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EMPIRICAL MODELS FOR PREDICTING PROTECTIVE PROPERTIES OF CONCRETE SHIELDS AGAINST HIGH-SPEED IMPACT

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We have accumulated practically all the information available in the literature regarding empirical models used for describing high-speed normal penetration of rigid strikers into concrete shields, including recently posed models. The description of the models is unified; this includes recommendations on the range of applicability of the models and additional restrictions implied by the used mathematical formulations. The description of the models is quite comprehensive, and allows their direct application. All the relevant formulas are presented in SI units. In addition to this extensive survey we include the results of original comparative investigations on the performance of various models for describing penetration into semiinfinite and finite-thickness shields.

Nomenclature

b	Thickness of the shield
b_{perf}	Perforation thickness
b_{scab}	Scabbing thickness
BLV	Ballistic limit velocity
c	Maximum aggregate size of concrete
$c_{\times}$	Half-size of the concrete aggregate
CRH	Caliber radius head of a ogive-nosed projectile
d	Maximum diameter of impactor, reference size
DOP	Depth of penetration
$\bar{e}$	Parameter, (24)
E	Young's modulus of material of projectile
E_{steel}	Young's modulus of steel
f_c'	Unconfined compressive strength (Pa)
H	Depth of penetration
$\overline{H}^{(i)}_{\min}, \overline{H}^{(i)}_{\max}$	Parameters in functions $\Psi_{\text{perf}}(\overline{H})$ and $\Psi_{\text{scab}}(\overline{H})$, (A.1)
$\overline{H}^{(i)}_{*}$	Parameter, (A.4)
H_{exp}	DOP obtained in experiment

Keywords: protection, concrete, shield, penetration, impact, scabbing, perforation, ballistic limit.

I	Parameter, (36)
I_*	Parameter, (45)
$I_{\min}^{(i)}, I_{\max}^{(i)}$	Parameters of penetration model, (35)
J	Parameter defined in (39)
K_{CRH}	$= \rho_{\mathrm{og}}/(2R)$, CRH of the ogive-nosed projectile
K_{shape}	Nose shape coefficient of impactor in Young's model
$K_{\mathrm{shape}}^{(1)}$	Nose shape coefficient of impactor, (18)
$K_{\mathrm{shape}}^{(2)}$	Nose shape coefficient of impactor, (25)
$K_{\mathrm{shape}}^{(3)}$	Nose shape coefficient of impactor, (54)
$K_{\mathrm{shape}}^{(4)}$	Nose shape coefficient of impactor, (39)
m	Mass of impactor
v_{imp}	Impact velocity
v_{res}	Residual velocity
v_0	Reference velocity, 1000 m/s
v_{sl}	Scabbing limit velocity
v_{bl}	Ballistic limit velocity
$\alpha_{\mathrm{pen}}, \beta_{\mathrm{pen}}, \gamma_{\mathrm{pen}}$	Parameters of penetration model, (41)
$\alpha_{\mathrm{perf}}, \beta_{\mathrm{perf}}, \gamma_{\mathrm{perf}}$	Parameters of perforation model, (42)
$\alpha_{\mathrm{scab}}, \beta_{\mathrm{scab}}, \gamma_{\mathrm{scab}}$	Parameters of scabbing model, (43)
$\alpha^{(i)}, \beta^{(i)}, \gamma^{(i)}$	Parameters in functions $\Psi_{\mathrm{perf}}(\bar{H})$ and $\Psi_{\mathrm{scab}}(\bar{H})$, (A.1)
$\tilde{\alpha}_{\mathrm{pen}}^{(i)}, \tilde{\beta}_{\mathrm{pen}}^{(i)}$	Parameters of penetration model, (35)
μ	Parameter of model (can be different for different models)
$\tilde{\mu}$	Parameter, (36)
Ξ_{perf}	Function determining dependence on $\bar{b}_{\mathrm{perf}}$ versus $\bar{v}_{\mathrm{imp}}$
Ξ_{scab}	Function determining dependence on $\bar{b}_{\mathrm{scab}}$ versus $\bar{v}_{\mathrm{imp}}$
ρ_{sh}	Density of concrete
ρ_{og}	Radius of the arc of the ogive
ϕ	Function determining dependence on $\bar{H}$ versus $\bar{v}_{\mathrm{imp}}$
Ψ_{perf}	Function determining dependence on $\bar{b}_{\mathrm{perf}}$ versus $\bar{H}$
Ψ_{scab}	Function determining dependence on $\bar{b}_{\mathrm{scab}}$ versus $\bar{H}$

A bar over a parameter indicates a dimensionless parameter. Parameters having dimensions of length are normalized by the diameter of impactor d while parameters having dimensions of velocity are normalized by the characteristic velocity $v_0 = 1000$ m/s.

1. Introduction

Hereafter we use the term "empirical model" for formulas describing penetration which have been obtained by statistical analysis of experimental results and are not based on physical laws. Relations between "integral characteristics" of penetration, namely, the impact velocity and the depth of penetration (DOP) for a semiinfinite shield and the ballistic limit velocity (BLV) and the thickness of the plate for a shield having a finite thickness, are examples of these empirical models.

Engineering models of penetration into concrete shields, including empirical models, can be found in the dedicated surveys [Kennedy 1976; Adeli and Amin 1985; Williams 1994; Teland 1998; Li et al. 2005]. Reviews and research papers [Walter and Wolde-Tinsae 1984; Brown 1986; Corbett et al. 1996; Dancygier and Yankelevsky 1996; Yankelevsky 1997; Dancygier 2000; Linderman et al. 1974; Ben-Dor et al. 2005; Vossoughi et al. 2007; Guirgis and Guirguis 2009; Daudeville and Malécot 2011], monographs [Bulson 1997; Bangash and Bangash 2006; Ben-Dor et al. 2006; Carlucci and Jacobson 2007; Bangash 2009; Szuladziński 2009], and a publication of the Department of Energy [2006] also include information on this topic. Few of the studies compare predictions of models with experimental results.

In this study we have accumulated practically all the available information in the literature regarding empirical models which are used for describing high-speed normal penetration of rigid strikers into concrete shields, including models formulated in recent years. This comprehensive survey also has the following characteristics: the description of the models is unified; it includes recommendations on the range of applicability of the models and additional restrictions implied by the used mathematical formulations; the descriptions of the models are quite comprehensive, allowing for their direct application; and all the relevant formulas are presented in SI units.

In addition to this extensive survey we include the results of original comparative investigations on the performance of various models for describing penetration into semiinfinite and finite-thickness shields.

2. Basic definitions

The local response of a shield is initiated with spalling (Figure 1a) and subsequently can result in penetration, scabbing of the shield material from the back face of the shield (Figure 1b), and eventual perforation of the shield (Figure 1c), transporting the projectile through the shield [DOE 2006].

We consider normal penetration (with zero angle of attack) of rigid (nondeformable) projectiles into a shield. If otherwise not indicated, we consider lightly reinforced concrete shields and flat-nosed projectiles. Formulas given for projectiles with a circular cross-section may be applied for projectiles having more complicated shapes by replacing the diameter by the equivalent diameter, based on the perimeter [Walter and Wolde-Tinsae 1984; Barr 1990].

Hereafter the following definitions are used [Kennedy 1976; DOE 2006].

The *depth of penetration (DOP)*, H, is defined as the depth to which a projectile penetrates into a massive (semiinfinite) concrete shield, for a given impact velocity.

The *scabbing thickness*, b_{scab}, for a given impact velocity, is defined as the shield thickness that is just large enough to prevent the peeling off of the back face of the panel opposite the face of impact. In other words, scabbing thickness is the minimum thickness of the shield required to prevent scabbing.

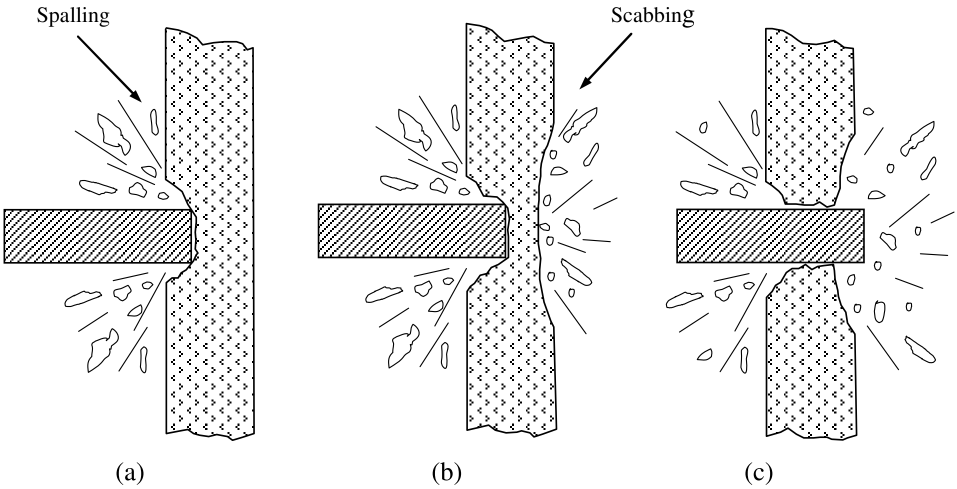

Figure 1. Local response of a shield: (a) penetration and spalling, (b) scabbing, and (c) perforation.

The *perforation thickness*, b_{perf}, for a given impact velocity, is defined as the shield thickness that is just large enough to allow a missile to emerge from the back face of the shield with zero exit velocity. In other words, the perforation thickness is the minimum thickness of the shield required to prevent perforation.

The *ballistic limit velocity (BLV)*, v_{bl}, is defined as the minimum impact (initial) velocity required to perforate a shield with a given thickness.

Similarly to the BLV, we introduce the *scabbing limit velocity*, v_{sl}, as the minimum impact (initial) velocity required for scabbing a shield with a given thickness.

Note that [DOE 2006] recommends, for practical calculations of the shield thicknesses that prevent scabbing and perforation, using values of b_{scab} and b_{perf} obtained from empirical formulas and increased by 10% and 20%, respectively.

We assume that $b_{\mathrm{scab}} < b_{\mathrm{perf}}$ in the range of validity of the models.

3. Unified approach

An empirical model is determined by a triad of equations having the form

$$\overline{H} = \phi(\bar{v}_{\mathrm{imp}}), \quad \bar{b}_{\mathrm{perf}} = \Psi_{\mathrm{perf}}(\overline{H}), \quad \bar{b}_{\mathrm{scab}} = \Psi_{\mathrm{scab}}(\overline{H}), \tag{1}$$

where ϕ, Ψ_{perf}, and Ψ_{scab} are known functions. Analysis of penetration into a finite-thickness shield involves the DOP into a semiinfinite shield, which is not directly related with the problem. Therefore, it is convenient to present the model for a finite-thickness shield by a pair of equations that are obtained after eliminating $\overline{H}$ from (1):

$$\bar{b}_{\mathrm{perf}} = \Xi_{\mathrm{perf}}(\bar{v}_{\mathrm{imp}}), \quad \bar{b}_{\mathrm{scab}} = \Xi_{\mathrm{scab}}(\bar{v}_{\mathrm{imp}}), \tag{2}$$

where

$$\Xi_{\mathrm{perf}}(z) = \Psi_{\mathrm{perf}}(\phi(z)), \quad \Xi_{\mathrm{scab}}(z) = \Psi_{\mathrm{scab}}(\phi(z)). \tag{3}$$

Taking into account that Ξ_{perf} and Ξ_{scab} are increasing functions, (2) and (3) yield the formulas for $\bar{v}_{\text{bl}}$ and $\bar{v}_{\text{sl}}$:

$$\bar{v}_{\text{bl}} = \Xi^{-1}_{\text{perf}}(\bar{b}), \quad \bar{v}_{\text{sl}} = \Xi^{-1}_{\text{scab}}(\bar{b}). \tag{4}$$

Figure 2 illustrates the derivation of the first relationship in (4). The curve described by the equation $\bar{b} = \Xi_{\text{perf}}(\bar{v}_{\text{imp}})$ separates the domain of parameters $\vec{v}_{\text{imp}}$ and $\vec{b}$ into two subdomains. The subdomain under the curve corresponds to perforation of the shield while the subdomain above the curve corresponds to nonperforation. Let $\bar{b}_0$ be an arbitrary thickness of the shield. Inspection of Figure 2 shows that the minimum value of $\vec{v}_{\text{imp}}$ for which perforation occurs (by definition this value equals the BLV) is $\Xi^{-1}_{\text{perf}}(\bar{b}_0)$, and, consequently, the first relationship in (4) is valid. The validity of the second relationship in (4) can be proved similarly.

Therefore a model for a plate having a finite thickness is determined by a pair of relationships given by (2) or (4). Hereafter we use this fact for shortening descriptions of models.

Some typical classes of penetration models are considered in the Appendix. This will allow us in describing particular models to present only the values of the coefficients and avoid rewriting bulky formulas.

4. Modified Pétry formulas

The Pétry formula [Pétry 1910] has a modified version [Kennedy 1976] which can be written similarly to [Li et al. 2005]:

$$\overline{H} = \phi(\bar{v}_{\text{imp}}) = \frac{0.0795 K_p m}{d^3} \log_{10}(1 + 50\bar{v}^2_{\text{imp}}), \tag{5}$$

where K_p is a coefficient depending on the type of the concrete. The coefficients in (5) are selected so that the dimensional parameters m and d are measured in SI units while British units (ft^3/lb) are retained for K_p in order to use generally adopted values of this parameter.

Amirikian [1950], with reference to [Samuely and Hamann 1939], prescribed the following values of the parameter K_p for different types of concrete: 0.00799 for massive concrete, 0.00426 for normal reinforced concrete, and 0.00284 for specially reinforced concrete. This version of the model is called the

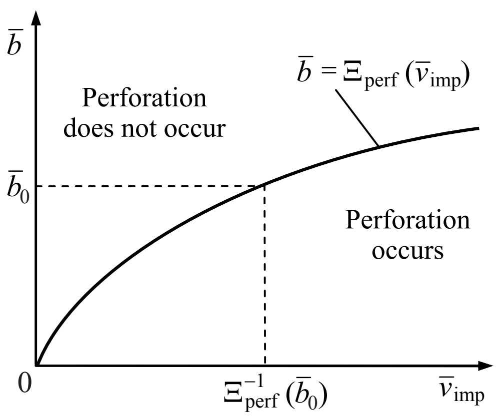

Figure 2. BLV and perforation thickness.

modified Pétry I model by Kennedy [1976], who recommended using an additional dependence between K_p and f'_c. For reinforced concrete this dependence has been presented in a graphical form in [Kennedy 1976], in what is called a modified Pétry II model. A convenient analytical form of this dependence was suggested in [Walter and Wolde-Tinsae 1984]:

$$K_p = 6.34 \cdot 10^{-3} \exp(-0.2937 \cdot 10^{-7} f'_c). \tag{6}$$

Amirikian [1950] suggested taking into account the thickness of the shield when the DOP is calculated. He recommended using (5) if $\bar{b} \geq 3\bar{H}$, assumed that perforation begins when $\bar{b} = 2\bar{H}$, that is,

$$\bar{b}_{\text{perf}} = \Psi_{\text{perf}}(\bar{H}) = 2\bar{H}, \tag{7}$$

and proposed, instead of (5), the following expression for the DOP, $\bar{H}_\times$, in the intermediate domain, $2\bar{H} \leq \bar{b} \leq 3\bar{H}$:

$$\bar{H}_\times = \Lambda(\bar{H}) = [1 + \exp(4(\bar{b}/\bar{H} - 2))]\bar{H}. \tag{8}$$

Kennedy [1976] proposed using the modified Pétry formulas for the scabbing thickness:

$$\bar{b}_{\text{scab}} = \Psi_{\text{scab}}(\bar{H}) = 2.2\bar{H}. \tag{9}$$

Equations (7) and (9) can be written in the form given by (A.5) with the following coefficients: $\alpha^{(1)}_{\text{perf}} = \alpha^{(1)}_{\text{scab}} = \gamma^{(1)}_{\text{perf}} = \gamma^{(1)}_{\text{scab}} = 0$, $\beta^{(1)}_{\text{perf}} = 2.0$, $\beta^{(1)}_{\text{scab}} = 2.2$, and $n = 1$.

Taking into account the modifications proposed by Amirikian [1950] the expression for the DOP for a shield with thickness $\bar{b}$ can be written as

$$\bar{H}_\times = \begin{cases} \phi(\bar{v}_{\text{imp}}) & \text{if } \bar{v}_{\text{imp}} \leq \phi^{-1}(\bar{b}/3), \\ \Lambda(\phi(\bar{v}_{\text{imp}})) & \text{if } \phi^{-1}(\bar{b}/3) < \bar{v}_{\text{imp}} < \phi^{-1}(\bar{b}/2), \\ \text{perforation} & \text{if } \bar{v}_{\text{imp}} \geq \phi^{-1}(\bar{b}/2). \end{cases} \tag{10}$$

Other basic ballistic characteristics are determined by (A.5), (A.6), and (A.9). Amde et al. [1997] presented the following the Pétry formula for the residual velocity referring to Gilbert Associates:

$$v_{\text{res}} = v_{\text{imp}}\sqrt{1 - (0.5b/H)}. \tag{11}$$

It should be noted that classifying the Pétry formulas, (5), as empirical models is done here partly in order to follow tradition. Indeed, their form implies that these formulas were derived by integrating the equation of motion of the penetrator under the assumption of two-term quadratic dependence (without the linear term) of drag force on velocity.

5. Ballistic research laboratory (BRL) formulas

The BRL formula does not take into account the influence of the unconfined compressive strength on the protective properties of a shield, and the value $f'_c = 20.7$ MPa was assumed [Kennedy 1976]. In order to remedy this shortcoming the modified BRL formula was suggested. Kennedy [1976] emphasized that this modified formula, in contrast to the common approach, directly predicts the perforation thickness:

$$\bar{b}_{\text{perf}} = \Xi_{\text{perf}}(\bar{v}_{\text{imp}}) = \frac{13m}{d^{2.8}\sqrt{f'_c}}\bar{v}^{1.33}_{\text{imp}}, \tag{12}$$

while the scabbing thickness can be estimated as

$$\bar{b}_{\text{scab}} = 2\bar{b}_{\text{perf}}. \tag{13}$$

The BLV and scrubbing limit velocity can be calculated using the formulas

$$\bar{v}_{\text{bl}} = \Xi^{-1}_{\text{perf}}(\bar{b}), \quad \bar{v}_{\text{sl}} = \Xi^{-1}_{\text{perf}}(\bar{b}/2). \tag{14}$$

Chelapati et al. [1972] obtained a similar result starting from the expression for the DOP. Their formula for the DOP, $\overline{H}$, coincides with the right-hand side of (12) divided by 1.3, which yields $\bar{b}_{\text{perf}} = 1.3\overline{H}$.

6. Whiffen formula

Bulson [1997], with reference to [Whiffen 1943], suggested the model

$$\overline{H} = \phi(\bar{v}_{\text{imp}}) = \frac{2.6m}{d^3\bar{c}^{0.1}\sqrt{f'_c}}(1.87\bar{v}_{\text{imp}})^{\tilde{n}}, \quad \tilde{n} = \frac{97.5}{\sqrt[4]{f'_c}}, \quad \bar{c} = \frac{c}{d}, \tag{15}$$

where c is the maximum aggregate size of concrete.

The latter equation is based on experiments with ogival-nosed projectiles conducted for the following ranges of parameters: $0.8 \le K_{\text{CRH}} \le 3.5$, $5\,\text{MPa} < f'_c < 70\,\text{MPa}$, $12\,\text{mm} < d < 965\,\text{mm}$, $0.02 < \bar{c} < 2$, $v_{\text{imp}} < 1130\,\text{m/s}$, $136\,\text{g} < m < 10,000\,\text{kg}$. This formula fits the experimental data within a scatter band of the order of $\pm 15\%$.

Note that Teland [1998] referred to this model as the "TBAA formula".

7. Army corporations of engineers (ACE) formula

The ACE formula [ACE 1946; Gwaltney 1968], written using variables $\overline{H}$ and $\bar{v}_{\text{imp}}$ and SI units, reads

$$\overline{H} = \phi(\bar{v}_{\text{imp}}) = \frac{11.1m}{d^{2.785}\sqrt{f'_c}}\bar{v}^{1.5}_{\text{imp}} + 0.5. \tag{16}$$

There are two versions of the dependencies between the perforation/scabbing thickness and the DOP which differ only slightly [Gwaltney 1968; Chelapati et al. 1972; Li et al. 2005]. Following [Kennedy 1976] we have selected the version of this formula with the coefficients shown in Table 1, which is associated with the model given by (A.1).

Formulas for the other basic ballistic characteristics are given by (A.5), (A.6), and (A.9).

	$\alpha^{(1)}$	$\beta^{(1)}$	$\gamma^{(1)}$	$\overline{H}^{(1)}_{\min}$	$\overline{H}^{(1)}_{\max}$
Perforation	1.24	1.32	0	1.35	13.5
Scabbing	1.36	2.12	0	0.65	11.75

Table 1. ACE model for perforation and scabbing. Coefficients in (A.1), $n = 1$.

8. Ammann and Whitney formula

The Ammann and Whitney formula was suggested for predicting perforation by small explosively generated fragments having impact velocity larger than 300 m/s [Kennedy 1976]:

$$\bar{H} = \phi(\bar{v}_{\mathrm{imp}}) = \frac{15 K^{(1)}_{\mathrm{shape}} m}{d^{2.8}\sqrt{f'_c}} \bar{v}^{1.8}_{\mathrm{imp}}, \tag{17}$$

where

$$K^{(1)}_{\mathrm{shape}} = \begin{cases} 0.72 & \text{for a flat nose,} \\ 0.84 & \text{for a blunt nose,} \\ 1.00 & \text{for an average nose (spherical end),} \\ 1.14 & \text{for a very sharp nose.} \end{cases} \tag{18}$$

9. Modified national defense research committee (NDRC) formula

The modified NDRC formula can be written in a commonly accepted form as an implicit function with respect to the DOP [NDRC 1946; Kennedy 1976]. Using the dimensionless variables $\bar{H}$ and $\bar{v}_{\mathrm{imp}}$ and SI units for the dimensional parameters this formula reads

$$G(\bar{H}) = \mu \bar{v}^{1.8}_{\mathrm{imp}}, \tag{19}$$

where

$$\mu = 9.55 \frac{K^{(1)}_{\mathrm{shape}} m}{d^{2.8}\sqrt{f'_c}}, \tag{20}$$

$$G(\bar{H}) = \begin{cases} 0.25 \bar{H}^2 & \text{if } \bar{H} \le 2, \\ \bar{H} - 1 & \text{if } \bar{H} > 2, \end{cases} \tag{21}$$

and the effect of the unconfined compressive strength is included in the model following the suggestion of Kennedy [1976].

In the universal form given by (1), this model can be rewritten as

$$\bar{H} = \phi(\bar{v}_{\mathrm{imp}}) = \begin{cases} 2\sqrt{\mu}\bar{v}^{0.9}_{\mathrm{imp}} & \text{if } \bar{v}_{\mathrm{imp}} \le 1/\mu^{5/9}, \\ \mu \bar{v}^{1.8}_{\mathrm{imp}} + 1 & \text{if } \bar{v}_{\mathrm{imp}} > 1/\mu^{5/9}. \end{cases} \tag{22}$$

The modified NDRC model for perforation and scabbing [Kennedy 1976] can be written in the form of (A.1) with coefficients from Table 2. The dependencies $\bar{b}_{\mathrm{perf}}$ and $\bar{b}_{\mathrm{scab}}$ versus $\bar{v}_{\mathrm{imp}}$ as well as $\bar{v}_{\mathrm{bl}}$ and $\bar{v}_{\mathrm{sl}}$ versus $\bar{b}$ are given by (A.5)–(A.9), where

$$\phi^{-1}(z) = \begin{cases} (0.5z/\sqrt{\mu})^{10/9} & \text{if } z \le 2, \\ [(z-1)/\mu]^{5/9} & \text{if } z \ge 2. \end{cases} \tag{23}$$

Inspection of Table 2 shows that the inequalities given by (A.4) are satisfied.

	i	$\beta^{(i)}$	$\alpha^{(i)}$	$\gamma^{(i)}$	$\overline{H}^{(i)}_{\min}$	$\overline{H}^{(i)}_{\max}$	$\overline{H}^{(i)}_{*}$
Perforation	1	0	3.19	0.718	–	1.35	2.22
	2	1.32	1.24	0	1.35	13.5	–
Scabbing	1	0	7.91	5.06	–	0.65	0.78
	2	2.12	1.36	0	0.65	11.75	–

Table 2. Modified NDRC model for perforation and scabbing. Coefficients in (A.1), $n = 2$.

According to [DOE 2006], the NDRC formula was derived for the following ranges of impact parameters: $v_{\text{imp}} > 152\,\text{m/s}$, $2.54\,\text{cm} < b < 40.6\,\text{cm}$ and $b/d \geq 3$. However, as suggested by Kennedy [1976], this formula can be extrapolated beyond these ranges.

10. Kar's formula

Kar's formula [Kar 1978; Bangash and Bangash 2006] is an improved modified NDRC formula that takes into account the size of the aggregates of concrete and the type of the projectile material.

In the universal form given by (1), Kar's formula is given by (22) where

$$\mu = 9.55\frac{K^{(2)}_{\text{shape}}m\bar{e}^{6.25}}{d^{2.8}\sqrt{f'_c}}, \qquad \bar{e} = \left(\frac{E}{E_{\text{steel}}}\right)^{0.2}, \tag{24}$$

and E and E_{steel} are the Young's moduli of the material of the projectile and steel. The nose shape parameter, $K^{(2)}_{\text{shape}}$, is calculated as

$$K^{(2)}_{\text{shape}} = \begin{cases} 0.72 & \text{for a flat-nosed projectile,} \\ \min(\hat{K}_{\text{nose}}, 1.17) & \text{for an ogive-nosed projectile,} \end{cases} \tag{25}$$

where

$$\hat{K}_{\text{nose}} = 0.72 + 0.25\sqrt{K_{\text{CRH}} - 0.25}. \tag{26}$$

Kar's model for perforation and scabbing can be written in the form of (A.1) where the coefficients are presented in Table 3 and $\bar{c}_{\times}$ is the half-size of the concrete aggregate.

The dependence of $\bar{b}_{\text{perf}}$ and $\bar{b}_{\text{scab}}$ on $\bar{v}_{\text{imp}}$, and of $\bar{v}_{\text{bl}}$ and $\bar{v}_{\text{sl}}$ on $\bar{b}$, is given by (A.5)–(A.9) with $\phi^{-1}(z)$ calculated from (23).

	i	$\beta^{(i)}$	$\alpha^{(i)}$	$\gamma^{(i)}$	$\overline{H}^{(i)}_{\min}$	$\overline{H}^{(i)}_{\max}$	$\overline{H}^{(i)}_{*}$
Perforation	1	$\bar{c}_{\times}$	3.19	0.718	–	1.35	2.22
	2	$\bar{c}_{\times} + 1.32$	1.24	0	1.35	13.5	–
Scabbing	1	$\bar{c}_{\times}$	$7.91\bar{e}$	$5.06\bar{e}$	–	0.65	0.71
	2	$\bar{c}_{\times} + 2.12\bar{e}$	$1.36\bar{e}$	0	0.65	11.75	–

Table 3. Kar's model for perforation and scabbing. Coefficients in (A.1), $n = 2$.

11. Healey–Weissman formula

The Healey–Weissman penetration formula [Healey and Weissman 1974; Li et al. 2005] can be considered as a version of Kar's formula where the expression for μ given by (24) is replaced by

$$\mu = 10.95\Big(\frac{E}{E_{\text{steel}}}\Big)\frac{K^{(2)}_{\text{shape}}m}{d^{2.8}\sqrt{f'_c}}. \tag{27}$$

The formula for the residual velocity reads [Healey et al. 1975; Kar 1979]:

$$\frac{v_{\text{res}}}{v_{\text{imp}}} = \begin{cases} [1-(b/b_{\text{perf}})^2]^{0.555} & \text{if } b \le 2d, \\ [1-(b/b_{\text{perf}})]^{0.555} & \text{if } b > 2d. \end{cases} \tag{28}$$

12. Bechtel formula

The Bechtel Corporation proposed the following formula for scabbing thickness that is valid for a hard cylindrical projectile [Rotz 1975; 1977; Bangash and Bangash 2006]:

$$\bar{b}_{\text{scab}} = \Xi_{\text{scab}}(\bar{v}_{\text{imp}}) = \frac{1.23\cdot 10^3 m^{0.4}}{d^{1.2}\sqrt{f'_c}}\bar{v}^{0.5}_{\text{imp}}. \tag{29}$$

This equation is based on twelve tests with solid missiles and nine tests with half-pipe missiles. The tests were conducted for the following ranges of parameters: [Teland 1998]: $37\,\text{m/s} < v_{\text{imp}} < 144\,\text{m/s}$, $20.3\,\text{cm} < d < 21.8\,\text{cm}$, and $30.5\,\text{cm} < b < 61\,\text{cm}$; and $7.6\,\text{cm} < b < 22.9\,\text{cm}$, $30\,\text{MPa} < f'_c < 40\,\text{MPa}$, and $3.6\,\text{kg} < m < 97.1\,\text{kg}$.

13. Stone and Webster formula

The following formula was proposed for calculating the scabbing thickness [Jankov et al. 1977; Li et al. 2005; Bangash and Bangash 2006]:

$$\bar{b}_{\text{scab}} = \Xi_{\text{scab}}(\bar{v}_{\text{imp}}) = \frac{100}{d}\Big(\frac{m}{K_{\bar{b}}}\Big)^{1/3}\bar{v}^{2/3}_{\text{imp}}, \tag{30}$$

where $K_{\bar{b}}$ is the dimensionless coefficient which can be approximated by the formula [Li et al. 2005]

$$K_{\bar{b}} = 0.013\bar{b} + 0.33, \quad 1.5 \le \bar{b} \le 3. \tag{31}$$

The equation is based on seven tests with solid missiles and 21 tests with half-pipe missiles. The tests were conducted for the following ranges of parameters [Teland 1998]: $27\,\text{m/s} < v_{\text{imp}} < 157\,\text{m/s}$, $4.1\,\text{cm} < d < 8.9\,\text{cm}$, $11.4\,\text{cm} < b < 15.2\,\text{cm}$, $22\,\text{MPa} < f'_c < 30\,\text{MPa}$, and $1.9\,\text{kg} < m < 12.8\,\text{kg}$.

14. CEA-EDF formula

The CEA-EDF (Commissariat à l'énergie atomique et Électricité de France) formula [Berriaud et al. 1978] reads:

$$\bar{b}_{\text{perf}} = \Xi_{\text{perf}}(\bar{v}_{\text{imp}}) = \frac{146 m^{0.5}}{d^{1.5}(f'_c)^{0.375}\rho_c^{0.125}}\bar{v}^{0.75}_{\text{imp}}, \tag{32}$$

where ρ_{sh} is the density of concrete.

This model is valid in the following ranges of parameters: $v_{\mathrm{imp}} < 200\,\mathrm{m/s}$, $150\,\mathrm{kg/m^3} < \rho_{\mathrm{sh}} < 300\,\mathrm{kg/m^3}$, $20\,\mathrm{kg} < m < 300\,\mathrm{kg}$, $0.35 < b/d < 4.17$, and $23\,\mathrm{MPa} < f_c' < 46\,\mathrm{MPa}$.

15. Degen's formula

Based on the available experimental data, Degen [1980] suggested a formula for the perforation thickness that can be written in the form given by (A.1) with the coefficients shown in Table 4, where $\bar{H}$ is determined from the modified NDRC model, (22). The dependence $\bar{v}_{\mathrm{sl}}$ versus $\bar{b}$ is given by (A.6) where $\phi^{-1}(z)$ is determined by (23).

The tests were conducted in the following ranges of parameters: $25\,\mathrm{m/s} < v_{\mathrm{imp}} < 310\,\mathrm{m/s}$, $15\,\mathrm{kg} < m < 134\,\mathrm{kg}$, $28\,\mathrm{MPa} < f_c' < 43\,\mathrm{MPa}$, $10\,\mathrm{cm} < d < 31\,\mathrm{cm}$, and $15\,\mathrm{cm} < b < 60\,\mathrm{cm}$. The concrete reinforcement varied in the range between $160\,\mathrm{kg/m^3}$ and $350\,\mathrm{kg/m^3}$, and penetrators having flat, conical, and hemispherical nose shapes were used.

16. Chang's formula

Chang's formulas [Chang 1981] for flat-nosed projectiles penetrating into a reinforced concrete shield read:

$$\bar{b}_{\mathrm{perf}} = \Xi_{\mathrm{perf}}(\bar{v}_{\mathrm{imp}}) = \frac{497}{d^{1.5}}\sqrt{\frac{m}{f_c'}}\,\bar{v}_{\mathrm{imp}}^{0.75}, \tag{33}$$

$$\bar{b}_{\mathrm{scab}} = \Xi_{\mathrm{scab}}(\bar{v}_{\mathrm{imp}}) = \frac{321}{d^{1.2}}\left(\frac{m}{f_c'}\right)^{0.4}\bar{v}_{\mathrm{imp}}^{2/3}. \tag{34}$$

These formulas are based on experiments conducted in the following ranges of parameters: $17\,\mathrm{m/s} < v_{\mathrm{imp}} < 312\,\mathrm{m/s}$, $110\,\mathrm{g} < m < 344\,\mathrm{kg}$, $23\,\mathrm{MPa} < f_c' < 46\,\mathrm{MPa}$, $5.1\,\mathrm{cm} < b < 61\,\mathrm{cm}$, and $2\,\mathrm{cm} < d < 30.5\,\mathrm{cm}$.

17. Haldar–Miller formula

The Haldar–Miller model for penetration [Haldar and Miller 1982] can be described as

$$\bar{H} = \phi(\bar{v}_{\mathrm{imp}}) = \begin{cases} \tilde{\alpha}_{\mathrm{pen}}^{(1)} I + \tilde{\beta}_{\mathrm{pen}}^{(1)} & \text{if } \sqrt{I_{\min}^{(1)}/\tilde{\mu}} \le \bar{v}_{\mathrm{imp}} \le \sqrt{I_{\max}^{(1)}/\tilde{\mu}}, \\ \tilde{\alpha}_{\mathrm{pen}}^{(2)} I + \tilde{\beta}_{\mathrm{pen}}^{(2)} & \text{if } \sqrt{I_{\min}^{(2)}/\tilde{\mu}} \le \bar{v}_{\mathrm{imp}} \le \sqrt{I_{\max}^{(2)}/\tilde{\mu}}, \\ \tilde{\alpha}_{\mathrm{pen}}^{(3)} I + \tilde{\beta}_{\mathrm{pen}}^{(3)} & \text{if } \sqrt{I_{\min}^{(3)}/\tilde{\mu}} \le \bar{v}_{\mathrm{imp}} \le \sqrt{I_{\max}^{(3)}/\tilde{\mu}}, \end{cases} \tag{35}$$

	i	$\beta^{(i)}$	$\alpha^{(i)}$	$\gamma^{(i)}$	$\bar{H}_{\min}^{(i)}$	$\bar{H}_{\max}^{(i)}$	$\bar{H}_*^{(i)}$
Perforation	1	0	2.2	0.3	–	1.52	3.67
	2	0.69	1.29	0	1.52	13.42	–

Table 4. Degen's model for perforation. Coefficients in (A.1), $n = 2$.

where

$$I = \tilde{\mu}\bar{v}_{\text{imp}}^2, \quad \tilde{\mu} = 10^6 \frac{K_{\text{shape}}^{(1)} m}{d^3 f_c'}, \tag{36}$$

with the coefficients presented in Table 5.

Haldar and Miller considered their model an improved modified NDRC model.

18. Haldar–Hamieh–Miller formula

This penetration model [Haldar et al. 1984; Haldar and Hamieh 1984] is described by (35) with the coefficients shown in Table 6.

The Haldar–Hamieh–Miller formula yields the following expression for the scabbing thickness:

$$\bar{b}_{\text{scab}} = 0.0342\tilde{\mu}\bar{v}_{\text{imp}}^2 + 3.3437, \quad 4.58/\sqrt{\tilde{\mu}} \le \bar{v}_{\text{imp}} \le 19.6/\sqrt{\tilde{\mu}}. \tag{37}$$

19. Hughes' formula

Hughes' formula for penetration [Hughes 1984] reads:

$$\overline{H} = 0.19 K_{\text{shape}}^{(4)} \frac{J}{1 + 12.3\ln(1 + 0.03J)}, \tag{38}$$

where

$$J = 10^6 \frac{m\bar{v}_{\text{imp}}^2}{d^3 f_t}, \quad K_{\text{shape}}^{(4)} = \begin{cases} 1.00 & \text{for a flat nose,} \\ 1.12 & \text{for a blunt nose,} \\ 1.26 & \text{for a spherical nose,} \\ 1.39 & \text{for a very sharp nose,} \end{cases} \tag{39}$$

and f_t is the tensile strength of concrete. With reference to [ACI 1978], Hughes [1984] recommended using the relationship between f_t and f_c':

$$f_t = 630\sqrt{f_c'}. \tag{40}$$

i	$\tilde{\alpha}_{\text{pen}}^{(i)}$	$\tilde{\beta}_{\text{pen}}^{(i)}$	$I_{\min}^{(i)}$	$I_{\max}^{(i)}$
1	0.22024	−0.02725	0.3	2.5
2	0.446	−0.592	2.5	3.0
3	0.06892	0.53886	3.0	21.0

Table 5. Haldar–Miller model for penetration. Coefficients in (35).

i	$\tilde{\alpha}_{\text{pen}}^{(i)}$	$\tilde{\beta}_{\text{pen}}^{(i)}$	$I_{\min}^{(i)}$	$I_{\max}^{(i)}$
1	0.2251	−0.0308	0.3	4.0
2	0.0567	0.6740	4.0	21.0
3	0.0299	1.1875	21.0	455

Table 6. Haldar–Hamieh–Miller model for penetration. Coefficients in (35).

The dependencies $\bar{b}_{\text{perf}}$ versus $\overline{H}$ and $\bar{b}_{\text{scab}}$ versus $\overline{H}$ are the same as given by (A.1) ($n = 2$) with the coefficients shown in Table 7.

Hughes [1984] noted that his formulas are valid in the range $J < 3500$ but they will be conservative for $J < 40$ and $\bar{b} < 3.5$.

20. Adeli–Amin formula

Using a large set of experimental data from [Sliter 1980], Adeli and Amin [1985] proposed a model that, similarly to the Haldar–Miller and Haldar–Hamieh–Miller models, is based on the factor I given by (36). This model can be written as

$$\overline{H} = \psi(\alpha_{\text{pen}}, \beta_{\text{pen}}, \gamma_{\text{pen}}; I), \tag{41}$$

$$\bar{b}_{\text{perf}} = \psi(\alpha_{\text{perf}}, \beta_{\text{perf}}, \gamma_{\text{perf}}; I), \tag{42}$$

$$\bar{b}_{\text{scab}} = \psi(\alpha_{\text{scab}}, \beta_{\text{scab}}, \gamma_{\text{scab}}; I), \tag{43}$$

where

$$\psi(\alpha, \beta, \gamma; I) = \beta + \alpha I - \gamma I^2 \tag{44}$$

and parameters α, β, and γ for penetration, perforation, and scabbing are given in Table 8. The parameter

$$I_* = 0.5\alpha/\gamma \tag{45}$$

is shown in the last column of Table 8. This parameter is the upper limit of the increasing function ψ versus I. The requirement similar to the inequality in (A.4) reads:

$$I < I_*. \tag{46}$$

Instead of (41), Adeli and Amin suggested using

$$\overline{H} = 0.0123 + 0.196I - 0.008I^2 + 0.0001I^3, \tag{47}$$

	i	$\beta^{(i)}$	$\alpha^{(i)}$	$\gamma^{(i)}$	$\overline{H}^{(i)}_{\text{min}}$	$\overline{H}^{(i)}_{\text{max}}$
Perforation	1	0	3.6	0	–	0.7
	2	1.4	1.58	0	0.7	–
Scabbing	1	0	5.0	0	–	0.7
	2	2.3	1.74	0	0.7	–

Table 7. Hughes' model for perforation and scabbing. Coefficients in (A.1), $n = 2$.

	β	α	γ	I_*
Penetration	0.0416	0.1698	0.0045	18.8
Perforation	0.906	0.3214	0.0106	15.1
Scabbing	1.8685	0.4035	0.0114	17.7

Table 8. Adeli–Amin model. Coefficients in (41)–(43).

where the function $\bar{H}(I)$ increases when $I < I_* = 19.1$.

The Adeli–Amin formulas are valid in the following ranges of parameters [Adeli and Amin 1985]: $27\,\mathrm{m/s} < v_{\mathrm{imp}} < 312\,\mathrm{m/s}$, $110\,\mathrm{g} < m < 344\,\mathrm{kg}$, $0.7 < b/d < 18$, $d < 30\,\mathrm{cm}$, $\bar{H} < 2.0$, and $0.3 < I < 21$. The range for the parameter I should be decreased taking into account (46) and values of I_* in Table 8.

21. CRIEPI formula

The model of the Central Research Institute of the Electric Power Industry (CRIEPI) of Japan gives the following relationship for the DOP [Ohnuma et al. 1985; Li et al. 2005]:

$$\bar{H} = \phi(\bar{v}_{\mathrm{imp}}) = \frac{26.1m}{d^{1.8}}\left(\frac{16.7\cdot 10^4}{(f_c')^{2/3}} - 1\right)\left[\frac{d+0.25}{(1.25\bar{b}+1)\bar{b}}\right]\bar{v}_{\mathrm{imp}}^2. \tag{48}$$

Ohnuma et al. [1985] recommended using (48) for $v_{\mathrm{imp}} \le 50\,\mathrm{m/s}$. Clearly, the constraints $f_c' < 68.2$ and $\bar{H} \le \bar{b}$ must be satisfied.

They also proposed the following formulas for calculating perforation and scabbing thicknesses:

$$\bar{b}_{\mathrm{perf}} = \Xi_{\mathrm{perf}}(\bar{v}_{\mathrm{imp}}) = \frac{447}{d^{1.5}}\sqrt{\frac{m}{f_c'}}\,\bar{v}_{\mathrm{imp}}^{0.75}, \tag{49}$$

$$\bar{b}_{\mathrm{scab}} = \Xi_{\mathrm{scab}}(\bar{v}_{\mathrm{imp}}) = \frac{306}{d^{1.2}}\left(\frac{m}{f_c'}\right)^{0.4}\bar{v}_{\mathrm{imp}}^{2/3}. \tag{50}$$

These formulas differ from (33) and (34) (Chang's model) by the values of the coefficients.

22. Vretblad (British) formula

Teland [1998], with reference to [Vretblad 1988], presented the following penetration model, also known as the "British formula":

$$\bar{H} = \phi(\bar{v}_{\mathrm{imp}}) = \frac{0.76\cdot 10^{-3}(1-0.6\cdot 10^{-8} f_c')m}{d^3\bar{c}_\times}\bar{v}_{\mathrm{imp}}^{1.5}. \tag{51}$$

23. UKAEA-CEBG-NNC formulas

This section is mainly based on the guidelines given in [Barr 1990]. Some additional information on this subject can be also found in [Fullard and Barr 1989; Fullard et al. 1991]. According to [Barr 1990], major contributions to this model were made by the UK Atomic Energy Authority (UKAEA), the Central Electricity Generating Board (CEGB), and the National Nuclear Corporation (NNC), so we refer to this model by the names of these organizations.

For calculation of the DOP of a solid missile penetrating into a reinforced concrete barrier with sufficient thickness so as to suffer no scabbing, the following model is recommended:

$$g(\bar{H}) = \mu\bar{v}_{\mathrm{imp}}^{1.8}, \tag{52}$$

where

$$\mu = 9.55 \frac{K^{(3)}_{\text{shape}} m}{d^{2.8}\sqrt{f'_c}}, \tag{53}$$

$$K^{(3)}_{\text{shape}} = \begin{cases} 0.72 & \text{for a flat nose,} \\ 0.84 & \text{for a spherical nose,} \\ 1.00 & \text{for a blunt (conic frustum or ogive) nose,} \\ 1.14 & \text{for a sharp (conical) nose,} \end{cases} \tag{54}$$

$$g(\overline{H}) = \begin{cases} 0.55\overline{H} - \overline{H}^2 & \text{if } \overline{H} < 0.22, \\ 0.25\overline{H}^2 + 0.0605 & \text{if } 0.22 \le \overline{H} \le 2, \\ \overline{H} - 0.9395 & \text{if } \overline{H} > 2. \end{cases} \tag{55}$$

In the unified form given by (1), the UKAEA-CEBG-NNC model can be rewritten as

$$\overline{H} = \phi(\bar{v}_{\text{imp}}) = \begin{cases} 0.275 - \sqrt{0.0756 - \mu \bar{v}^{1.8}_{\text{imp}}} & \text{if } \bar{v}_{\text{imp}} < 0.233/\mu^{5/9}, \\ 2\sqrt{\mu \bar{v}^{1.8}_{\text{imp}} - 0.0605} & \text{if } 0.233/\mu^{5/9} \le \bar{v}_{\text{imp}} \le 1.033/\mu^{5/9}, \\ \mu \bar{v}^{1.8}_{\text{imp}} + 0.9395 & \text{if } \bar{v}_{\text{imp}} > 1.033/\mu^{5/9}. \end{cases} \tag{56}$$

The ranges of parameters in this formula are as follows: $+20\%$ to -20% for $\overline{H} > 0.75$ and $+100\%$ to -50% for $\overline{H} < 0.75$, $25\,\text{m/s} < v_{\text{imp}} < 300\,\text{m/s}$, $5 \cdot 10^3\,\text{kg/m}^3 < m/d^3 < 2 \cdot 10^5\text{kg/m}^3$, and $22\,\text{MPa} < f'_c < 44\,\text{MPa}$.

For predicting the scabbing thickness the following formula was suggested:

$$\bar{b}_{\text{scab}} = \Xi_{\text{scab}}(\bar{v}_{\text{imp}}) = 5.3\mu^{1/3}\bar{v}^{0.6}_{\text{imp}}. \tag{57}$$

The accuracy of this formula is $\pm 40\%$ for $2 < \bar{b}_{\text{scab}} < 5.6$ within the following ranges of parameters: $29\,\text{m/s} < v_{\text{imp}} < 238\,\text{m/s}$, $26\,\text{MPa} < f'_c < 44\,\text{MPa}$, and $1.5 \cdot 10^3\,\text{kg/m}^3 < m/(d^2 b_{\text{scab}}) < 4 \cdot 10^4\,\text{kg/m}^3$.

The formula for the BLV of a flat-nosed missile with a circular or noncircular cross-section against a reinforced concrete shield reads:

$$\bar{v}_{\text{bl}} = \begin{cases} \bar{v}^*_{\text{bl}} & \text{if } \bar{v}^*_{\text{bl}} \le 0.07, \\ \bar{v}^*_{\text{bl}}(4 \cdot \bar{v}^{*2}_{\text{bl}} + 1) & \text{if } \bar{v}^*_{\text{bl}} > 0.07, \end{cases} \tag{58}$$

where

$$\bar{v}^*_{\text{bl}} = \frac{1.3 \cdot 10^{-3} k_R k_p^{2/3} \rho_{\text{sh}}^{1/6} d^2 \sqrt{f'^*_c}}{m^{2/3}} \sqrt{r + 0.3}\,\bar{b}^{4/3}, \tag{59}$$

$$f'^*_c = \begin{cases} f'_c & \text{if } f'_c \le 37\,\text{MPa}, \\ 37\,\text{MPa} & \text{if } f'_c > 37\,\text{MPa}, \end{cases} \tag{60}$$

$$k_R = \begin{cases} 1.2 - 0.6c_R/b & \text{if } 0.12 \le c_R/b \le 0.49, \\ 1 & \text{if } c_R/b < 0.12 \text{ or } c_R/b > 0.49, \end{cases} \tag{61}$$

$$r = \frac{100}{b}\left(\frac{a_F}{3c_F} + \frac{2a_R}{3c_R}\right), \qquad k_p = p/(\pi d). \tag{62}$$

Here a_F (a_R) is the cross-sectional area of a single front (rear) rebar; c_F (c_R) is a front (rear) rebar spacing (see Figure 3; a steel plate at the distal face of the shield is lacking in the considered model); p is the perimeter of the impactor; f_c' is the characteristic compressive strength of concrete measured for 150 mm diameter, 300 mm long cylinders; and $k_p = 1$ for a missile with a circular cross-section.

This model is valid in the following ranges of parameters: $11\,\mathrm{m/s} < v_{\mathrm{bl}} < 300\,\mathrm{m/s}$, $0 < r < 0.75$, $0.2 < p/(\pi b) < 3$, $150\,\mathrm{kg/m^3} < m/(p^2 b) < 10^4\,\mathrm{kg/m^3}$, and $f_c' > 15\,\mathrm{MPa}$.

In the case in which a steel plate is installed at the distal face of the shield (Figure 3) to improve its protective effectiveness, the following formula is recommended instead of (88):

$$v_{\mathrm{bl}}^* = \frac{1.3\rho_{\mathrm{sh}}^{1/6}\sqrt{f'^*_c}}{m^{2/3}}\left(\frac{p}{\pi}\right)^{2/3}\sqrt{B + 0.3}\,b^{4/3}, \tag{63}$$

where

$$B = r + 100 b_{\mathrm{steel}}/b \tag{64}$$

and b_{steel} is the steel plate thickness.

This formula is valid in the following ranges of parameters: $45\,\mathrm{m/s} < v_{\mathrm{bl}} < 300\,\mathrm{m/s}$, $0 < r < 0.75$, $150\,\mathrm{kg/m^3} < m/(p^2 b) < 10^4\,\mathrm{kg/m^3}$, $1.2 < B < 4.3$, $0.2 < p/(\pi b) < 3$, and $f_c' > 15\,\mathrm{MPa}$.

24. Young's formula

24.1. *Original model.* After some algebra, Young's equation [Young 1997] can be written as

$$\bar{H} = \phi(\bar{v}_{\mathrm{imp}}) = \frac{K_S K_{\mathrm{shape}} K_m}{d^{2.4}} P(\bar{v}_{\mathrm{imp}}), \tag{65}$$

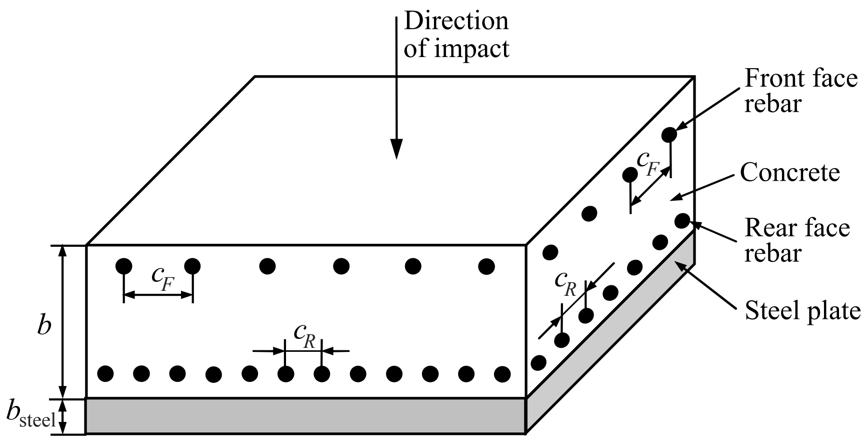

Figure 3. Reinforced concrete barrier, with notation.

where the coefficients K_S and K_{shape} will be defined subsequently, and

$$P(\bar{v}_{\text{imp}}) = \begin{cases} P_a(\bar{v}_{\text{imp}}) & \text{if } \bar{v}_{\text{imp}} < \bar{v}_*, \\ P_b(\bar{v}_{\text{imp}}) & \text{if } \bar{v}_{\text{imp}} \geq \bar{v}_*, \end{cases} \tag{66}$$

$$P_a(\bar{v}_{\text{imp}}) = \alpha_1 \ln(1+\alpha_2 \bar{v}_{\text{imp}}^2), \quad \alpha_1 = 9.48 \cdot 10^{-4}, \quad \alpha_2 = 215, \tag{67}$$

$$P_b(\bar{v}_{\text{imp}}) = k(\bar{v}_{\text{imp}} - \bar{v}_0), \quad k = 0.0213, \quad \bar{v}_0 = 0.0305, \tag{68}$$

$$\bar{v}_* = 0.061, \tag{69}$$

$$K_m = \begin{cases} 0.46m^{0.85} & \text{if } m < 182, \\ m^{0.7} & \text{if } m \geq 182. \end{cases} \tag{70}$$

Coefficient K_{shape} depends on the shape of the impactor and is determined as

$$K_{\text{shape}} = \begin{cases} 0.18\bar{L}_{\text{nose}} - 0.09\Delta\bar{L}_{\text{nose}} + 0.56 & \text{for an ogive nose}, \\ 0.25\bar{L}_{\text{nose}} - 0.125\Delta\bar{L}_{\text{nose}} + 0.56 & \text{for a conic nose}, \end{cases} \tag{71}$$

where $\bar{L}_{\text{nose}}$ and $\Delta\bar{L}_{\text{nose}}$ are the dimensionless (measured in impactor's diameter units) length of the nose and reduction of this length because of the bluntness (if any), respectively.

For sharp ogive-nosed shapes, the following equation can also be used:

$$K_{\text{shape}} = 0.56 + 0.18\sqrt{K_{\text{CRH}} - 0.25}. \tag{72}$$

If the nose of the impactor is neither ogive nor cone, Young [1997] recommended approximating the actual nose shape with ogive or conic shapes. If the bluntness is less than 10% of the penetrator diameter, it can be ignored.

Coefficient K_S depends on the properties of the concrete and is determined as

$$K_S = \frac{0.247 K_e (11 - K_D)}{(t_c' \bar{b}')^{0.06} (f'_c)^{0.3}}, \tag{73}$$

where

$$t_c' = \min(t_c, 1), \quad \bar{b}' = \min(\bar{b}, 6), \quad K_e = \max\left[\left(\frac{K_F F}{\overline{W}}\right)^{0.3}, 1\right], \tag{74}$$

$$K_F = \begin{cases} 0.5 & \text{if } \bar{b} \leq 2, \\ 1 & \text{if } \bar{b} > 2, \end{cases} \qquad F = \begin{cases} 20 & \text{for reinforced concrete}, \\ 30 & \text{for nonreinforced concrete}. \end{cases} \tag{75}$$

K_D is the volumetric rebar content, t_c is the cure time in days, and $\overline{W}$ is the shield thickness. Bars over variables denote dimensionless parameters which are normalized as indicated in the beginning of this section.

If there is no available data for calculating the coefficient K_S, the default value, $K_S = 0.9$, can be used.

This model is recommended for $7\,\text{MPa} < f_c' < 124\,\text{MPa}$, $m > 5\,\text{kg}$, $\overline{H} \geq 3$, and $v_{\text{imp}} < 1220\,\text{m/s}$.

24.2. ***Modifications of the models.*** Function $P(\bar{v}_{\mathrm{imp}})$ is discontinuous at the point $\bar{v}_{\mathrm{imp}} = \bar{v}_*$, which has no physical meaning and can cause problems when applying these relations. Ben-Dor et al. [2008a; 2008b] proposed two modifications of the model that improve the smoothness of $P(\bar{v}_{\mathrm{imp}})$.

The first modification [Ben-Dor et al. 2008a] is based on the correction of the parameters α_1, α_2, and $\bar{v}_*$ in (67)–(69). In order to satisfy the requirements that $P_a(\bar{v}_*) = P_b(\bar{v}_*)$ and $P'_a(\bar{v}_*) = P'_b(\bar{v}_*)$, their new values of these parameters are selected as

$$\bar{v}_* = 0.065, \quad \alpha_1 = 6.092 \cdot 10^{-3}, \quad \alpha_2 = 30.34. \tag{76}$$

In the second modification [Ben-Dor et al. 2008b], a more smooth approximation of the function $P(\bar{v}_{\mathrm{imp}})$ is proposed. This approximation is continuous and has continuous first and second derivatives for $\bar{v} = \bar{v}_*$ (and, consequently, for all $\bar{v}_{\mathrm{imp}} > 0$). Toward this end, Ben-Dor et al. [2008b] modified the model for relatively small $\bar{v}_{\mathrm{imp}}$ while keeping the Young's approximation for large $\bar{v}_{\mathrm{imp}}$. Specifically, the function $P_b(\bar{v}_{\mathrm{imp}})$ is given by (68) while the following approximation of $P_a(\bar{v}_{\mathrm{imp}})$ for small values of $\bar{v}_{\mathrm{imp}}$ is used:

$$P_a(\bar{v}_{\mathrm{imp}}) = (-4.95\bar{v}_{\mathrm{imp}}^2 + 0.196)\bar{v}_{\mathrm{imp}}^2, \quad \bar{v}_{\mathrm{imp}} \le \bar{v}_*, \quad \bar{v}_* = 0.0813. \tag{77}$$

25. UMIST formulas

This section is based on [Li et al. 2005; 2006], which refer to the approach suggested in the studies conducted at the University of Manchester Institute of Science and Technology (UMIST) [Reid and Wen 2001; BNFL 2003].

25.1. ***Penetration model.*** The formula for the DOP reads [Li et al. 2005]:

$$\bar{H} = \phi(\bar{v}_{\mathrm{imp}}) = \frac{0.88 \cdot 10^6 K^{(3)}_{\mathrm{shape}} m}{\sigma_t(\bar{v}_{\mathrm{imp}}) d^3} \bar{v}_{\mathrm{imp}}^2, \tag{78}$$

where σ_t is the rate-dependent characteristic strength of concrete,

$$\sigma_t(\bar{v}) = \zeta_0(f'_c) + \zeta_1(f'_c)\bar{v}, \tag{79}$$

$$\zeta_0(f'_c) = 4.2 f'_c + 1.35 \cdot 10^8, \quad \zeta_1(f'_c) = 14 f'_c + 0.45 \cdot 10^8. \tag{80}$$

Equation (78) has been validated in the following ranges of parameters: $\bar{H} < 2.5$, $3\,\mathrm{m/s} < v_{\mathrm{imp}} < 66\,\mathrm{m/s}$, $35\,\mathrm{kg} < m < 2500\,\mathrm{kg}$, and $5\,\mathrm{cm} < d < 60\,\mathrm{cm}$.

25.2. ***Perforation and scabbing model and its analysis.*** On the basis of formulas presented in [Li et al. 2005] (in [Li et al. 2006] a similar model is described for flat-nosed projectiles when the concrete reinforcement is ignored), the relations for the BLV and the scabbing limit velocity can be written as

$$\frac{\bar{v}_{\mathrm{bl}}^2}{\kappa \sigma_t(\bar{v}_{\mathrm{bl}})} = \begin{cases} \chi^{(1)}_{\mathrm{perf}}(\bar{b}) & \text{if } 0.5 < \bar{b} \le 1, \\ \chi^{(2)}_{\mathrm{perf}}(\bar{b}) & \text{if } 1 \le \bar{b} < 5, \\ \chi^{(3)}_{\mathrm{perf}}(\bar{b}) & \text{if } \bar{b} \ge 5, \end{cases} \tag{81}$$

$$\frac{\bar{v}_{\mathrm{sl}}^2}{\kappa \sigma_t(\bar{v}_{\mathrm{sl}})} = \begin{cases} \chi^{(1)}_{\mathrm{scab}}(\bar{b}) & \text{if } 0.5 < \bar{b} < 5, \\ \chi^{(2)}_{\mathrm{scab}}(\bar{b}) & \text{if } \bar{b} \ge 5, \end{cases} \tag{82}$$

where

$$\begin{aligned} &\chi^{(1)}_{\text{perf}}(\bar{b}) = \eta(1.506\bar{b}^2 - 0.506\bar{b}), && \chi^{(2)}_{\text{perf}}(\bar{b}) = \eta(2\bar{b}^3 - \bar{b}),\\ &\chi^{(3)}_{\text{perf}}(\bar{b}) = 78.5\bar{b} - 235.5, && \chi^{(2)}_{\text{scab}}(\bar{b}) = (56.52\bar{b} - 24.3)K^{(3)}_{\text{shape}},\\ &\chi^{(1)}_{\text{scab}}(\bar{b}) = \eta(1.386\bar{b}^2 - 0.5441\bar{b})/K^{(3)}_{\text{shape}}, && \kappa = 2\cdot 10^{-8} d^3/m. \end{aligned} \tag{83}$$

Parameter η is associated with the reinforcement of the shield and is given by

$$\eta = \begin{cases} 1.5r(d/c_{FR}) + 0.5 & \text{if } d/c_{FR} < \sqrt{d/d_{FR}},\\ 1.5r(d/d_{FR}) + 0.5 & \text{if } d/c_{FR} \geq \sqrt{d/d_{FR}}, \end{cases} \tag{84}$$

where $r = 100a_{FR}/(c_{FR}b)$ is determined by (62) with $a_F = a_R = a_{FR} = \pi d^2_{FR}/4$ and $c_F = c_R = c_{FR}$; a_{FR} and d_{FR} are the cross-sectional area and the diameter of the rebar, respectively; and c_{FR} is the rebar spacing (see Figure 3).

These scabbing and perforation models are valid for $2.2\,\text{cm} < d < 60\,\text{cm}$, $1\,\text{kg} < m < 2622\,\text{kg}$, $20\,\text{MPa} < f'_c < 79\,\text{MPa}$, $0 < r < 4$, $5.1\,\text{cm} < b < 64\,\text{cm}$, and $0 < v_{\text{imp}} < 427\,\text{m/s}$ [Li et al. 2005].

Since σ_t is a linear function of $\bar{v}$, determining $\bar{v} = \bar{v}_{\text{bl}}$ or $\bar{v} = \bar{v}_{\text{sl}}$ in terms of $\bar{b}$ requires solving the quadratic equation $f(\bar{v}) = 0$, where

$$f(\bar{v}) = \bar{v}^2 - (\kappa\zeta_1\Theta)\bar{v} - \kappa\zeta_0\Theta, \tag{85}$$

and Θ is the right-hand side of (81) or (82). It can be easily shown that the equation $f(\bar{v}) = 0$ has a single positive root:

$$\bar{v} = \omega(\Theta), \quad \omega(\Theta) = 0.5\kappa\zeta_1\Theta + \sqrt{\kappa\Theta(0.25\kappa\zeta_1\Theta + \zeta_0)}. \tag{86}$$

Taking into account (86), $\bar{v}_{\text{bl}}$ and $\bar{v}_{\text{sl}}$, which are determined by (81) and (82), can be expressed as increasing functions of $\bar{b}$. Consequently, the suggested model adequately describes the physics of penetration, and the single-valued inverse functions $\bar{b}_{\text{perf}}(\bar{v}_{\text{imp}})$ and $\bar{b}_{\text{scab}}(\bar{v}_{\text{imp}})$ exist. Since the functions determined by (81) and (82) are discontinuous at the point $\bar{b} = 5$, the function $\bar{v}_{\text{bl}} = \bar{v}_{\text{bl}}(\bar{b})$ assumes different values, $\bar{v}'_{\text{bl}}$ and $\bar{v}''_{\text{bl}}$, to the left and to the right of this point. Therefore the inverse function, $\bar{b}_{\text{perf}} = \Xi_{\text{perf}}(\bar{v}_{\text{imp}})$, cannot be defined in the interval $\bar{v}'_{\text{bl}} < \bar{v}_{\text{imp}} < \bar{v}''_{\text{bl}}$. The discontinuity of the function $\bar{v}_{\text{sl}} = \bar{v}_{\text{sl}}(\bar{b})$ causes similar problems.

26. Malaysia models

Zaidi et al. [2010], of the Universiti Tun Hussein Onn Malaysia (UTHM), proposed a linear model (hereafter, UTHM model) describing the dependence between the DOP, $\overline{H}$, and the parameter $mv^2_{\text{imp}}/(f'_c d^3)$ which is valid for ogive-nosed impactors:

$$\overline{H} = \frac{0.5\cdot 10^6 m\bar{v}^2_{\text{imp}}}{f'_c d^3} q_1 + q_0, \tag{87}$$

where the coefficients depend on the CRH of the impactor, K_{CRH}:

$$
\begin{aligned}
q_0 &= -0.6256K_{\mathrm{CRH}}^3 + 7.9656K_{\mathrm{CRH}}^2 - 32.892K_{\mathrm{CRH}} + 48.008,\\
q_1 &= 0.014K_{\mathrm{CRH}}^3 - 0.1585K_{\mathrm{CRH}}^2 + 0.5606K_{\mathrm{CRH}} - 0.5405.
\end{aligned}
\tag{88}
$$

This formula can be applied in the following ranges of parameters: $139\,\mathrm{m/s} < v_{\mathrm{imp}} < 1225\,\mathrm{m/s}$, $13.5\,\mathrm{MPa} \le f_c' \le 108\,\mathrm{MPa}$, $64\,\mathrm{g} < m < 13.2\,\mathrm{kg}$, $2 \le K_{\mathrm{CRH}} \le 6$, and $13\,\mathrm{mm} < d < 76.2\,\mathrm{mm}$.

Both Rahman et al. [2010] and Latif et al. [2011] considered perforation by flat-nosed (cylindrical) impactors. Rahman et al. proposed the scabbing limit velocity

$$
\bar{v}_{\mathrm{sl}} = 10^{-3} d \sqrt{\frac{2 f_c' d}{m} \psi_1(\bar{b})}, \tag{89}
$$

where

$$
\psi_1(\bar{b}) = \begin{cases} 0.87\bar{b} - 0.29 & \text{if } 0.69 \le \bar{b} \le 3.0,\\ 3.31\bar{b} - 7.58 & \text{if } 3.0 < \bar{b} \le 6.0,\\ 4279.181\bar{b} - 25662.82 & \text{if } 6.0 < \bar{b} \le 14.86. \end{cases} \tag{90}
$$

The latter formula is valid in the following ranges of parameters: $0.69 < b/d < 14.86$, $24.15\,\mathrm{MPa} \le f_c' \le 50.2\,\mathrm{MPa}$, $17.5\,\mathrm{mm} < d < 305.0\,\mathrm{mm}$, $0.92\,\mathrm{kg} < m < 309\,\mathrm{kg}$, $29\,\mathrm{m/s} < v_{\mathrm{sl}} < 427\,\mathrm{m/s}$, and $50.8\,\mathrm{mm} < b < 609.6\,\mathrm{mm}$.

Latif et al. [2011] suggested a formula for the BLV that can be written as

$$
\bar{v}_{\mathrm{bl}} = 10^{-3} d \sqrt{\frac{2 f_c' d}{m} \psi_2(\bar{b})}, \tag{91}
$$

where

$$
\psi_2(\bar{b}) = 0.174\bar{b}^3 + 0.169\bar{b}^2 + 0.0577\bar{b} + 0.2969. \tag{92}
$$

This formula is valid in the following ranges of parameters: $0.92\,\mathrm{kg} < m < 309\,\mathrm{kg}$, $29\,\mathrm{m/s} < v_{\mathrm{bl}} < 427\,\mathrm{m/s}$, $0.69 < b/d < 14.86$, $24.15\,\mathrm{MPa} \le f_c' \le 50.2\,\mathrm{MPa}$, and $17.5\,\mathrm{mm} < d < 305.0\,\mathrm{mm}$.

27. TM 5-855-1 formulas

The formulas of [TM 5-855-1 1986] are proposed for describing penetration by a standard fragment in the shape of a cylinder of diameter d and length $0.5d$ with a hemispherical nose. The mass of the fragment that is used in the formulas is calculated as

$$
m = (5\pi/24)\rho_{\mathrm{imp}} d^3 = 0.654\rho_{\mathrm{imp}} d^3. \tag{93}
$$

The model of [TM 5-1300 1990] employs the following formula for the fragment mass (after conversion to SI units): $m = 5.149 \cdot 10^3 d^3$, that is, the density is assumed to be $\rho_{\mathrm{imp}} = 7873\,\mathrm{kg/m^3}$.

After some transformations the formulas of [TM 5-855-1 1986] can be written in the form

$$H = \begin{cases} \dfrac{13.46m^{0.37}\bar{v}_{\text{imp}}^{0.9}}{(f_c')^{0.25}} & \text{if } \bar{v}_{\text{imp}} \le \bar{v}_{\text{imp}}^*, \\ \dfrac{1.09\cdot 10^3 m^{0.4}\bar{v}_{\text{imp}}^{1.8}}{(f_c')^{0.5}} + 0.395m^{0.33} & \text{if } \bar{v}_{\text{imp}} > \bar{v}_{\text{imp}}^*, \end{cases} \tag{94}$$

$$b_{\text{perf}} = 0.0311Hm^{0.033} + 2.95m^{0.33}, \tag{95}$$

$$b_{\text{scab}} = 0.0334Hm^{0.033} + 4.465m^{0.33}, \tag{96}$$

$$\bar{v}_{\text{res}} = \begin{cases} \bar{v}_{\text{imp}}[1-(b/b_{\text{perf}})^2]^{0.555} & \text{if } \bar{v}_{\text{imp}} \le \bar{v}_{\text{imp}}^*, \\ \bar{v}_{\text{imp}}[1-(b/b_{\text{perf}})]^{0.555} & \text{if } \bar{v}_{\text{imp}} > \bar{v}_{\text{imp}}^*, \end{cases} \tag{97}$$

where

$$\bar{v}_{\text{imp}}^* = 5.13\cdot 10^{-3}(f_c')^{0.278}/m^{0.044}. \tag{98}$$

28. Folsom's model for penetration into a shield with a predrilled hole

Folsom [1987] proposed a formula for the DOP of ogive-nosed projectile into a predrilled shield that can be written, in the interpretation of [Teland 2001], in the form

$$\bar{H} = \frac{17.3m\bar{v}_{\text{imp}}^{1.5}}{f_c' d^{2.785}}\cdot\frac{1-038\eta^2}{1-\eta^2} - \frac{4\Omega(K_{\text{CRH}},\eta)}{1-\eta^2} + \sqrt{K_{\text{CRH}}-0.25}, \tag{99}$$

where d_0 is diameter of the hole and

$$\begin{aligned} \Omega(z,\eta) &= \left(\frac{1-\eta^2}{4} - z + 2z^2\right)g + \frac{1-2z}{2}\left[g\sqrt{z^2-g^2} + z^2\sin^{-1}(g/z)\right] - \frac{g^3}{3}, \\ g(z,\eta) &= \sqrt{(1-\eta)[z-0.25(1-\eta)]}, \qquad \eta = d_0/d. \end{aligned} \tag{100}$$

29. Some other models and related problems

Walter and Wolde-Tinsae [1984] proposed a modified version of several empirical formulas that account for the presence of a steel plate at the distal face of a shield by introducing an artificial thickness of the shield. Riera [1989] presented the relationship between H and v_{imp} as an implicit function and suggested and discussed equations for determining v_{bl} and v_{sl}. Al-Hachamee and Azeez [2010] proposed formulas for H, b_{perf}, and b_{scab} on the basis of the results of 20 experiments from the literature. Dancygier [1997] proposed a method that allowed for including the reinforcement ratio as a parameter in the existing semiempirical perforation formulas.

Me-Bar [1997] proposed a method for scaling ballistic penetration into concrete shields using energy balance whereby the energy absorbed by a shield during penetration was expressed as a sum of the energy expended for surface effects and the energy expended for volume effects. Dancygier [2000] investigated the effects of impact by similarly (though not necessarily geometrically similar) shaped impactors on the reinforced concrete barriers. Based on the widely known empirical formulae he derived expressions for the velocity ratios between the simulator (reference impactor) and the simulated impactors that are required in order to yield the same DOP and the same perforation limit thickness.

30. Comparison between models and their experimental validations

30.1. ***Brief review.*** Plots which allow comparing the dependencies obtained using various phenomenological models can be found in [Kennedy 1976; Yankelevsky 1997; Teland 1999; Li and Tong 2003] as well as other studies.

Analysis of the models based on comparison of their predictions with experimental results is complicated for the following reasons: the small number of performed experiments, the narrow range of parameters within which the experiments are usually conducted, and the lack of information concerning the properties of the concrete in the shield (reinforcement, aggregate size, etc.). Also, in some cases the accuracy of the model is evaluated using the same experimental data as used in the derivation of the model.

Kennedy [1976] examined the ACE, NDRC, BBL, Ammann–Whitney and Pétry models using experimental data and recommended the modified NDRC formula for calculating b_{scab} and b_{perf} in the following ranges of the parameters: $d < 41\,\mathrm{cm}$, $0.02\,\mathrm{kg/m^3} < m/d^3 < 0.54\,\mathrm{kg/m^3}$, and $30\,\mathrm{m/s} < v_{imp} < 900\,\mathrm{m/s}$. He concluded that this formula generally agrees with the test results within an accuracy of ±20% in these ranges of the parameters.

Sliter [1980] collected experimental data on the penetration of cylindrical projectiles into finite-thickness shields and calculated the DOP using the NDRC model, the scabbing thickness using the NDRC, Bechtel, and Stone–Webster models, and the perforation thickness using the NDRC and CEA-EDF models. He found that the values of the DOP predicted by the NDRC formula are within an experimental scatter of ±25% for $v_{imp} > 152\,\mathrm{m/s}$ while the agreement between the predicted and the observed values of the DOP is unsatisfactory for relatively low impact velocities. As far as scabbing thickness prediction concerns, Sliter [1980] noted that the NDRC, Bechtel, and Stone–Webster models may all be used equally well for missiles having relatively small diameters. Regarding perforation, he indicated that damage is predicted better by the CEA-EDF formula than by the NDRC model.

On the basis of experimental data collected by Sliter [1980], Adeli and Amin [1985] examined the modified NDRC, Haldar–Miller, Hughes, and Adeli–Amin models for penetration, the Pétry I, Pétry II, ACE, modified NDRC, BRL, Bechtel, Chang, Hughes, and Adeli–Amin models for scabbing, and the Pétry I, Pétry II, ACE, modified NDRC, BRL, CEA-EDF, Degen, Chang, Hughes, and Adeli–Amin models for perforation. Their conclusions are as follow.

Penetration modeling. For $\overline{H}_{exp} \geq 0.6$, where $\overline{H}_{exp} = H_{exp}/d$ and H_{exp} is the experimentally observed DOP, the modified NDRC, Haldar–Miller, Hughes, and Adeli–Amin models "tend to agree with the experimental results within ±25%". For $\overline{H}_{exp} < 0.6$, the Pétry II, Haldar–Miller, and cubic Adeli–Amin models perform better than the other models. The ACE and Pétry I models considerably overestimate the DOP. Consequently, for estimating the DOP Adeli and Amin [1985] recommended applying the quadratic Adeli–Amin formula for $v_{imp} < 145\,\mathrm{m/s}$ and the quadratic Adeli–Amin model or modified NDRC formula for $145\,\mathrm{m/s} \leq v_{imp} < 305\,\mathrm{m/s}$.

Scabbing modeling. The Adeli–Amin, Bechtel, and Chang models perform better and, generally, are the least conservative among all the models. The Hughes model is the most conservative while the

Pétry I, Pétry II, and BRL equations performance is the poorest in predicting the scabbing thickness. Consequently, for scabbing modeling quadratic the Adeli–Amin, Bechtel, or Chang's formulas for $v_{\mathrm{imp}} < 310\,\mathrm{m/s}$ are recommended.

Perforation modeling. The CEA-EDF, Degen, Chang, and Adeli–Amin models perform better than the other models although the predictions of the modified NDRC and Pétry I formulas also agree with the experimental data. The predictions of the Pétry II, BRL, and ACE formulas do not exhibit good agreement with the results of experiments. Consequently, the CEA-EDF, Degen, Chang, and Adeli–Amin models for $v_{\mathrm{imp}} < 310\,\mathrm{m/s}$ are recommended for perforation modeling.

Walter and Wolde-Tinsae [1984] used the results of 45 experiments for comparing the performance of the basic empirical models. They concluded that among nine empirical models the best predictions of perforation/nonperforation events are provided by the Pétry I, Pétry II, Degen, BRL, and CEA-EDF formulas whereby the percentage of the correct prediction varies in the range 71–75% while the percentage of the correct prediction is small for the NDRC and Kar's formulas (53%).

In [DOE 2006] Chang's formula is recommended for calculating b_{scab} and b_{perf}.

30.2. ***Evaluation of performance of different models: Finite-thickness shields.*** The evaluation of the performance of different empirical models in this section is based on the results of experiments collected in [Sliter 1980]. The input data for the evaluated models are also adopted from [ibid.], and only those experiments that satisfy the conditions for the validity of the model are used for evaluating each of the models. The UMIST model is not included in the list of analyzed models because most of the experiments from [ibid.] fall outside the range of validity of this model. Comparison of the UMIST model and the NDRC model with other sets of experimental data can be found in [Li et al. 2006].

General results on the performance of the various models are summarized in Table 9. The denominator in the fraction in the penetration (scabbing, perforation) column denotes the number of experiments in which penetration (scabbing, perforation) was observed, while the numerator is the number of calculations which predicted the corresponding phenomenon. The last column contains similar characteristics for all types of damage (penetration, scabbing, and perforation) so that the numerator of the fraction equals the sum of the numerators of the fractions in the preceding columns and the denominator is the sum of the denominators of the fractions in the preceding columns. Clearly, only the experiments that were calculated using the empirical models are taken into account. Parameters b_{perf} and b_{scab} in the average model are determined as arithmetic means of the values of the corresponding parameters in the models which predicted these parameters. It is assumed that penetration, scabbing, or perforation occurs if $b < b_{\mathrm{scab}}$, $b_{\mathrm{scab}} \le b < b_{\mathrm{perf}}$, or $b \ge b_{\mathrm{perf}}$, respectively.

Analysis of the data presented in Table 9, conducted taking into account the number of the experiments included in the evaluation of each model, reveals the advantages of Chang's model and the CRIEPI model over the other models.

30.3. ***Evaluation of performance of different models: Semiinfinite shields.*** Evaluation of the performance of different models for semiinfinite shields is based on the experimental data on ogive-nosed impactors collected in [Hansson 2003]. These experiments were conducted in the following ranges of parameters: $13\,\mathrm{mm} \le d \le 365\,\mathrm{mm}$, $132\,\mathrm{m/s} \le v_{\mathrm{imp}} \le 1050\,\mathrm{m/s}$, $64\,\mathrm{g} \le m \le 485\,\mathrm{kg}$, $21.6\,\mathrm{MPa} \le f_c' \le 140\,\mathrm{MPa}$, $55\,\mathrm{mm} \le H_{\mathrm{exp}} \le 1.96\,\mathrm{m}$, and $1.5 \le K_{\mathrm{CRH}} \le 6$.

Model	Penetration	Scabbing	Perforation	Penetration, scabbing, perforation
Pétry II	$\frac{31}{35} = 89\%$	$\frac{3}{51} = 6\%$	$\frac{2}{16} = 13\%$	$\frac{36}{102} = 35\%$
BRL	$\frac{29}{35} = 83\%$	$\frac{9}{51} = 18\%$	$\frac{4}{16} = 25\%$	$\frac{42}{102} = 41\%$
ACE	$\frac{17}{19} = 89\%$	$\frac{4}{6} = 67\%$	$\frac{5}{5} = 100\%$	$\frac{26}{30} = 87\%$
NDRC	$\frac{28}{35} = 80\%$	$\frac{11}{51} = 22\%$	$\frac{12}{16} = 75\%$	$\frac{51}{102} = 50\%$
Chang	$\frac{30}{35} = 86\%$	$\frac{41}{51} = 80\%$	$\frac{8}{16} = 50\%$	$\frac{79}{102} = 77\%$
Adeli–Amin	$\frac{20}{22} = 91\%$	$\frac{9}{18} = 50\%$	$\frac{4}{5} = 80\%$	$\frac{33}{45} = 73\%$
CRIEPI	$\frac{31}{35} = 89\%$	$\frac{48}{51} = 94\%$	$\frac{4}{16} = 25\%$	$\frac{83}{102} = 81\%$
Average	$\frac{34}{35} = 97\%$	$\frac{32}{51} = 63\%$	$\frac{6}{16} = 31\%$	$\frac{71}{102} = 70\%$

Table 9. Comparison of empirical models based on type of shield damage with experimental data from [Sliter 1980].

In the calculations for evaluating the performances of different empirical models, we included only those models which are applicable for ogive-shaped impactors: the Ammann–Whitney model, the modified NDRC model, Degen's model, Kar's model, the Healey–Weissman model, the Haldar–Hamieh–Miller model, Hughes' model, the UKAEA-CEBG-NNC model, Young's model, the Whiffen formula, and the UTHM formula. The UMIST, Haldar–Miller, and Adeli–Amin formulas are not included because most or all the experiments fall outside the range of validity of these models. A value of $\bar{c} = 1$ is used in Whiffen's model.

Detailed results of the calculations are shown in Figure 4, where circles indicate experiments for which the constraints for applicability of the model (if any) are satisfied while triangles denote experiments that fall outside the range of validity of the particular model. The average model is constructed using values of the DOP equal to the arithmetic mean of the values of the DOPs for all models where the model's constraints are satisfied. The error, $\tilde{\epsilon}$, is calculated for each experiment as

$$\tilde{\epsilon} = \frac{H - H_{\text{exp}}}{H_{\text{exp}}} 100\%, \tag{101}$$

where H_{exp} is the DOP observed in the experiment. Horizontal lines denote the values of $\tilde{\epsilon}$ which are considered the minimum $\tilde{\epsilon}_{\min}$ and maximum $\tilde{\epsilon}_{\max}$ values of $\tilde{\epsilon}$; in some cases we excluded outliers with anomalously high deviations from the mean. Hereafter in calculating $\tilde{\epsilon}_{\min}$ and $\tilde{\epsilon}_{\max}$ (and in other calculations) for each model, we take into account only those experiments that correspond to the range of the validity of the model (denoted by the circles in Figure 4).

The results of calculations are summarized in Table 10, where the name of the model is indicated in the first column. In some cases the whole range of variation of $\overline{H}_{\text{exp}}$ is divided into two subranges. The number of experiments in each subrange or the whole range, where the value of the parameter $\tilde{\epsilon}$ falls in the interval between $\tilde{\epsilon}_{\min}$ and $\tilde{\epsilon}_{\max}$, is indicated in column 2 (index N); the end-points of the range (or subrange) are presented in column 3; $\tilde{\epsilon}_{\min}$ and $\tilde{\epsilon}_{\max}$ in columns 4 and 5, respectively. The indexes in the rest of the columns are described below.

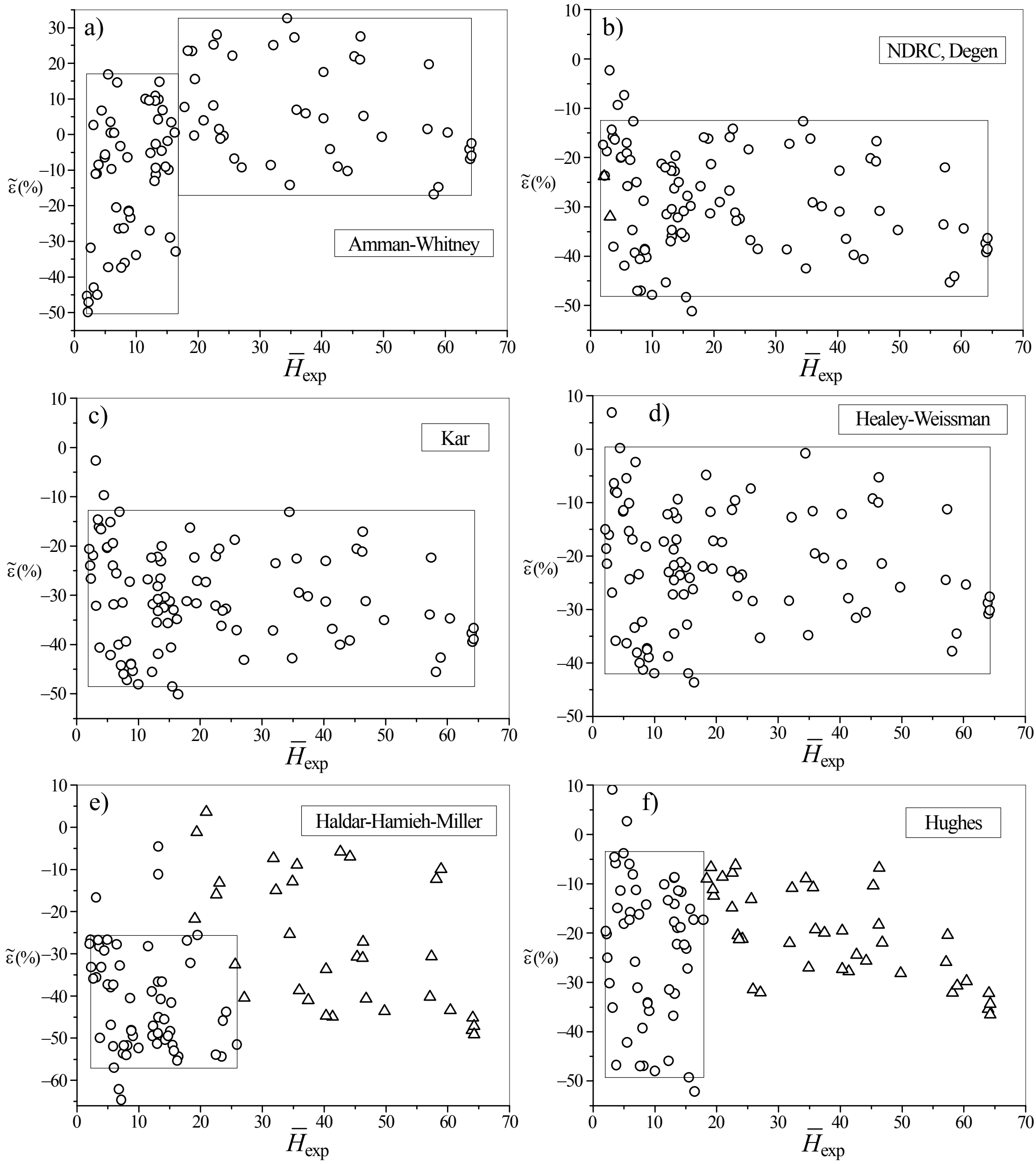

Figure 4. Comparison of DOPs predicted using empirical model with results of experiments, taking (circles) and not taking (triangles) into account model constraints (continued on next page).

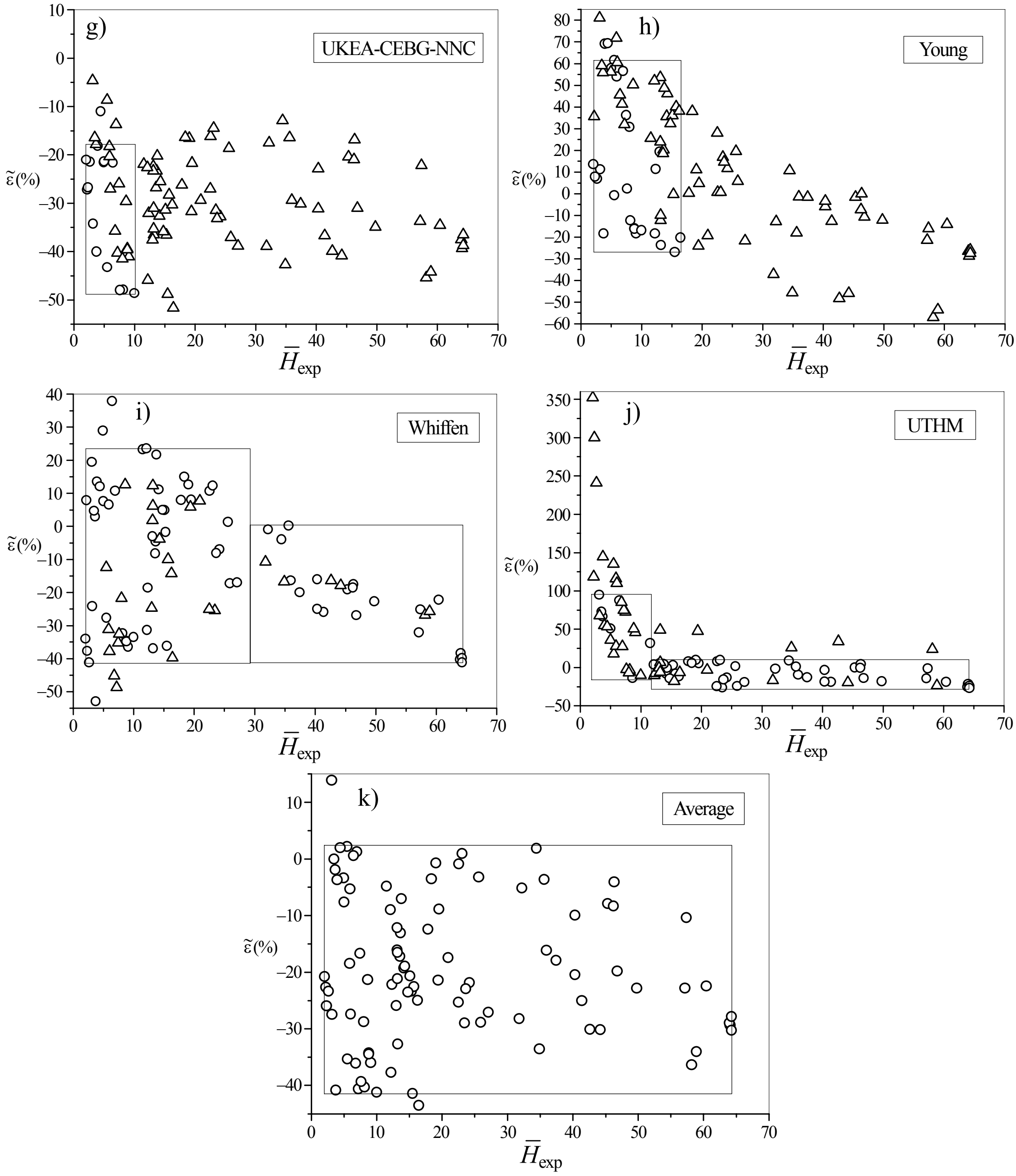

Figure 4 (cont.)

Inspection of Table 10 and Figure 4 shows that for the modified NDRC, Degen, Kar, Haldar–Hamieh–Miller, UKAEA-CEBG-NNC, Hughes, and average models, $\tilde{\epsilon}_{max} < 0$ or slightly larger than 0. Consequently, practically all predicted values of the DOP are smaller than the experimental values, and the predictions of the models are expected to be biased towards smaller magnitudes of the DOP. Since for the other models as well the interval of variation of $\tilde{\epsilon}$ is strongly asymmetrical with respect to $\tilde{\epsilon} = 0$

Model	N	$\overline{H}_{\text{exp}}$	$\tilde{\epsilon}_{\text{min}}$	$\tilde{\epsilon}_{\text{max}}$	$\hat{k}$	$\hat{\epsilon}_*$
Ammann–Whitney	53	$2\div 18$	−50%	+17%	1.20	40%
	41	$18\div 65$	−17%	+33%	0.93	23%
NDRC, Degen	88	$2\div 65$	−49%	−12%	1.44	26%
Kar	91	$2\div 65$	−49%	−13%	1.44	26%
Healey–Weissman	92	$2\div 65$	−42%	0%	1.26	27%
Haldar–Hamieh–Miller	56	$2\div 26$	−57%	−25%	1.70	27%
Hughes	51	$2\div 18$	−50%	−3%	1.36	31%
UKEA-CEBG-NNC	14	$2\div 10$	−50%	−18%	1.50	23%
Young's	24	$2\div 17$	−30%	+62%	0.85	38%
Whiffen	44	$2\div 28$	−41%	+24%	1.10	35%
	20	$28\div 65$	−41%	0%	1.26	26%
UTHM	8	$2\div 12$	−14%	+95%	0.71	39%
	43	$12\div 65$	−27%	+9%	1.10	20%
Average	92	$2\div 65$	−42%	+2%	1.25	27%

Table 10. Summary of data for evaluating performance of penetration models on the basis of the indexes $\hat{k}$ and $\hat{\epsilon}_*$.

($\tilde{\epsilon}_{\text{max}}$ strongly deviates from $-\tilde{\epsilon}_{\text{min}}$), predictions of the models are also biased in these cases. Therefore in practice it is not recommended to directly use the predictions of any one of these models for the indicated range of penetration conditions.

Let us introduce for each model the correction factor, $\hat{k} > 0$, which allows us to reduce or increase the predicted value of the DOP, that is, replace the calculated values, H, by $\hat{k}H$. Then, taking into account (101), the *new* errors of the model's predictions, $\hat{\epsilon}$, can be expressed through the *old*, $\tilde{\epsilon}$, as

$$\hat{\epsilon} = \frac{\hat{k}H - H_{\text{exp}}}{H_{\text{exp}}} \cdot 100\% = \left[\hat{k}\left(\frac{H}{H_{\text{exp}}}\right) - 1\right] \cdot 100\% = [\hat{k}(0.01\tilde{\epsilon} + 1) - 1] \cdot 100\%. \tag{102}$$

The value of $\hat{k}$ is selected by applying the following criterion: all values of $\hat{\epsilon}$ must fall in the interval between the minimum, $-\hat{\epsilon}_*$, and the maximum, $+\hat{\epsilon}_*$, values of the errors of the corrected model. Equation (102) implies that $\hat{\epsilon}$ increases when $\tilde{\epsilon}$ is increased. Therefore, the above requirement yields the following system of equations:

$$\hat{k}(0.01\tilde{\epsilon}_{\text{min}} + 1) - 1 = -0.01\hat{\epsilon}_*, \quad \hat{k}(0.01\tilde{\epsilon}_{\text{max}} + 1) - 1 = 0.01\hat{\epsilon}_*, \tag{103}$$

which has the solution

$$\hat{k} = \frac{2}{2 + 0.01(\tilde{\epsilon}_{\text{min}} + \tilde{\epsilon}_{\text{max}})}, \quad \hat{\epsilon}_* = \frac{\tilde{\epsilon}_{\text{max}} - \tilde{\epsilon}_{\text{min}}}{2 + 0.01(\tilde{\epsilon}_{\text{min}} + \tilde{\epsilon}_{\text{max}})}. \tag{104}$$

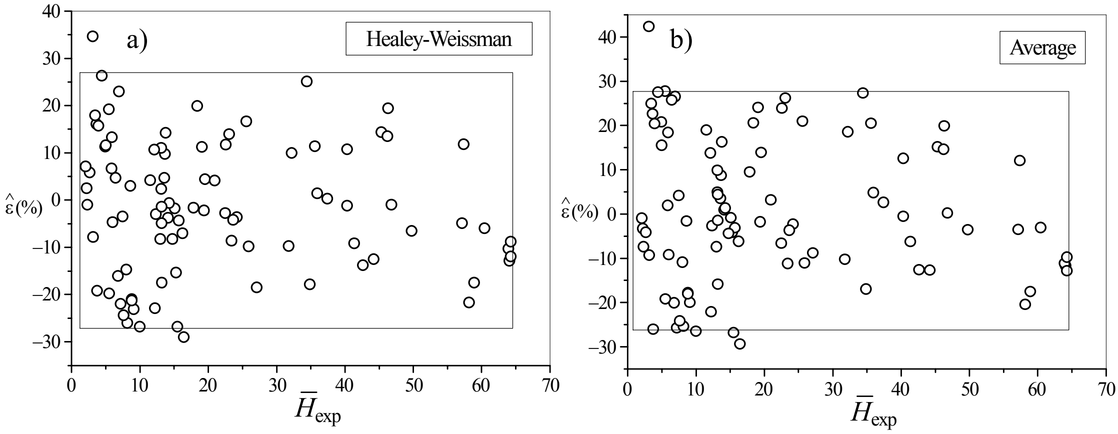

Figure 5. Comparison of the DOPs predicted using the recommended corrected empirical model with the results of experiments.

The values of parameters $\hat{k}$ and $\hat{\epsilon}_*$ for all models are presented in Table 10. Now the performances of the corrected models can be evaluated by using only one fundamental parameter, $\tilde{\epsilon}$, which is related with their errors. In the study of the performance of a certain empirical model which is based on the same experimental data, Hansson [2003] also suggested introducing a correction factor, although he did not devise an unambiguous procedure for determining the correction coefficient.

In selecting the best model, we take into account the following considerations: the number of subranges; the error interval of the corrected model, $2\hat{\epsilon}_*$; the proximity of the absolute value of the coefficient $\hat{k}$ to 1; and the number of points, N. Inspection of Table 10 shows that at the first stage it is appropriate to select for further consideration those models which span the whole range $2 \le \overline{H}_{\mathrm{exp}} \le 65$ without dividing it into subranges and having a parameter $\hat{\epsilon}_*$ that only slightly differs from the minimal (among all models) value. These models include the modified NDRC, Degen, Kar, Healey–Weissman, and average models, for which $26\% \le \hat{\epsilon}_* \le 27\%$. Among the remaining five models the performance of the modified NDRC, Degen, and Kar models is inferior to the rest as they are characterized by comparatively large values of $\hat{k}$.

Therefore, the most appropriate models are the Healey–Weissman and average models, which are corrected using the correction factors 1.26 and 1.25, respectively, and which are expected to have error $\pm 27\%$. Figure 5 illustrates the distribution of errors for these corrected models.

We used a comparatively simple approach for the comparison and evaluation of the performance of different penetration models. It is feasible to employ other approaches that are more heavily based on the methods of mathematical statistics. In particular, different models can be interpreted as regression equations, and the correction factor for each model can be interpreted as a regression coefficient. Clearly, models with several regression coefficients can be also considered. However, in this case as well, unambiguous procedures for solving the problem cannot be devised [Draper and Smith 1998].

31. Concluding remarks

We would like to draw attention to some peculiarities of the procedures for evaluating the performance and accuracy of empirical penetration models.

Firstly, analysis of the accuracy of a model which employs the same data used for the model derivation is questionable. Secondly, while a model with multiple parameters, based on a few experimental points, can be quite accurate at these points, good predictive properties of the model cannot be guaranteed. In the limiting case, one can suggest a model where the number of parameters is equal to the number of the experimental points. The errors of the approximation at these experimental points vanish while the accuracy of the model at other points can be inadequate. Thirdly, it is not worthwhile to overvalue statistical estimates of the reliability of a model since many of these estimates are based on a number of questionable assumptions.

Appendix

Most of the types of dependencies between $\bar{b}_{\text{perf}}$ and $\overline{H}$ and between $\bar{b}_{\text{scab}}$ and $\overline{H}$ can be described as

$$\bar{b}_{p/s} = \Psi_{p/s}(\overline{H}) = \begin{cases} \beta^{(1)} + \alpha^{(1)}\overline{H} - \gamma^{(1)}\overline{H}^2 & \text{if } \overline{H}^{(1)}_{\text{min}} \le \overline{H} \le \overline{H}^{(1)}_{\text{max}}, \\ \vdots & \vdots \\ \beta^{(i)} + \alpha^{(i)}\overline{H} - \gamma^{(i)}\overline{H}^2 & \text{if } \overline{H}^{(i)}_{\text{min}} \le \overline{H} \le \overline{H}^{(i)}_{\text{max}}, \\ \vdots & \vdots \\ \beta^{(n)} + \alpha^{(n)}\overline{H} - \gamma^{(n)}\overline{H}^2 & \text{if } \overline{H}^{(n)}_{\text{min}} \le \overline{H} \le \overline{H}^{(n)}_{\text{max}}, \end{cases} \tag{A.1}$$

where it is assumed that all parameters $\alpha^{(i)}$, $\beta^{(i)}$, and $\gamma^{(i)}$ are nonnegative and functions $\Psi_{p/s}$ are continuous at the joint points of the adjacent segments:

$$\overline{H}^{(i)}_{\text{max}} = \overline{H}^{(i+1)}_{\text{min}}, \qquad i = 1, 2, \ldots, n-1, \tag{A.2}$$

$$\beta^{(i)} + \alpha^{(i)}\overline{H}^{(i)}_{\text{max}} - \gamma^{(i)}[\overline{H}^{(i)}_{\text{max}}]^2 = \beta^{(i)} + \alpha^{(i)}\overline{H}^{(i+1)}_{\text{min}} - \gamma^{(i)}[\overline{H}^{(i+1)}_{\text{min}}]^2, \qquad i = 1, 2, \ldots, n-1. \tag{A.3}$$

The subscript p/s denotes "perforation" or "scabbing". Clearly, the parameters $\alpha^{(i)}$, $\beta^{(i)}$, $\gamma^{(i)}$, $\overline{H}^{(i)}_{\text{min}}$, and $\overline{H}^{(i)}_{\text{max}}$ should have the same additional subscripts, p or s, which are omitted for simplicity.

If $\gamma^{(i)} = 0$, then $\Psi_{p/s}(\overline{H})$ is an increasing function on the i-th interval ($i = 1, 2, \ldots, n$). If $\gamma^{(i)} > 0$, then $\Psi_{p/s}(\overline{H})$ is an increasing function when

$$\overline{H}^{(i)}_{\text{max}} < \overline{H}^{(i)}_{*}, \qquad \overline{H}^{(i)}_{*} = 0.5\alpha^{(i)}/\gamma^{(i)}, \qquad i = 1, 2, \ldots, n \tag{A.4}$$

The constraints given by (A.4) must be taken into account when the admissible ranges of the parameters are indicated.

Taking into account (A.1), (2) and (4) can be rewritten as

$$\bar{b}_{p/s} = \Xi_{p/s}(\bar{v}_{\mathrm{imp}}) = \begin{cases} \beta^{(1)}+\alpha^{(1)}\phi(\bar{v}_{\mathrm{imp}})-\gamma^{(1)}[\phi(\bar{v}_{\mathrm{imp}})]^2 & \text{if } \phi^{-1}(\overline{H}^{(1)}_{\min}) \le \bar{v}_{\mathrm{imp}} \le \phi^{-1}(\overline{H}^{(1)}_{\max}), \\ \vdots & \vdots \\ \beta^{(i)}+\alpha^{(i)}\phi(\bar{v}_{\mathrm{imp}})-\gamma^{(i)}[\phi(\bar{v}_{\mathrm{imp}})]^2 & \text{if } \phi^{-1}(\overline{H}^{(i)}_{\min}) \le \bar{v}_{\mathrm{imp}} \le \phi^{-1}(\overline{H}^{(i)}_{\max}), \\ \vdots & \vdots \\ \beta^{(n)}+\alpha^{(n)}\phi(\bar{v}_{\mathrm{imp}})-\gamma^{(n)}[\phi(\bar{v}_{\mathrm{imp}})]^2 & \text{if } \phi^{-1}(\overline{H}^{(n)}_{\min}) \le \bar{v}_{\mathrm{imp}} \le \phi^{-1}(\overline{H}^{(n)}_{\max}), \end{cases} \quad \text{(A.5)}$$

$$\begin{aligned} \bar{v}_{bl/sl} &= \Xi^{-1}_{\mathrm{perf}}(\bar{b}) \\ &= \begin{cases} \phi^{-1}(\overline{H}^{(1)}) & \text{if } \beta^{(1)}+\alpha^{(1)}\overline{H}^{(1)}_{\min}-\gamma^{(1)}[\overline{H}^{(1)}_{\min}]^2 \le \bar{b} \le \beta^{(1)}+\alpha^{(1)}\overline{H}^{(1)}_{\max}-\gamma^{(1)}[\overline{H}^{(1)}_{\max}]^2, \\ \vdots & \vdots \\ \phi^{-1}(\overline{H}^{(i)}) & \text{if } \beta^{(i)}+\alpha^{(i)}\overline{H}^{(i)}_{\min}-\gamma^{(i)}[\overline{H}^{(i)}_{\min}]^2 \le \bar{b} \le \beta^{(i)}+\alpha^{(i)}\overline{H}^{(i)}_{\max}-\gamma^{(i}[\overline{H}^{(i)}_{\max}]^2, \\ \vdots & \vdots \\ \phi^{-1}(\overline{H}^{(n)}) & \text{if } \beta^{(n)}+\alpha^{(n)}\overline{H}^{(n)}_{\min}-\gamma^{(1)}[\overline{H}^{(n)}_{\min}]^2 \le \bar{b} \le \beta^{(n)}+\alpha^{(n)}\overline{H}^{(n)}_{\max}-\gamma^{(n)}[\overline{H}^{(n)}_{\max}]^2, \end{cases} \end{aligned} \quad \text{(A.6)}$$

where the subscript bl/sl denotes the BLV and scabbing limit velocity, and $\overline{H}^{(i)}$ is a root of the equation

$$\beta^{(i)}+\alpha^{(i)}\overline{H}^{(i)}-\gamma^{(i)}[\overline{H}^{(i)}]^2=\bar{b}, \quad \text{(A.7)}$$

which is given by

$$\overline{H}^{(i)} = \sqrt{\left[\frac{\alpha^{(i)}}{2\gamma^{(i)}}\right]^2+\frac{\bar{b}-\beta^{(i)}}{\gamma^{(i)}}}-\frac{\alpha^{(i)}}{2\gamma^{(i)}} \quad \text{if } \gamma^{(i)}>0, \quad \text{(A.8)}$$

$$\overline{H}^{(i)} = \frac{\bar{b}-\beta^{(i)}}{\alpha^{(i)}} \quad \text{if } \gamma^{(i)}=0. \quad \text{(A.9)}$$

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