 1$ rad/m, with (a) uniform initial stresses Q and (b) uniform initial stresses P.

these two figures also reflect the relation between the wave speed and initial stress because of the fixed wave number. It can be seen that with the increase of the initial stresses in both directions, the frequency variation is not linear and the varying trends of each mode are different. The effects of the compressive stress and tensile stress do not always contrast. For instance, for mode 2, the highest frequency is at about the initial stress $Q = 0$ and $P = 0$; both compressive stress and tensile stress make the wave speed and frequency lower.

In the above examples, the initial stresses in the wave-propagation direction are uniform, that is, $P_1 = P_2 = P_3$. Now we illustrate the differences in the effects of P_1 and P_2. In order to facilitate the comparison, we take an equal-thickness layered plate, N/N/N-1/1/1, as an example. We calculate three cases: $P_1 = P_2 = P_3 = 0$; $P_1 = -80$ GPa, $P_2 = P_3 = 0$; and $P_2 = -80$ GPa, $P_1 = P_3 = 0$. The obtained dispersion curves and the frequency-stress curves are shown in Figures 5 and 6. It can be seen that the effects of P_1 and P_2 are different. We cannot conclude that the effect of one is stronger than that of another one. For different modes, the extents of their influence are different, and vary as the frequency increases.

3.3. *Effects of the initial stresses on SH waves.* This section concerns the effects of the initial stresses on SH waves. Figures 7 and 8 show the dispersion curves for the N/S/N-1/2/1 plate under different uniform initial stresses P and Q. It can be seen that the effects of initial stresses are very regular. For the case of initial stress in the wave-propagation direction, compressive stress always makes the wave speed lower but tensile stress makes the wave speed higher. At small wavenumbers, the effect is weak. As the wavenumber increases, the effect becomes stronger. The effect of initial stresses Q is quite different from that of initial stresses P except that compressive stress still makes the wave speed lower. For a plate with the initial stress in the thickness direction, the effect of initial stresses does not change significantly as the wavenumber increases, but it becomes stronger as the mode order increases and it is very weak on the first mode.
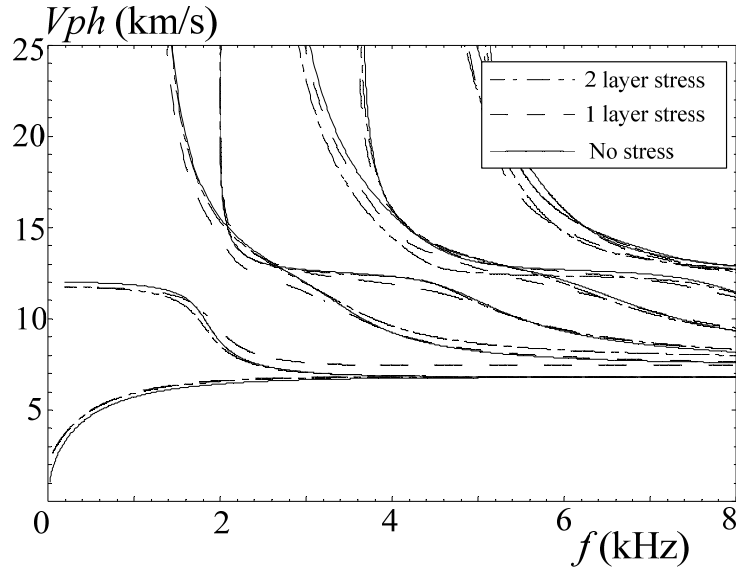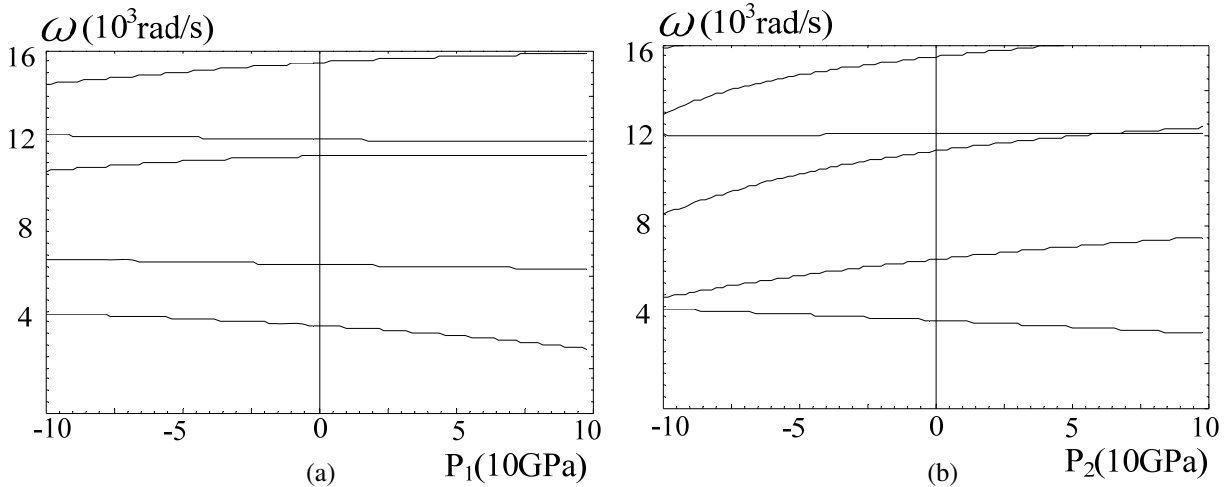
Figure 5. Lamb-like wave phase velocity spectra for the N/N/N-1/1/1 plate under initial stresses P; 1 layer stress represents $P_1 = -80\,\text{GPa}$, $P_2 = P_3 = 0$; and 2 layer stress represents $P_2 = -80\,\text{GPa}$, $P_1 = P_3 = 0$.

Figure 6. Lamb-like wave frequency curves versus initial stresses for the N/N/N-1/1/1 plate at $k = 1\,\text{rad/m}$, with (a) initial stresses P_1 and (b) initial stresses P_2.

Figure 9 illustrates the curves of frequency with initial stresses P and Q for the N/S/N-1/2/1 plate at $k = 1\,\text{rad/m}$. It can be seen that with the increase of the initial stresses in both directions, the frequencies of all modes approximately linearly lower. The effect of compressive stress is entirely contrary to that of tensile stress. For the initial stresses in the wave-propagation direction, the slopes of the frequency curves
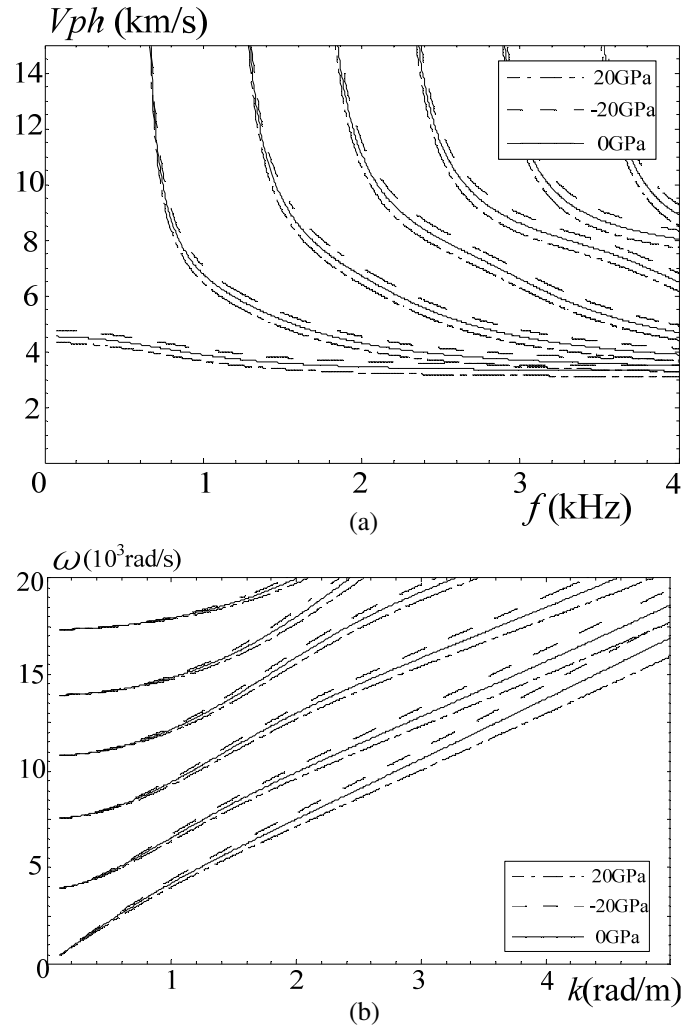
Figure 7. SH wave dispersion curves for the N/S/N-1/2/1 plate under uniform initial stresses P; (a) phase velocity spectra and (b) frequency spectra.

for all modes are similar. For the initial stresses in the thickness direction, the slope of the frequency curve for the first mode is very small. As the mode order increases, the slope becomes larger.

4. Conclusions

Following [Biot 1965], guided wave propagation in layered plates under initial stresses in the thickness and wave-propagation directions are investigated. The Legendre polynomial series method is used to solve the coupled wave equations. The effects of the initial stresses on the dispersion curves are discussed. Based on the calculated results, the following conclusions can be drawn:

(1) The effect of the initial stress on Lamb-like waves is quite different from that on the SH waves. The effect on SH waves is very regular.
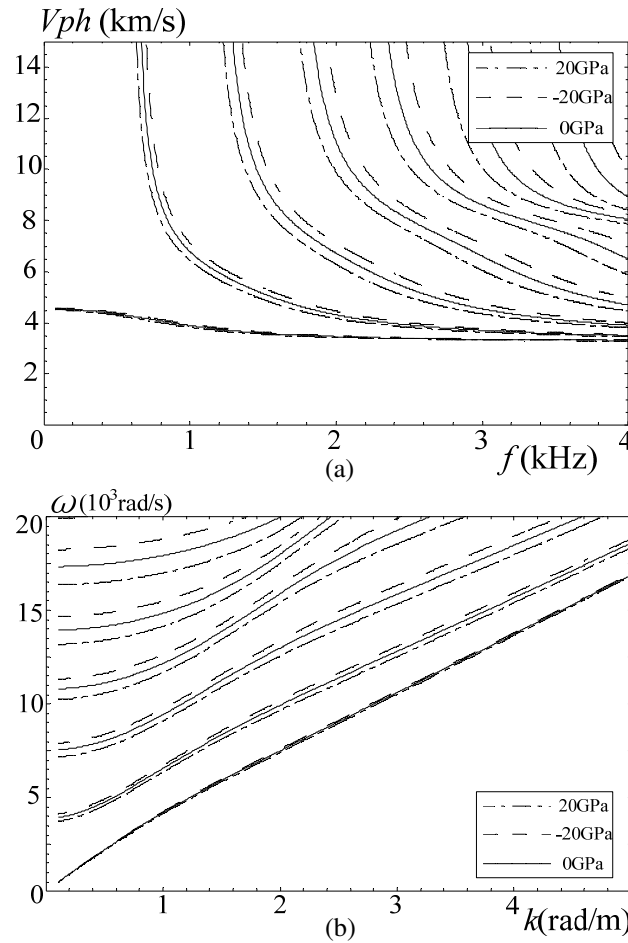
Figure 8. SH wave dispersion curves for the N/S/N-1/2/1 plate under initial stresses Q; (a) phase velocity spectra and (b) frequency spectra.
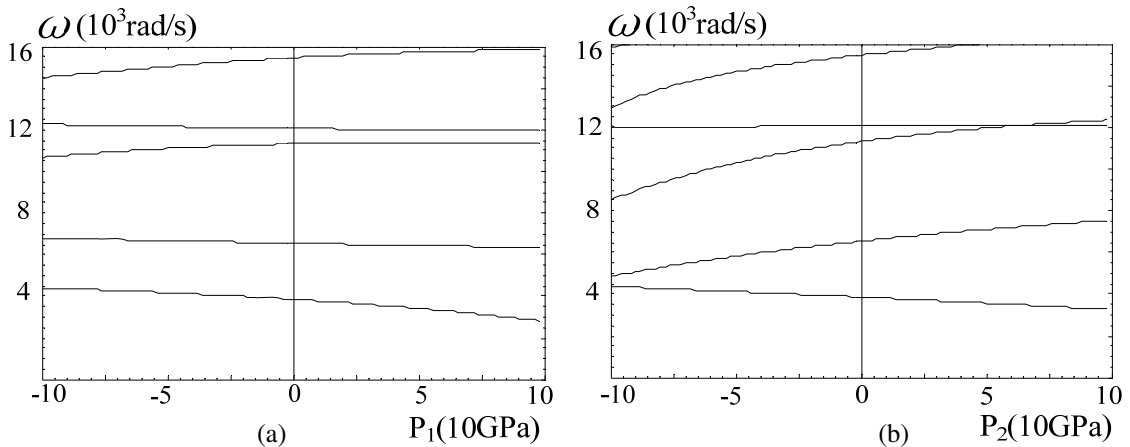
Figure 9. SH wave frequency curves versus initial stresses for the N/S/N-1/2/1 plate at $k = 1\,\text{rad/m}$, with (a) uniform initial stresses Q and (b) uniform initial stresses P.

(2) The effect of the compressive stress on Lamb-like waves is not always contrary to that of the tensile stress. But the effect of the compressive stress on SH waves is almost entirely contrary to that of the tensile stress.

(3) For Lamb-like waves, the effect of the initial stress in the thickness direction usually is contrary to that of the initial stress in the wave-propagation direction. However, for SH waves, the effects of initial stresses in the two directions do not contrast, but their influential patterns are different.

(4) For Lamb-like waves, with the increase of the initial stresses in both directions, the frequency variation is not linear and the varying trends of every mode are different. For SH waves, the curves of the frequency versus the stresses are approximately linear for all modes.

Acknowledgment

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