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LAMINAR FLOW OF A POWER-LAW FLUID BETWEEN CORRUGATED PLATES

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This paper deals with the problem of a steady, fully developed, laminar flow of a power-law fluid between corrugated plates. A nonlinear governing equation is transformed into a sequence of linear inhomogeneous equations by the Picard iteration method. At each iteration step, the inhomogeneous equation is solved using the method of particular solutions in which the solution consists of two parts: the general solution and the particular solution. The right-hand side of the inhomogeneous equation is interpolated using the radial basis functions and monomials, and simultaneously unknown coefficients of the particular solution are obtained. The method of fundamental solutions is applied in order to obtain the general solution. Unknown coefficients of the general solution are calculated by fulfilling the boundary conditions. In this paper, dimensionless velocity of the fluid and the product of the friction factor and Reynolds number $f\mathrm{Re}$ are presented for different values of corrugation amplitude and different parameters of the power-law fluid model.

1. Introduction

The problem of a steady, fully developed, laminar flow in ducts of different cross-sectional shapes has received quite extensive attention over the years. A wide range of problems was researched in this area. For instance, a laminar flow between cylinders arranged in a regular array by means of the eigenfunction expansion and the boundary collocation method was investigated by Sparrow and Loeffler [1959]. Zarling [1976] considered flow in different complexly shaped ducts (a circular duct, a square duct, a rectangular duct, an elliptical duct) using the Schwarz–Neumann alternating method along with the boundary collocation method in the least-square sense. Flow in a channel with longitudinal ribs was examined by Wang [1994]. He solved the problem by means of the eigenfunction expansion and the boundary collocation method. The same method was applied by Hu and Yeh [2009] in order to obtain the solution for the problem of a laminar flow in a channel with moving bars. Fluid flow and heat transfer in internally finned tubes were often analyzed in the literature because of their importance from a practical point of view; see for example [Tien et al. 2012].

Also structures with corrugated boundaries have a wide range of practical applications in technology, and one can find many examples in nature. It is worth mentioning the application of this class of flows to corrugated walls in heat exchangers. An experimental comparison of heat and mass transfer between different heat exchangers with corrugated walls was conducted Zimmerer et al. [2002]. Another example of practical problems with corrugated boundaries is peristaltic pumping, which is the transport of fluid induced by a progressive wave of contraction along the distensible duct. Such phenomena exist in many biological systems, e.g., the gastrointestinal tract, the ureter and the small blood vessels. Peristaltic pumps are also used in industry (to transport corrosive or aggressive fluids) and medicine (to transport

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bodily fluids outside the human body). See [Yin and Fung 1971] for a comparison between theory and experiment in peristaltic transport.

Recently more researchers have dealt with the mechanics of non-Newtonian fluids because of their great importance in practical issues, e.g., in plastic processing (molten polymers), the food industry (chocolate, ketchup, yogurt), the personal care industry (shampoo, shaving foam or cream, toothpaste) or medicine (blood, synovial fluid, saliva) [Astarita and Marrucci 1974; Chhabra and Richardson 2008; Irgens 2014]. One of the simplest non-Newtonian fluid models is the power-law fluid. This model is very common in the literature. It was investigated using different numerical methods. Schechter [1961] analyzed flow of a power-law fluid in a rectangular duct using the Ritz method for solving the momentum equation. More accurate results for the same problem were obtained by Wheeler and Wissler [1965], who employed the finite-difference scheme based on the over-relaxation method. The finite-element method was applied to isothermal slow channel flow of power-law fluids by Palit and Fenner [1972]. Their results were compared to the results obtained using the finite-difference method (for rectangular channels) and to an exact solution (Newtonian fluid flow). Liu et al. [1988] presented a comparison of the Galerkin finite-element method and the boundary-fitted coordinate transformation method. The power-law fluid flow in a circular pipe and square and triangular ducts was analyzed. Kostic [1993] investigated flow of a power-law fluid in rectangular ducts by means of the finite-difference method. Fully developed, laminar flow of a power-law fluid in rectangular ducts was also examined by Syrjälä [1995]. He used the finite-element method to solve the momentum equations numerically. Madhav and Malin [1997] analyzed the same problem with application of the single-slab solution procedure. Lima et al. [2000] investigated two-dimensional, laminar flow of a power-law fluid inside the rectangular ducts by means of the generalized integral transform technique.

The problem of fully developed, laminar flow of a Newtonian fluid between corrugated plates was first investigated by Wang [1976]. Ng and Wang [2010] analyzed Darcy–Brinkman flow between corrugated plates. To our best knowledge, there are no other published works on the flow between corrugated plates. The purpose of this paper is to analyze the flow of a power-law fluid between corrugated plates using the method of fundamental solutions and the radial basis functions.

2. Method of fundamental solutions and its applications to solving nonlinear problems

The method of fundamental solutions (MFS) is a meshless method. The method can be applied to solve problems described by partial differential equations for which the fundamental solutions are known. The fundamental solution is a function of a distance between a point inside the considered region and a source point. The source points are located on a *pseudoboundary* that is outside the considered region. The boundary of the considered region and the pseudoboundary do not have any common points. The approximate solution in the MFS is assumed to be a linear combination of fundamental solutions. The governing equation is satisfied exactly by the fundamental solution at any point in the considered region, which also ensures that the approximate solution fulfills the governing equation at any point of the region. The boundary conditions are fulfilled approximately using the boundary collocation technique. The MFS was originally proposed by Kupradze and Aleksidze [1964]. A numerical implementation of the MFS was presented in [Mathon and Johnston 1977]. Some noteworthy review articles can be found in the literature. A review of the MFS applications to elliptic boundary problems was presented in [Fairweather and Karageorghis 1998] while a review of applications of the MFS to scattering and radiation problems was included in [Fairweather et al. 2003]. Applications of the MFS to inverse problems, in turn, were reviewed in [Karageorghis et al. 2011].

The MFS can also be found in the literature under other names: the superposition method [Burgess and Mahajerin 1984], the boundary point method [Johnson 1987], the fundamental solutions method [Bogomolny 1985], the source functions method [de Mey 1978], the fundamental collocation method [Burgess and Mahajerin 1987], the charge simulation method [Amano 1998] or the regular indirect boundary element method [Wearing and Sheikh 1988].

The problem of a steady, fully developed, laminar flow of a power-law fluid is governed by a nonlinear equation. There are only a few examples of applications of the MFS to such problems, and the first attempt to use the MFS for a nonlinear Poisson problem was probably given in [Burgess and Mahajerin 1987]. In that paper, the particular solution was expressed as an integral over the considered region and as a sum of the right-hand-side function times the fundamental solution. Then the Picard iteration method was applied. In [Balakrishnan and Ramachandran 1999; Balakrishnan et al. 2002; Chen 1995; Wang and Qin 2006; Wang et al. 2006], an original nonlinear Poisson-type differential equation in a two-dimensional domain was converted into a sequence of linear Poisson equations. Then the radial basis functions (RBF) and the MFS were applied respectively to construct the expression of the particular and the homogeneous solutions at each iteration step. This procedure was used for more complicated problems of applied mechanics: heat conduction problems in anisotropic and inhomogeneous media [Wang et al. 2005], large deflection of plates [Klekiel and Kołodziej 2006], isothermal gas flow in porous medium [Uściłowska and Kołodziej 2006], thermoelasticity of functionally graded materials [Wang and Qin 2008], nonlinear elliptic problems [Li and Zhu 2009], determination of effective thermal conductivity of unidirectional composites with linearly temperature-dependent conductivity of constituents [Kołodziej and Uściłowska 2012], two-dimensional nonlinear elasticity [Al-Gahtani 2012], elastoplastic torsion of prismatic rods [Kołodziej and Gorzelańczyk 2012], dynamic response of von Karman nonlinear plate model [Uściłowska and Berendt 2013] and some inverse problems [Kołodziej et al. 2013; Mierzwiczak and Kołodziej 2011]. Balakrishnan and Ramachandran [2001] solved nonlinear Poisson problems by means of the method of fundamental solutions and radial basis functions called the osculatory radial basis functions.

Application of the MFS to a Laplace equation with a nonlinear boundary condition was presented by Karageorghis and Fairweather [1989], who considered nonlinear plane potential problems. Steady-state heat conduction with temperature-dependent thermal conductivity and mixed boundary conditions involving radiation was investigated in [Karageorghis and Lesnic 2008a]. In that paper, the classical Kirchhoff transformation was employed. In this way, the governing equation was transformed to the Laplace equation and the only nonlinearity in the new boundary value problem was included in nonlinear boundary conditions. The nonlinear system of algebraic equations was then solved by a standard procedure. The same numerical algorithm was applied for steady-state nonlinear heat conduction in composite materials [Karageorghis and Lesnic 2008b]. A similar approach with the MFS was used for the water wave problem [Kołodziej and Mierzwiczak 2008; Mollazadeh et al. 2011; Wu and Tsay 2009; Wu et al. 2006; 2008].

A linearization scheme for an inhomogeneous term in terms of a dependent variable and the first or second derivative with respect to time, resulting in a Helmholtz-type equation (for which the fundamental

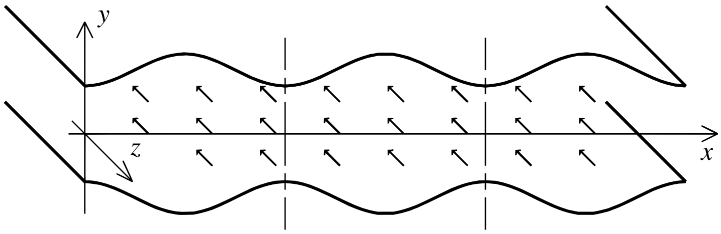

Figure 1. Geometry of the considered problem.

solution is known), was proposed in [Fallahi 2012; Fallahi and Hosami 2011]. Consequently, the particular solutions are no longer needed and the MFS can be directly applied to the linearized equation. In [Tri et al. 2011; 2012], the perturbation technique was combined with the MFS in order to solve a nonlinear Poisson-type equation. The nonlinear problem was transformed into a sequence of inhomogeneous linear problems that can be solved by the MFS and the RBF. The homotopy analysis method combined with the MFS was applied to solve a nonlinear Poisson-type problem in [Tsai 2012]. The Eulerian–Lagrangian method in combination with the MFS was used by Young et al. [2008] in order to solve the nonlinear unsteady Burgers equation. In [Young et al. 2009], unsteady Navier–Stokes equations were transformed into simple advection-diffusion and Poisson equations by the operator-splitting scheme. The obtained advection-diffusion equations and pressure Poisson equation were then solved using the MFS together with the Eulerian–Lagrangian method and the method of particular solution. Feng et al. [2013] solved potential flow for predicting ship motion responses in the frequency domain. The MFS was also successfully applied to nonlinear functionally graded materials [Li et al. 2014; Marin and Lesnic 2007]. Moreover, there are examples of using the MFS in combination with the hybrid finite-element model for solving nonlinear Poisson-type problems [Wang et al. 2012].

In this paper, power-law fluid flow between corrugated plates is investigated by employing the MFS and the RBF. This nonlinear problem is transformed into a sequence of linear inhomogeneous problems using the Picard iteration method. Then the method of particular solution is applied at each iteration step. The RBF and monomials are used in order to interpolate the right-hand side of the inhomogeneous equation and to obtain the particular solution while the MFS is employed to obtain the general solution.

3. Statement of the problem

The geometry of the considered problem is illustrated in Figure 1. The flow is limited by two symmetrical, corrugated plates. The fluid flows between these plates in the direction parallel to the z axis.

The upper and lower walls (plates) can be represented by

$$y = \pm\left[h + \varepsilon \cdot \cos\left(\frac{2\pi x}{2\lambda}\right)\right], \tag{3-1}$$

where h denotes average distance between the plate and the x axis, ε is corrugation amplitude and λ denotes length of the repeating part of the considered region Ω.

After introducing dimensionless quantities

$$X = \frac{x}{\lambda}, \qquad Y = \frac{y}{\lambda}, \qquad H = \frac{h}{\lambda}, \qquad E = \frac{\varepsilon}{\lambda}, \qquad L = \frac{\lambda}{\lambda} = 1, \tag{3-2}$$

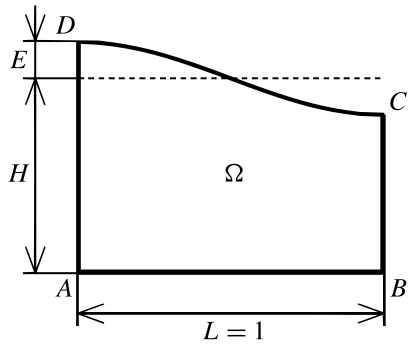

Figure 2. The repeating part of the considered region Ω with characteristic dimensionless quantities.

(3-1) takes the form

$$Y = \pm[H + E \cdot \cos(\pi X)]. \tag{3-3}$$

The repeating part of the considered region Ω with characteristic dimensionless quantities is depicted in Figure 2.

The following equation can be written in the Cartesian coordinate system for steady, fully developed, laminar, axial flow of a power-law fluid:

$$\frac{\partial}{\partial x}\left(\eta(\gamma)\frac{\partial w(x, y)}{\partial x}\right) + \frac{\partial}{\partial y}\left(\eta(\gamma)\frac{\partial w(x, y)}{\partial y}\right) = \frac{dp}{dz}, \tag{3-4}$$

where $w(x, y)$ is axial velocity (velocity of the fluid has only one component), $\frac{dp}{dz}$ is the constant pressure gradient, $\eta(\gamma)$ is the viscosity function (which in the literature is also called apparent viscosity) and

$$\gamma = \sqrt{\left(\frac{\partial w(x, y)}{\partial x}\right)^2 + \left(\frac{\partial w(x, y)}{\partial y}\right)^2}. \tag{3-5}$$

The viscosity function for the power-law fluid takes the form

$$\eta(\gamma) = K \cdot \gamma^{m-1}, \tag{3-6}$$

where K is the consistency factor and m is the power-law index. For $m < 1$, the fluid shows shear-thinning (pseudoplastic) behavior, and for $m > 1$, the fluid shows shear-thickening (dilatant) behavior. If $m = 1$, the fluid shows Newtonian behavior.

For the considered region Ω, the following boundary conditions are formulated:

$$\frac{\partial w(x, y)}{\partial y} = 0 \quad \text{on AB} \qquad \text{(symmetry condition)}, \tag{3-7}$$

$$\frac{\partial w(x, y)}{\partial x} = 0 \quad \text{on BC and DA} \qquad \text{(symmetry condition)}, \tag{3-8}$$

$$w(x, y) = 0 \quad \text{on CD} \qquad \text{(nonslip condition)}. \tag{3-9}$$

Introducing dimensionless quantities (3-2) as well as dimensionless velocity

$$W(X, Y) = \frac{w(x, y)}{-\frac{\lambda^2}{\mu_r}\frac{dp}{dz}} \tag{3-10}$$

(where μ_r is reference viscosity) and dimensionless viscosity function

$$E(\chi) = \frac{\eta(\gamma)}{\mu_r}, \tag{3-11}$$

(3-4) takes the form

$$\frac{\partial}{\partial X}\left(E(\chi)\frac{\partial W(X, Y)}{\partial X}\right) + \frac{\partial}{\partial Y}\left(E(\chi)\frac{\partial W(X, Y)}{\partial Y}\right) = -1, \tag{3-12}$$

where

$$\chi = \sqrt{\left(\frac{\partial W(X, Y)}{\partial X}\right)^2 + \left(\frac{\partial W(X, Y)}{\partial Y}\right)^2}. \tag{3-13}$$

Dimensionless viscosity function $E(\chi)$ for the power-law fluid can be represented by

$$E(\chi) = B_1 \cdot \chi^{m-1}, \tag{3-14}$$

where B_1 is a dimensionless consistency factor defined as

$$B_1 = \frac{K}{\mu_r}\left(\frac{K}{\mu_r}\frac{dp}{dz}\right)^{m-1}. \tag{3-15}$$

Finally after some mathematical operations, the considered problem is defined by the governing equation

$$\nabla^2 W(X, Y) = -\frac{1}{E(\chi)}\left(1 + \frac{\partial E(\chi)}{\partial X}\frac{\partial W(X, Y)}{\partial X} + \frac{\partial E(\chi)}{\partial Y}\frac{\partial W(X, Y)}{\partial Y}\right) \tag{3-16}$$

with the boundary conditions

$$\frac{\partial W(X, Y)}{\partial Y} = 0 \quad \text{on AB}, \tag{3-17}$$

$$\frac{\partial W(X, Y)}{\partial X} = 0 \quad \text{on BC and DA}, \tag{3-18}$$

$$W(X, Y) = 0 \quad \text{on CD}. \tag{3-19}$$

4. The proposed method of solution

In this paper, the Picard iteration method is used in order to solve the nonlinear equation (3-16). Then the nonlinear governing equation (3-16) is transformed into a sequence of inhomogeneous problems in which the value of velocity from the previous iteration step is used on the right-hand side of the equation. At the i-th iteration step, the inhomogeneous problem is described by

$$\nabla^2 W^{[i]}(X, Y) = -\frac{1}{E^{[i-1]}(\chi)}\left(1 + \frac{\partial E^{[i-1]}(\chi)}{\partial X}\frac{\partial W^{[i-1]}(X, Y)}{\partial X} + \frac{\partial E^{[i-1]}(\chi)}{\partial Y}\frac{\partial W^{[i-1]}(X, Y)}{\partial Y}\right). \tag{4-1}$$

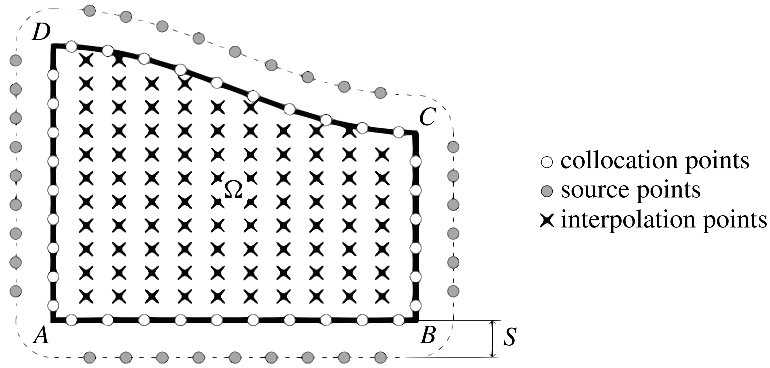

Figure 3. Distribution of the collocation points, the source points and the interpolation points.

The first approximation is obtained for a Newtonian fluid ($B_1 = 1$ and $m = 1$). Then the problem is described by the governing equation

$$\nabla^2 W^{[1]}(X, Y) = -1 \tag{4-2}$$

with boundary conditions (3-17)–(3-19). In order to solve the above equation, the following additional function is introduced:

$$\Phi(X, Y) = W^{[1]}(X, Y) + \tfrac{1}{4}(X^2 + Y^2 - 1). \tag{4-3}$$

Then (4-2) is transformed into the Laplace equation

$$\nabla^2 \Phi(X, Y) = 0. \tag{4-4}$$

The boundary conditions formulated for the additional function $\Phi(X, Y)$ take the forms

$$\frac{\partial \Phi(X, Y)}{\partial Y} = \tfrac{1}{2} Y \quad \text{on AB,} \tag{4-5}$$

$$\frac{\partial \Phi(X, Y)}{\partial X} = \tfrac{1}{2} X \quad \text{on BC and DA,} \tag{4-6}$$

$$\Phi(X, Y) = \tfrac{1}{4}(X^2 + Y^2 - 1) \quad \text{on CD.} \tag{4-7}$$

The problem described by (4-4) with boundary conditions (4-5)–(4-7) in the considered region Ω can easily be solved using the MFS.

In this method, the approximate solution is assumed to be a linear combination of fundamental solutions. The fundamental solution for the Laplace operator is given by

$$f_S(r_j) = \ln r_j, \tag{4-8}$$

where r_j is the distance between the point (X, Y) and the j-th source point (X_j, Y_j):

$$r_j = \sqrt{(X - X_j)^2 + (Y - Y_j)^2}. \tag{4-9}$$

The source points are located outside the considered region Ω on the pseudoboundary at a distance S from the domain boundary as shown in Figure 3.

The approximate solution of the problem described by (4-4) and boundary conditions (4-5)–(4-7) takes the form

$$\Phi(X, Y) = \sum_{j=1}^{N_S} c_j^{[1]} \ln r_j, \tag{4-10}$$

where N_S is the number of source points. The unknown coefficients $c_j^{[1]}$ ($j = 1, \ldots, N_S$) are calculated using the boundary collocation technique (by fulfilling boundary conditions at N_C collocation points) [Kołodziej and Zieliński 2009].

Thus, the solution of the original problem described by the governing equation (4-2) and boundary conditions (3-17)–(3-19) can be written as

$$W^{[1]}(X, Y) = \sum_{j=1}^{N_S} c_j^{[1]} \ln r_j - \tfrac{1}{4}(X^2 + Y^2 - 1). \tag{4-11}$$

At subsequent iteration steps, the problem described by the inhomogeneous equation (4-1) with boundary conditions (3-17)–(3-19) is solved using the method of particular solutions. In this method, the solution of the considered problem consists of two parts:

$$W^{[i]}(X, Y) = W_g^{[i]}(X, Y) + W_p^{[i]}(X, Y), \tag{4-12}$$

where $W_g^{[i]}(X, Y)$ is the general solution and $W_p^{[i]}(X, Y)$ is the particular solution.

The right-hand side of (4-1) at the i-th iteration step can be denoted by

$$b^{[i]}(X, Y) = -\frac{1}{E^{[i-1]}(\chi)}\left(1 + \frac{\partial E^{[i-1]}(\chi)}{\partial X}\frac{\partial W^{[i-1]}(X, Y)}{\partial X} + \frac{\partial E^{[i-1]}(\chi)}{\partial Y}\frac{\partial W^{[i-1]}(X, Y)}{\partial Y}\right). \tag{4-13}$$

The particular solution is obtained by interpolation of the right-hand side of (4-1) using the RBF and monomials:

$$\sum_{m=1}^{N_m} \alpha_m^{[i]} \varphi(r_m) + \sum_{k=1}^{N_k} \beta_k^{[i]} p_k(X, Y) = b^{[i]}(X, Y), \tag{4-14}$$

where N_m is the number of interpolation points, N_k is the number of monomials, $\varphi(r_m)$ is the form of the RBF, $p_k(X, Y)$ is the form of the k-th monomial and

$$r_m = \sqrt{(X - X_m)^2 + (Y - Y_m)^2} \tag{4-15}$$

is the distance between the point (X, Y) and the m-th interpolation point (X_m, Y_m). The unknown coefficients $\alpha_m^{[i]}$ ($m = 1, \ldots, N_m$) and $\beta_k^{[i]}$ ($k = 1, \ldots, N_k$) are calculated by solving the set of equations

$$\begin{cases} \displaystyle\sum_{m=1}^{N_m} \alpha_m^{[i]} \varphi(r_{m\,\mathrm{int}}) + \sum_{k=1}^{N_k} \beta_k^{[i]} p_k(X_{\mathrm{int}}, Y_{\mathrm{int}}) = b(X_{\mathrm{int}}, Y_{\mathrm{int}}), & 1 \le \mathrm{int} \le N_m, \\ \displaystyle\sum_{m=1}^{N_m} \alpha_m^{[i]} p_k(X_m, Y_m) = 0, & 1 \le k \le N_k. \end{cases} \tag{4-16}$$

k	$p_k(X,Y)$	$\hat{p}_k(X,Y)$
1	1	$\frac{1}{4}(X^2+Y^2)$
2	X	$\frac{1}{8}(X(X^2+Y^2))$
3	Y	$\frac{1}{8}(Y(X^2+Y^2))$
4	$X \cdot Y$	$\frac{1}{12}(XY(X^2+Y^2))$
5	X^2	$\frac{1}{14}(X^4+X^2Y^2-\frac{1}{6}Y^4)$
6	Y^2	$\frac{1}{14}(Y^4+X^2Y^2-\frac{1}{6}X^4)$

Table 1. Forms of the monomials and their particular solutions for the Laplace operator.

The particular solution is represented by

$$W_p^{[i]}(X,Y) = \sum_{m=1}^{N_m} \alpha_m^{[i]} \hat{\varphi}(r_m) + \sum_{k=1}^{N_k} \beta_k^{[i]} \hat{p}_k(X,Y), \tag{4-17}$$

where $\hat{\varphi}(r_m)$ is the particular solution that corresponds to the m-th RBF and $\hat{p}_k(X,Y)$ is the particular solution related to the k-th monomial.

In this paper, the multiquadric function (MQ) is used as the RBF:

$$\varphi(r_m) = \sqrt{r_m^2 + c^2}, \tag{4-18}$$

where c is the shape parameter. The particular solution corresponding to the MQ for the Laplace operator takes the form

$$\hat{\varphi}(r_m) = -\tfrac{1}{3}c^3 \ln\Bigl(\sqrt{r_m^2+c^2}+c\Bigr) + \tfrac{1}{9}(4c^2+r_m^2)\sqrt{r_m^2+c^2}. \tag{4-19}$$

The forms of monomials and their particular solutions used in the paper are presented in Table 1.

The general solution is a solution of the Laplace equation

$$\nabla^2 W_g^{[i]}(X,Y) = 0 \tag{4-20}$$

and can be easily found by the MFS in the form

$$W_g^{[i]}(X,Y) = \sum_{j=1}^{N_S} d_j^{[i]} \ln r_j. \tag{4-21}$$

The distance between the point (X, Y) and the j-th source point is defined by (4-9). The unknown coefficients $d_j^{[i]}$ ($j = 1, \dots, N_S$) are calculated using the boundary collocation technique.

Thus, the whole solution of the considered problem at the i-th iteration step takes the form

$$W^{[i]}(X,Y) = \sum_{j=1}^{N_S} d_j^{[i]} \ln r_j + \sum_{m=1}^{N_m} \alpha_m^{[i]} \hat{\varphi}(r_m) + \sum_{k=1}^{N_k} \beta_k^{[i]} \hat{p}_k(X,Y). \tag{4-22}$$

The same numerical procedure was successfully applied for other problems and can be found, e.g., in [Golberg et al. 1998].

Step 1	Input parameters of the considered region: H, E and m.
Step 2	Input parameters of the method: N_C, N_S, S, N_m, N_k, c and tolerance of the convergence error tol.
Step 3	Calculate the first approximation (for a Newtonian fluid): $W^{[1]}(X,Y)=\sum_{j=1}^{N_S} c_j^{[1]} \ln r_j - \frac{1}{4}(X^2+Y^2-1).$
Step 4	Take i = 2.
Step 5	Interpolate the right-hand side of (4-1): calculate unknown coefficients of the particular solution $\alpha_m^{[i]}$ and $\beta_k^{[i]}$.
Step 6	Fulfill boundary conditions: calculate unknown coefficients of the general solution $d_j^{[i]}$.
Step 7	Calculate the whole solution at the i-th iteration step at selected control points: $W^{[i]}(X,Y)=\sum_{j=1}^{N_S} d_j^{[i]} \ln r_j + \sum_{m=1}^{N_m} \alpha_m^{[i]}\hat{\varphi}(r_m) + \sum_{k=1}^{N_k} \beta_k^{[i]} \hat{p}_k(X,Y).$
Step 8	Check the condition for stopping of the iteration process: If $\lVert W^{[i]}(X,Y) - W^{[i-1]}(X,Y)\rVert <$ tol, then STOP. Else, take $i=i+1$ and go to Step 5.
Step 9	Calculate average velocity W_{av} and product of the friction factor and Reynolds number $f\mathrm{Re}$.

Table 2. Numerical algorithm of the proposed method of solution.

The dimensionless average velocity is defined as

$$W_{\mathrm{av}}=\frac{\int_\Omega W(X,Y)\,d\Omega}{\int_\Omega d\Omega}=\frac{\int_0^1\int_0^{H+E\cos(\pi X)} W(X,Y)\,dY\,dX}{\int_0^1\int_0^{H+E\cos(\pi X)} dY\,dX}=\frac{\int_0^1\int_0^{H+E\cos(\pi X)} W(X,Y)\,dY\,dX}{H}. \tag{4-23}$$

The above quantity is calculated numerically using the obtained approximate solution (4-22) and the trapezoidal rule.

For noncircular ducts, the friction factor can be defined as

$$f=\left(-\frac{dp}{dz}\right)\frac{2D_h}{w_{\mathrm{av}}^2\rho}, \tag{4-24}$$

where D_h is hydraulic diameter, w_{av} is dimensional average velocity and ρ is fluid density.

Reynolds number Re for noncircular ducts is given by

$$\mathrm{Re}=\frac{\rho w_{\mathrm{av}} D_h}{\mu_r}. \tag{4-25}$$

Let us introduce dimensionless hydraulic diameter

$$\widetilde{D}_h=\frac{D_h}{\lambda}, \tag{4-26}$$

which is defined as

$$\widetilde{D}_h=\frac{4\tilde{A}}{\widetilde{P}}, \tag{4-27}$$

where

$$\tilde{A}=\int_0^1\int_0^{H+E\cos(\pi X)} dY\,dX = H \tag{4-28}$$

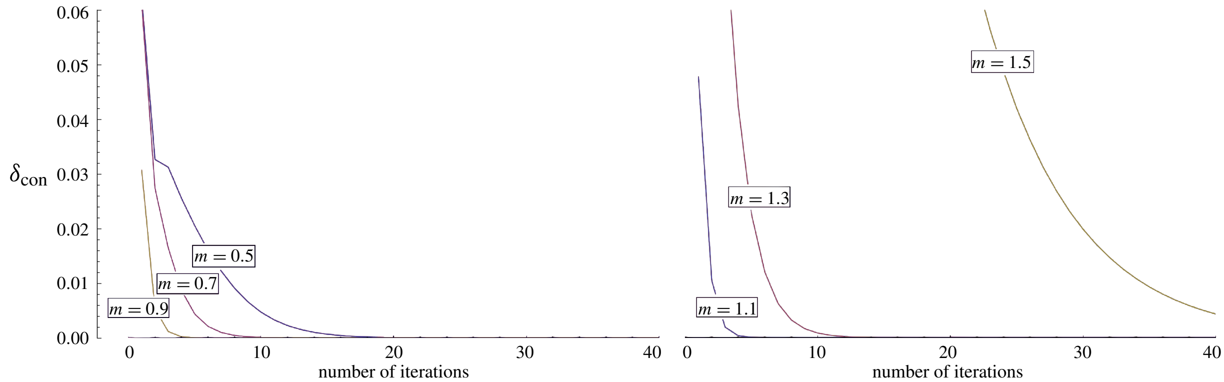

Figure 4. Convergence of the iteration process: (left) for power-law index below 1 and (right) for power-law index above 1.

is the dimensionless area of the considered region Ω and

$$\widetilde{P} = \int_0^1 \sqrt{1 + [E\pi \sin(\pi X)]^2}\, dX. \tag{4-29}$$

is the dimensionless wetted perimeter. The above integral is calculated numerically using the trapezoidal rule.

Thus, the product of the friction factor and Reynolds number can be expressed by

$$f\mathrm{Re} = \frac{32\widetilde{A}^2}{W_{\mathrm{av}}\widetilde{P}^2}. \tag{4-30}$$

In order to summarize this part of the paper, i.e., the proposed method of solution, the numerical algorithm of the method is presented in Table 2.

5. Results

In the first numerical experiment, convergence of the iteration process is investigated. In Figure 4, the maximal error of the convergence of the iteration process at subsequent iteration steps is presented. The error of the convergence of the iteration process is defined as

$$\delta_{\mathrm{con}} = \| W^{[i]}(X, Y) - W^{[i-1]}(X, Y) \|. \tag{5-1}$$

It can be observed that, if power-law index m is less than 1, the convergence process is faster for greater values of m (Figure 4, left). If m is greater than 1, in turn, the convergence process is faster for smaller values of m (Figure 4, right). In general, the convergence process is faster if m is closer to 1. However, the convergence for all the presented values of the power-law index is satisfactory.

The effect of corrugation amplitude E on the error of the convergence of the iteration process δ_{con} is shown in Table 3. The error δ_{con} at subsequent steps for different values of E varies in the same range. This implies that E has very little effect on the convergence of the proposed iteration process.

In Figure 5, equivelocity lines for different values of corrugation amplitude E are presented. It can be observed that, with increasing value of corrugation amplitude, equivelocity lines move in the direction

Iteration step	$E = 0.2$	$E = 0.4$	$E = 0.6$
1	$9.7009 \cdot 10^{-2}$	$7.3963 \cdot 10^{-2}$	$8.4669 \cdot 10^{-2}$
2	$2.8207 \cdot 10^{-2}$	$3.4205 \cdot 10^{-2}$	$3.5555 \cdot 10^{-2}$
3	$1.6416 \cdot 10^{-2}$	$1.6635 \cdot 10^{-2}$	$1.6297 \cdot 10^{-2}$
4	$6.5646 \cdot 10^{-3}$	$8.1569 \cdot 10^{-3}$	$7.6351 \cdot 10^{-3}$
5	$3.3554 \cdot 10^{-3}$	$3.9547 \cdot 10^{-3}$	$3.5712 \cdot 10^{-3}$
6	$1.4004 \cdot 10^{-3}$	$1.8884 \cdot 10^{-3}$	$1.6563 \cdot 10^{-3}$
7	$6.6425 \cdot 10^{-4}$	$8.8845 \cdot 10^{-4}$	$7.6053 \cdot 10^{-4}$
8	$2.7702 \cdot 10^{-4}$	$4.1252 \cdot 10^{-4}$	$3.4597 \cdot 10^{-4}$
9	$1.2521 \cdot 10^{-4}$	$1.8939 \cdot 10^{-4}$	$1.5600 \cdot 10^{-4}$
10	$5.2102 \cdot 10^{-5}$	$8.6167 \cdot 10^{-5}$	$6.9937 \cdot 10^{-5}$

Table 3. The error of the convergence of the iteration process for different values of corrugation amplitude E in subsequent iteration steps.

of the bottom-left corner of the considered region Ω where the value of fluid velocity is maximal. Thus, the value of velocity (also the maximal value) decreases with increasing values of E.

Equivelocity lines for different values of power-law index m are shown in Figure 6. As shown, the density of equivelocity lines increases and the value of fluid velocity increases with increasing m.

Figure 7 presents average velocity W_{av} for various values of power-law index m and corrugation amplitude E. It can be observed that W_{av} decreases with increasing E (it can be observed also in Figure 5). The value of W_{av} for the same E increases with increasing m. For steady, fully developed, laminar flow of a Newtonian fluid between parallel plates, $W_{\mathrm{av}} = \frac{1}{3}$. One can observe that, for pseudoplastic fluids

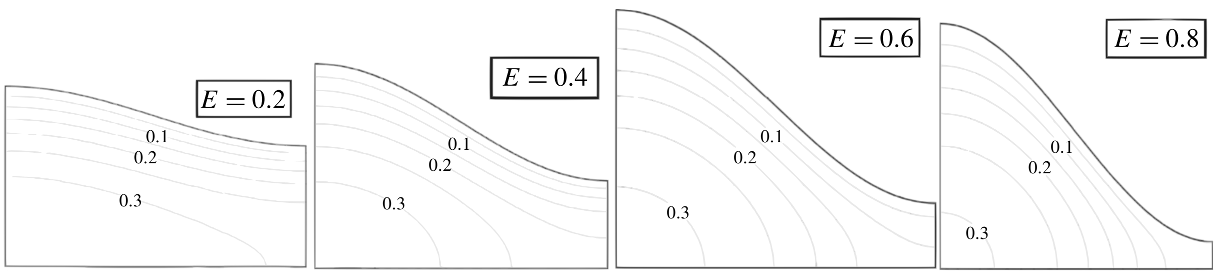

Figure 5. Equivelocity lines for different values of corrugation amplitude.

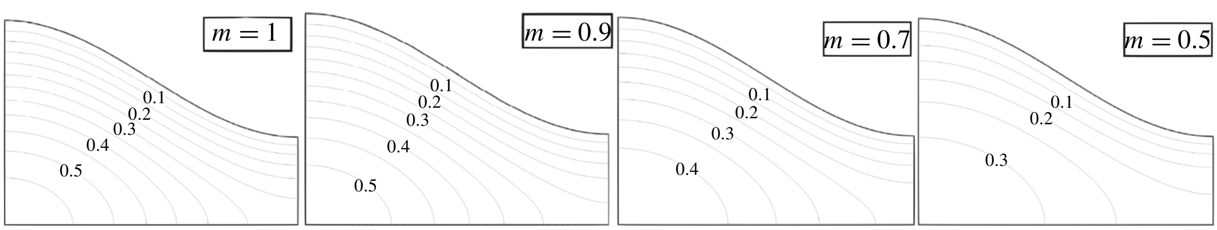

Figure 6. Equivelocity lines for different values of power-law index m.

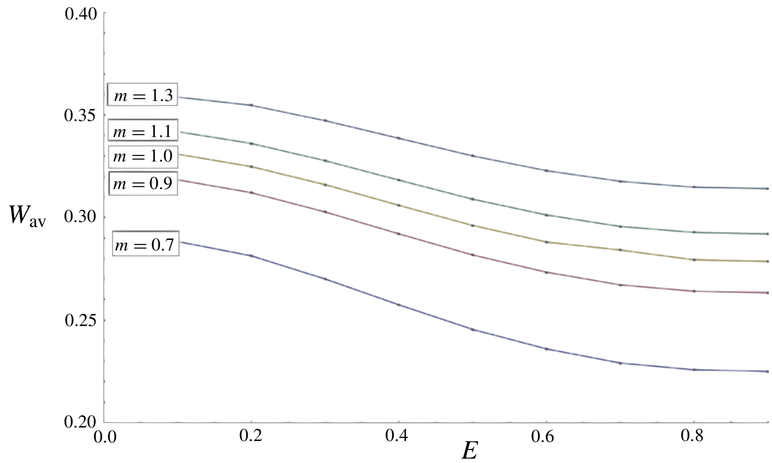

Figure 7. Average velocity W_{av} for different values of corrugation amplitude E and different values of power-law index m.

($m < 1$), W_{av} is always less than for Newtonian flow between parallel plates. In the case of dilatant fluids ($m > 1$), W_{av} is greater than for Newtonian flow between parallel plates but only for smaller values of E.

In Figure 8, the effect of corrugation amplitude E and power-law index m on the product of the friction factor and Reynolds number $f\mathrm{Re}$ is illustrated. For the same E, $f\mathrm{Re}$ increases with decreasing m. For the same m, $f\mathrm{Re}$ decreases with increasing E. For steady, fully developed, laminar flow of a Newtonian fluid between parallel plates, $f\mathrm{Re} = 96$. It can be observed that $f\mathrm{Re}$ is always less than for the case of Newtonian flow between parallel plates for dilatant fluids $m > 1$, and for pseudoplastic fluids ($m < 1$), $f\mathrm{Re}$ is greater than for the case of Newtonian flow between parallel plates only for smaller values of E.

Dimensionless average velocity W_{av} for different dimensionless consistency factors B_1 and different values of dimensionless corrugation amplitude E is shown in Figure 9. It can be seen that W_{av} decreases with increasing B_1.

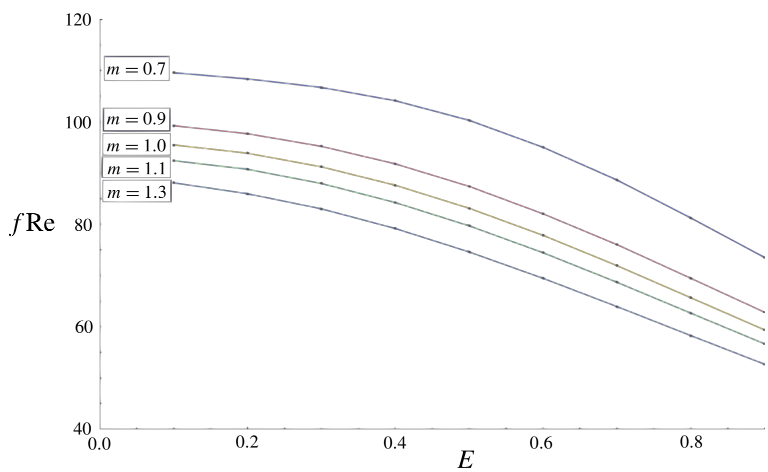

Figure 8. Product of friction factor and Reynolds number $f\mathrm{Re}$ for different values of corrugation amplitude E and different values of power-law index m.

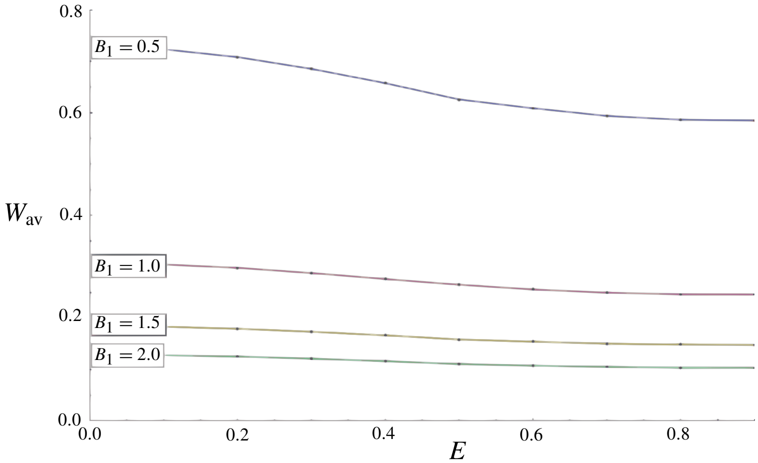

Figure 9. Average velocity W_{av} for different values of corrugation amplitude E and different values of dimensionless consistency factor B_1.

Figure 10 presents the product of the friction factor and Reynolds number $f\mathrm{Re}$ for different dimensionless consistency factors B_1 and different values of dimensionless corrugation amplitude E. It can be observed that $f\mathrm{Re}$ increases with increasing B_1.

6. Conclusions

On the basis of the performed numerical experiments, the following conclusions can be drawn:

(1) The application of the MFS in combination with the RBF for the problem of fully developed, laminar flow of a power-law fluid between corrugated plates gives satisfactory results.

(2) Amplitude of corrugation E has little effect on the iteration process convergence.

(3) Satisfactory convergence is obtained faster if power-law index m is closer to 1.

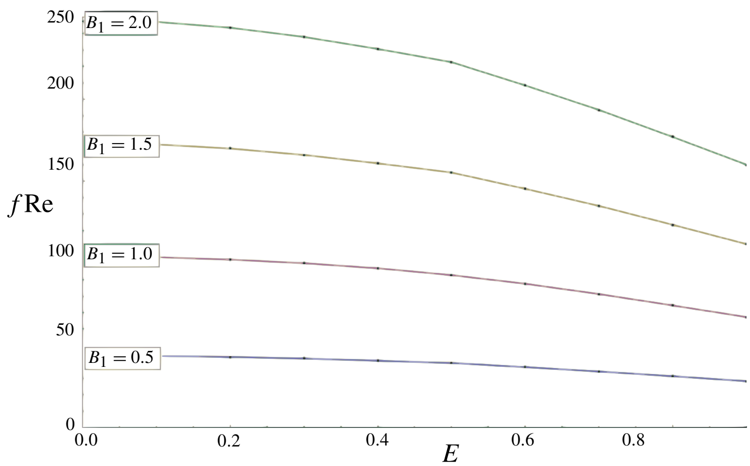

Figure 10. Product of friction factor and Reynolds number $f\mathrm{Re}$ for different values of corrugation amplitude E and different values of dimensionless consistency factor B_1.

(4) Average velocity W_{av} and the product of the friction factor and Reynolds number $f\mathrm{Re}$ decrease with increasing E.

(5) Average velocity W_{av} for a power-law fluid takes lower values than average velocity for a Newtonian fluid ($B_1 = 1$ and $m = 1$). Average velocity W_{av} increases with increasing m and decreases with increasing value of dimensionless consistency factor B_1.

(6) Product $f\mathrm{Re}$ is greater for a power-law fluid than a Newtonian fluid and decreases with increasing E. Product $f\mathrm{Re}$ decreases with increasing m and increases with increasing B_1.

(7) For pseudoplastic fluids ($m < 1$), W_{av} is less than for Newtonian flow between parallel plates and $f\mathrm{Re}$ is greater than in the case of Newtonian flow between parallel plates only for smaller values of E.

(8) For dilatant fluids ($m > 1$), W_{av} is greater than for Newtonian flow between parallel plates but only for smaller E and $f\mathrm{Re}$ is less than for the case of Newtonian flow between parallel plates.

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