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A HYSTERETIC BINGHAM MODEL FOR MR DAMPERS TO CONTROL CABLE VIBRATIONS

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This paper proposes a new parametric dynamic model for magnetorheological (MR) fluid dampers. The proposed model adapts the regularized Bingham model to accurately reproduce the hysteretic behavior of such dampers. The optimized model parameters are then obtained by making the model predictions as close as possible to the reported experimental measurements. As one application of the model, the performances of MR dampers in mitigating free and forced cable vibrations are numerically investigated and evaluated. Numerical simulations results demonstrate the accuracy and the effectiveness of the proposed dynamic hysteretic regularized Bingham model (HRB) in comparison with a standard Bingham model.

1. Introduction

It is widely acknowledged today that long-span cables are prone to vibrations because of their relatively small mass, high flexibility and very low levels of inherent mechanical damping. Large amplitude vibrations of stay cables in cable-stayed structures are one of the major issues that structural engineers aim to obviate. This complex phenomenon is still not fully understood and is most commonly known to be caused by wind [Matsumotoa et al. 1998], by a combination of wind and rain [Hikami and Shiraishi 1988], or by parametric excitation (movements of the pylons or/and the deck in the case of the cable-stayed bridges) [Costa et al. 1996].

In order to mitigate stay cable vibrations, various control techniques and strategies have been developed and some of them have been used in real cable-stayed structures. These techniques include the use of crossing ties or spacers [Langsoe and Larsen 1987], treatment of the stay cable surface to improve its aerodynamic characteristics [Flamand 1995] and use of external dampers. The latter technique is the most widely used of the three nowadays because of its effectiveness in reducing vibrations to an allowable level.

Among the control devices that have been used successfully, the magnetorheological fluid dampers seem to be one of the most reliable and practical. Such dampers are filled with MR fluids which are controllable suspensions that exhibit reversible, rapid and drastic changes in their rheological characteristics when subjected to the application of external magnetic fields.

In order to successfully achieve desirable control performance, it is essential to have a damping force model which can accurately represent the inherent highly nonlinear and hysteretic dynamic behavior of MR dampers. Several quasi-static and dynamic models have been developed for describing the behavior of MR dampers. Quasi-static approaches are based on the study of non-newtonian yield stress fluids flow by using the Bingham model [Hong et al. 2008b; Stanway et al. 1987] or the Herschel–Bulkley model

[Chooi and Oyadiji 2008; Hong et al. 2008a]. Even though quasi-static models are unable to capture the hysteretic force-velocity relationship characterizing the dynamic response of MR dampers, they remain very useful in the preliminary design process and performance prediction of such dampers.

On the other hand, numerous kinds of dynamic models have been proposed in the recent literature. These models focus on the accurate description of the damper nonlinear hysteretic force-velocity relationship. They may be grouped into two distinct categories: nonparametric and parametric models (see the review articles [Wang and Liao 2011; Zhu et al. 2012], which contains an extensive bibliography). The first category includes, among others, neural network models [Chang and Roschke 1998], fuzzy models [Kim et al. 2006] and polynomial models [Choi et al. 2001]. The second category of models consists of those which are based on mechanical constitutive laws such as the viscoelastic-plastic model [Gamota and Filisko 1991], the nonlinear biviscous model [Wereley et al. 1998], the Bouc–Wen model [Piccirillo and Tusset 2014; Spencer et al. 1997], the Dahl model [Zhou et al. 2006], the hyperbolic tangent function model [Guo et al. 2006], the sigmoid function model [Ma et al. 2007], and many others.

In this paper a novel and concise parametric dynamic model for MR fluid dampers is proposed. The optimized model parameters are obtained by best fitting the model predictions with reported experimental data. Based on Galerkin method, the vibration equations for a stay cable/MR damper system are established. Numerical simulations associated with a stay cable, belonging to the Rades-La Goulette cable-stayed bridge, are carried out. The effectiveness of the proposed HRB model, compared to the standard Bingham model, for the MR damper on the vibration mitigation of the considered cable subjected to the free and forced vibration is investigated.

2. The proposed model for MR dampers

The damping force F_d^B generated by the MR dampers, dependent on the time variable t and using the rigid-viscoplastic Bingham model, is given by a relationship of the form

$$
\begin{aligned}
F_d^B(t) - F_0 &= C_v v_d + F_y \operatorname{sgn}(v_d) && \text{if } F_d^B(t) - F_0 \geq F_y, \\
v_d &= 0 && \text{if } 0 \leq F_d^B(t) - F_0 \leq F_y,
\end{aligned}
\tag{1}
$$

where v_d denotes the piston velocity, F_0 the offset in the damping force due to the presence of the accumulator, F_y the frictional force related to the yield stress of the MR fluid (generally depending on the applied magnetic field) and C_v the damping coefficient. The latter is defined as the slope of the damping force versus the piston velocity and is related to the plastic viscosity of the MR fluid. The above relationship, illustrated through the dashed bilinear diagram of Figure 1, expresses the fact that the viscoplastic flow of the MR fluid occurs only if the damping force exceeds the frictional force.

Furthermore, an approximation to the strict Bingham model by means of a regularization technique may be introduced, similar to that employed in [Papanastasiou 1987]. In such a case, the discontinuous Bingham equation (1) is replaced by a continuous equation characterized by a regularization parameter and the damping force F_d^{RB} may be expressed in the form

$$
F_d^{RB}(t) - F_0 = C_v v_d + F_y \left[1 - \exp\left(-\operatorname{sgn}(v_d) \frac{v_d}{v_0} \right) \right] \operatorname{sgn}(v_d),
\tag{2}
$$
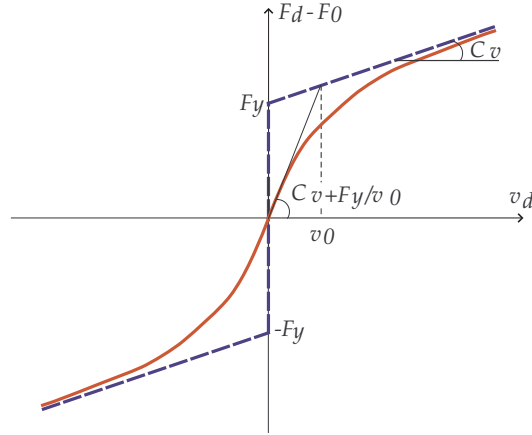
Figure 1. Bingham (dashed line) and regularized Bingham (continuous line) models.

where v_0 denotes the regularization parameter, which has a velocity dimension and controls the exponential growth of the damping force. It is clear from the above formula, that the smaller is this parameter, the better is the approximation of the initial Bingham model, as illustrated in Figure 1.

To take into account the nonlinear hysteretic behavior of the MR damper, we transform (2) to describe a hysteresis loop depicted in Figure 2 and defined for every piston velocity $v_d \in [-v_m, v_m]$ by

$$F_d^{\mathrm{HRB}}(t) - F_0 = \begin{cases} C_v(v_d - v_h) + F_y\left[1 - \exp\left(-\mathrm{sgn}(v_d - v_h)\dfrac{v_d - v_h}{v_0}\right)\right]\mathrm{sgn}(v_d - v_h) & \text{for } \dot{v}_d < 0, \\ C_v(v_d + v_h) + F_y\left[1 - \exp\left(-\mathrm{sgn}(v_d + v_h)\dfrac{v_d + v_h}{v_0}\right)\right]\mathrm{sgn}(v_d + v_h) & \text{for } \dot{v}_d > 0, \end{cases} \tag{3}$$

where v_h is a scale factor having the dimension of a velocity which defines the width of the hysteresis loop and v_m denotes the maximum reached velocity of the damper piston. The obtained hysteresis loop is composed of an upper curve relating to the force variation with respect to the decreasing velocities, and a lower curve corresponding to the force variation with increasing velocities.
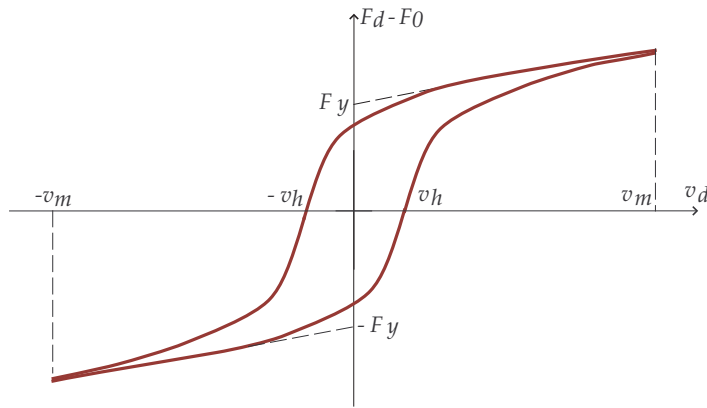
Figure 2. The hysteretic regularized Bingham model (HRB).

3. Identification procedure and model validation through experimental results

In order to accurately model the performance of MR dampers, the proposed dynamic hysteretic regularized Bingham model requires identification of a set of 6 parameters

$$\Xi = \{F_0, F_y, C_v, v_0, v_h, v_m\}. \tag{4}$$

These parameters are obtained by best fitting the model predictions with reported experimental data. This may be accomplished using a multidimensional optimization procedure by means of least-squares.

The experimental data obtained from a series of tests conducted to measure the mechanical response of the MRF132-LD type MR damper system (developed and manufactured by the LORD Corporation) and reported in [Choi et al. 2001] have been first considered. The MR damper was excited sinusoidally with an excitation frequency of 1.4 Hz and an exciting magnitude of ±20 mm at various input currents I (0.0, 1.2 and 2.0 A).

The values of the parameters defined in Equation (4) are adjusted by fitting numerical MR damper force-velocity responses computed from simulations, using the proposed dynamic hysteretic regularized Bingham model, to experimental data measured at different input currents. The obtained values are listed in Table 1. Figure 3 shows good agreement between the measured data and the results of simulations carried out using the identified parameters.

The results of a second series of tests reported in [Guo et al. 2006] have also been analyzed. The MR damper, designed by the Shijiazhuang Railway Institute, was tested under a 1.5 Hz sinusoidal excitation with an amplitude of ±5 mm at various input currents I (0.0, 0.5, 1.0, 1.5 and 2.0 A). It appears from Figure 4 that the MR damper force-velocity responses obtained from numerical simulations, using the identified parameters listed in Table 1, match the recorded experimental measurements very well.

We point out that the values of the identified parameters F_0, C_v, v_0, v_h and v_m remain almost constant,
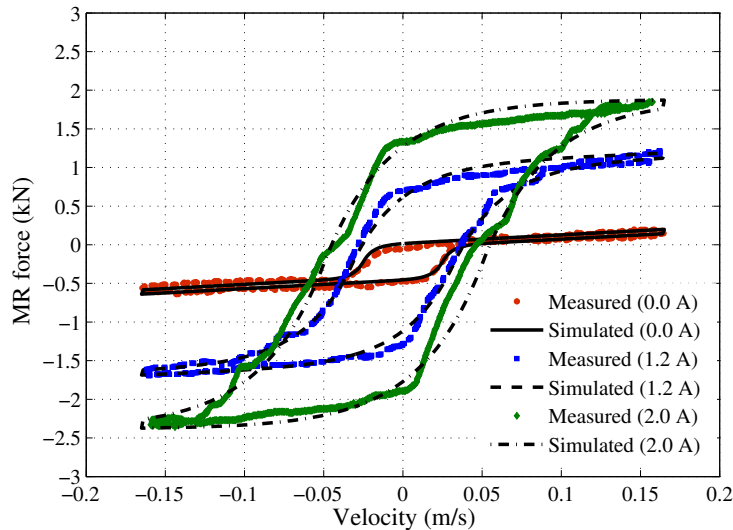
Figure 3. Comparison of model predictions and experimental results reported in [Choi et al. 2001].

Model parameters	Damper from [Choi et al. 2001]			Damper from [Guo et al. 2006]				
I (A)	0	1.2	2.0	0	0.5	1.0	1.5	2.0
F_0 (N)	240	250	250	20	25	20	30	30
F_y (N)	210	1280	2130	250	370	620	850	1070
C_v (N.s/mm)	0.9	0.8	0.7	0.9	1	1	1	1
v_0 (mm/s)	36	30	32	9	8	9	8	9
v_h (mm/s)	29	32	38	8	6	6	6	5
v_m (mm/s)	165	165	165	46.5	46.5	46.5	46.5	46.5

Table 1. Identified model parameters.
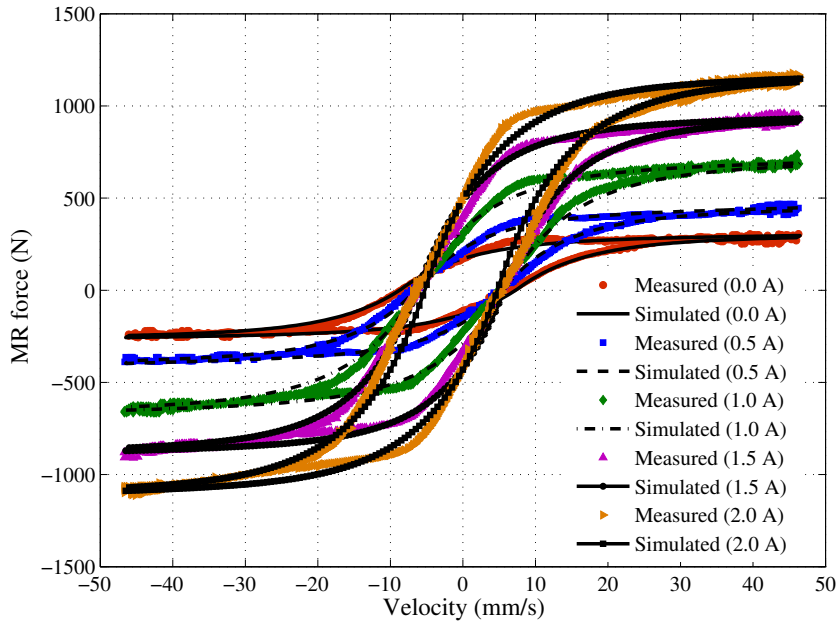
Figure 4. Comparison of model predictions and experimental results reported from [Guo et al. 2006].

while the value of F_y is sensitive to the change of the electrical current I supplied to the MR damper. Furthermore, the frictional force F_y can be linearized with respect to the input current as

$$F_y(I) = \alpha I + \beta, \tag{5}$$

where α and β are constant coefficients. $\alpha = 956.6\,\text{N.A}^{-1}$ and $\beta = 196.3\,\text{N}$ for the Damper from [Choi et al. 2001], while $\alpha = 414\,\text{N.A}^{-1}$ and $\beta = 228\,\text{N}$ for the Damper from [Guo et al. 2006].

4. Control of cable transverse vibration with an attached MR damper

4A. *Problem formulation for a stay cable/MR damper system.* Consider a uniform and flexible inclined stay cable subjected to its self-weight (m_c denotes the cable's mass per unit length) and hanged at its fixed
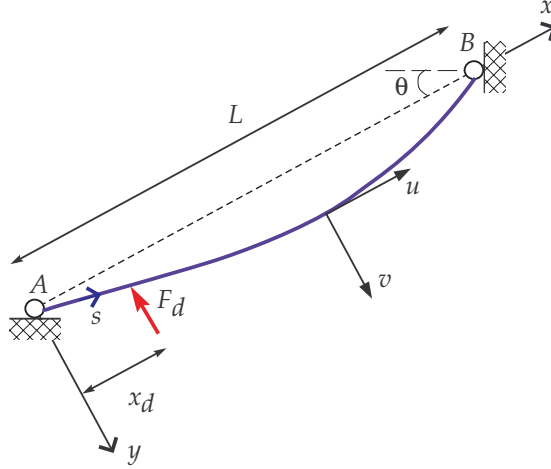
Figure 5. An inclined cable with an attached damper.

end-points A and B that are situated at different levels, as sketched in Figure 5. The stay cable, assumed to be homogeneous and constituted by an isotropic linearly elastic material with Young's modulus E_c, has a chord length L and is inclined at an angle θ to the horizontal. The cable's cross-section is undeformable and its area is denoted by A_c. The strained static profile of the cable is spanned by the curvilinear co-ordinate s with $s = 0$ at A.

Planar oscillations are considered in the Cartesian frame (A, x, y) where the y axis is perpendicular to the x axis taken along the cable's chord. The cable is subjected to a distributed external excitation F_{ext} as well as a transverse concentrated force denoted by F_d due to the presence of a MR damper attached at the location x_d from the support A in the x direction (Figure 5).

The equations governing the dynamic equilibrium of an inclined cable may be written as

$$\begin{cases} \dfrac{\partial}{\partial s}\left[(T+\tau)\left(\dfrac{dx}{ds}+\dfrac{\partial u}{\partial s}\right)\right] + F_{\text{ext},x}(x,t) = m_c\left(\dfrac{\partial^2 u}{\partial t^2}+g\sin\theta\right) \\[2mm] \dfrac{\partial}{\partial s}\left[(T+\tau)\left(\dfrac{dy}{ds}+\dfrac{\partial v}{\partial s}\right)\right] + F_{\text{ext},y}(x,t) + F_d(t)\delta(x-x_d) = m_c\left(\dfrac{\partial^2 v}{\partial t^2}-g\cos\theta\right) \\[2mm] \dfrac{\partial}{\partial s}\left[(T+\tau)\dfrac{dw}{ds}\right] + F_{\text{ext},z}(x,t) = m_c\dfrac{\partial^2 w}{\partial t^2} \end{cases} \tag{6}$$

where $\tau = E_c A_c \epsilon^D$ is the additional dynamic cable tension, ϵ^D is the additional dynamic axial strain, T is the static stay's tension, g is the acceleration of gravity, $\delta(.)$ is the Dirac's delta function, while u, v and w are the cable dynamic displacement components in the x, y and z directions, respectively.

Considering a Green–Lagrange strain measure, the contribution ϵ^D can be described as

$$\epsilon^D = \frac{\partial u}{\partial x} + \frac{dy}{dx}\frac{\partial v}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right]. \tag{7}$$

In the foregoing, the following simplifying assumptions are enforced:

- Due to the high tension levels in the stay cable, small sag effects appear (i.e., $ds \simeq dx$). Consequently, the static equilibrium configuration can be described by a parabolic profile (see [Ben Mekki and Auricchio 2011; Irvine 1981] for more details).

- The transversal frequency is smaller than the longitudinal frequency. Thus, the longitudinal (i.e., along the x-direction) inertial force vanishes and the additional dynamic tension τ is approximated as a constant with respect to x.

- By assuming small deformations, second order terms of ϵ^D can be neglected and the additional dynamic tension is considered as a small perturbation with respect to the static tension T.

Making use of these simplifying assumptions, the transverse deflection $v(x, t)$ of the stay cable satisfies the differential equation of motion for the combined stay cable/MR damper system

$$m_c \frac{\partial^2 v}{\partial t^2} - T \frac{\partial^2 v}{\partial x^2} + \frac{E_c A_c}{L} \left(\frac{m_c g \cos\theta}{T} \right)^2 \int_0^L v\, dx = F_{\text{ext}, y} - F_d \delta(x - x_d). \tag{8}$$

The transverse deflection can be approximated using a finite modal superposition of the form

$$v(x, t) = \sum_{i=1}^{N} \xi_i(t) \varphi_i(x) \tag{9}$$

where $\xi_i(t)$ are non-dimensional modal participation factors and $\varphi_i(x)$ are a set of modal shape functions, assumed to be continuous and to satisfy the geometric boundary conditions at the cable's ends A and B.

By assuming sinusoidal modal shape functions as

$$\varphi_i(x) = \sin\left(\frac{i\pi x}{L} \right) \tag{10}$$

and performing a Galerkin-type approximation of (8), the time-dependent generalized modal coordinates $\xi_i(t)$ can be determined by solving the differential equations

$$m_i \ddot{\xi}_i(t) + \left(k_i + \eta m_i \right) \xi_i(t) = F_i - F_d(t) \varphi_i(x_d) \quad \text{for} \quad i = 1 \ldots N \tag{11}$$

where

$$\eta = \frac{E_c A_c}{L} \left(\frac{m_c g \cos\theta}{T} \right)^2,$$

$$m_i = m_c \int_0^L \varphi_i(x) \varphi_i(x)\, dx = m_c \frac{L}{2},$$

$$k_i = T \int_0^L \frac{d\varphi_i}{dx} \frac{d\varphi_i}{dx}\, dx = \frac{T \pi^2 i^2}{2L},$$

$$F_i = m_c \int_0^L F_{\text{ext}, y}(t) \varphi_i(x)\, dx.$$

It is to be noticed that the presence of the MR damper introduces non-linearity into the stay cable/MR damper system. The Newmark scheme is adopted to obtain the numerical solutions of the system of dynamical equations (11).

L [m]	θ [°]	T [kN]	E_c [GPa]	A_c [cm^2]	m_c [kg/m]
55.4	16.5	4313.5	190	55.5	44

Table 2. Geometrical and mechanical design properties of the stay cable (S16) of the Rades-La Goulette cable-stayed bridge.

4B. *An illustrative numerical application.* The longest cable (S16) in the Rades-La Goulette cable-stayed bridge is considered to investigate the effects of the MR damper parameters on the damping capability and control efficacy.

Opened for traffic in 2009, the Rades-La Goulette cable-stayed bridge was built over the Tunis Lake Canal in Tunisia. The total length of the bridge is 260 m divided into three spans, with a 120 m long central span. Two 40 m tall towers carry a pre-stressed concrete deck. Each pylon is equipped with a stay curtain comprising 8 pairs of stays, arranged following a semi-harp scheme and leading to a symmetric bridge scheme with respect to both the axial vertical plane and the mid-span cross-plane.

The geometrical and material properties of the considered stay cable (S16) are summarized in Table 2. The internal damping of the cable is not considered. A MRF132-LD type MR damper, whose characteristics are listed in Table 1, is attached to the stay cable (S16) at the location $x_d = 0.1L$.

Figure 6 compares the modal contributions of the first mode vibration $\alpha_1(t)$ as well as the damper force evolutions computed by considering the proposed HRB model and the Bingham model. Both modal contributions have been obtained by solving system (11) in which the corresponding model of the MR damper has been introduced for a supplied current of 2.0 A. It is worth noting that the cable vibration is damped more effectively when using the proposed HRB model.

According to Figure 6, it can also be observed that the largest vibration response using the proposed HRB model is around 70% of the one obtained by means of the Bingham model and that the vibration decay speed is much faster. An equivalent viscous damping value of the Bingham model is 2.5%. However, the equivalent viscous damping value of the proposed HRB model is 9.2%. It turns out that the proposed HRB model is very effective to reduce structural response. This could be explained by the fact that it can provide a large damper force under small velocities and displacements.

The stay cable/MR damper system is then subjected to an external harmonic excitation of the form

$$F_{\text{ext}, y}(t) = 1.5 \cos \left(\sqrt{\frac{k_1}{m_1}} t \right). \tag{12}$$

After a brief transitory period, the stay cable/MR damper (of 2.0 A current) system reaches a periodic regime and the modal coordinate exhibits a sinusoidal behavior in time at frequency $f_{\text{ext}} = \sqrt{k_1/m_1}$. Figure 7 shows that the proposed HRB model is again able to damp the harmonic excitation better than the Bingham model.

Vibrations of stay cables in cable stayed-bridges are mainly caused by two phenomena: cable support motion from either the bridge-girder or the pylon mainly coming from heavy vehicles, and aerodynamic actions on the cable itself from wind or a combination of wind and rain. The augmented system damping due to the attached external actuator is often used to measure the efficacy of a vibration control device. For a long stay cable, damping ratio is the most important parameter that dominates aerodynamic stability.
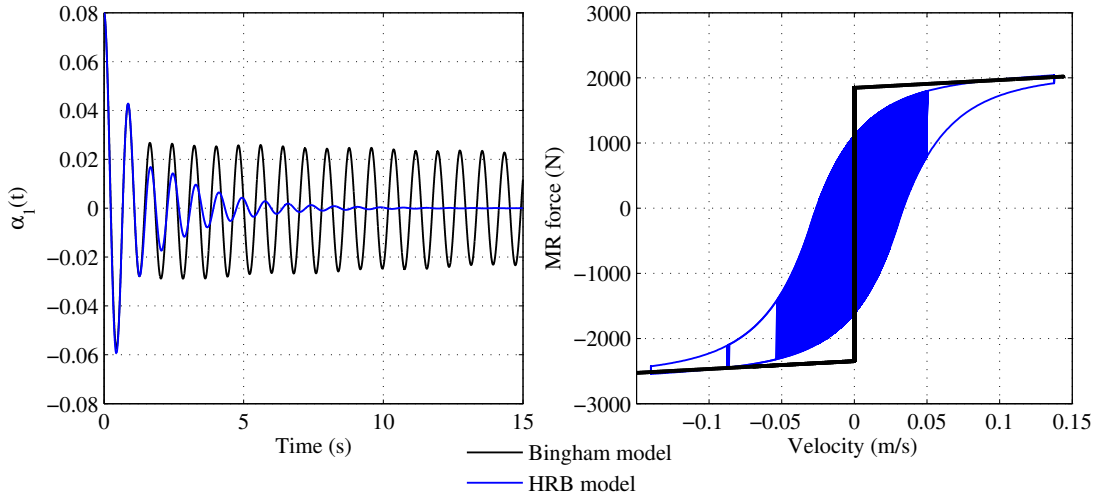
Figure 6. Control of the stay cable free vibration: comparison between the proposed HRB model and the Bingham model.
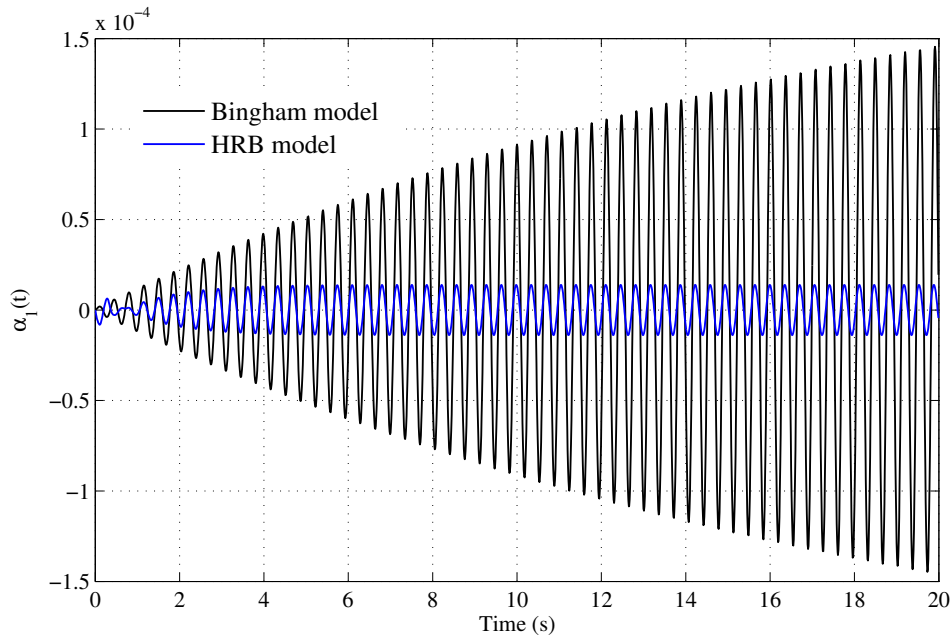
Figure 7. Cable response under a harmonic excitation.

This added damping coefficient by means of an external damping device can be tuned on a wind velocity. The responses of a stay cable will be analyzed with a stepping switch strategy under various constant input currents.

In order to show its influence on the achieved damping of the cable, several values of the supplied electric current are used in the numerical simulations. The force-velocity relationships using the dynamic
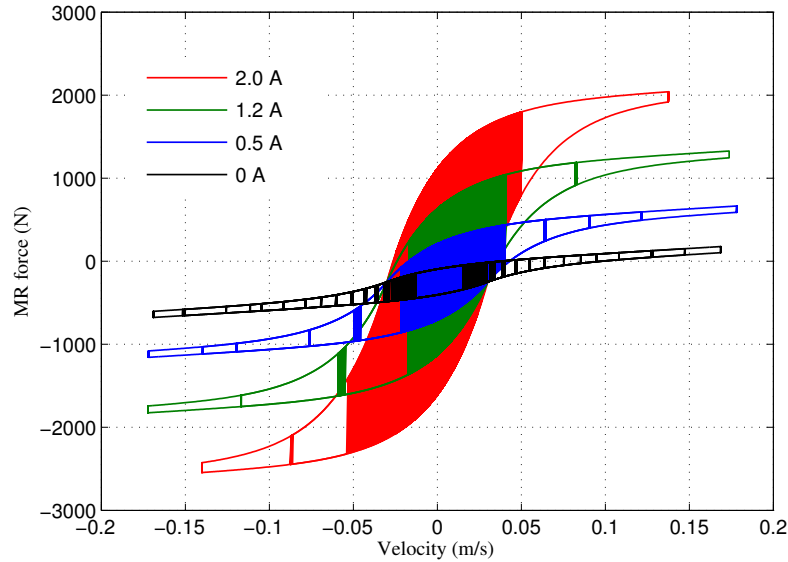
Figure 8. Damper force-velocity relationships for different values of supplied current.

HRB model, for different values of input electric current, are plotted in Figure 8. It can be seen that the MR damper exhibits strong nonlinear hysteretic behavior as well as an excellent property of energy dissipation. Furthermore, the area of the hysteresis loop increases significantly corresponding to the raise of the input current supplied to the MR damper.

In Figure 9 several responses evaluated according to the complete system (11) using the HRB model in correspondence of different values of input electric current are depicted. The effect of the supplied electric current in regulating the equivalent viscous damping clearly appears, as predicted by (5). In particular, the values of the equivalent viscous damping are 1.6%, 4.2%, 7.7% and 9.2% in correspondence of different values of input electric current, respectively, 0 A, 0.5 A, 1.2 A and 2.0 A. The evaluation of the
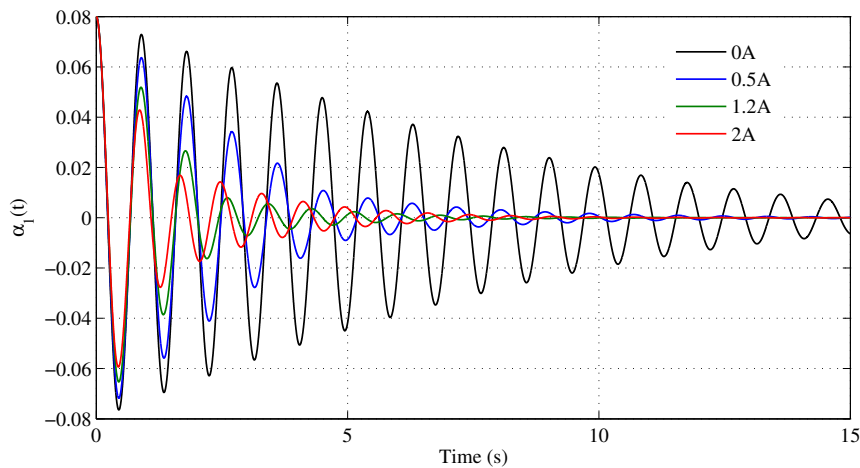
Figure 9. Control of the stay cable free vibration: influence of the input electric current.

equivalent viscous damping has been carried out by using a numerical optimization with the conjugate gradient method. Accordingly, the equivalent viscous damping is chosen by minimizing the square difference between the free response of the system (11) using the proposed HRB model for the MR damper, and the one obtained using a viscous damping.

5. Conclusion

This paper has addressed a new nonlinear dynamic model for a magnetorheological damper to accurately reconstruct the hysteretic relationship between the damping force and the velocity. The optimized model parameters are obtained by reproducing numerically the hysteretic responses of the reported experimental damper force-velocity data. Vibration equations for the stay cable/MR damper system are derived for the analysis of cable vibration mitigation using the proposed HRB model and the standard Bingham model. The longest cable in the Rades-La Goulette cable-stayed bridge is considered. The numerical simulations demonstrate that the proposed HRB model is able to control free and forced vibrations better than the Bingham model.

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