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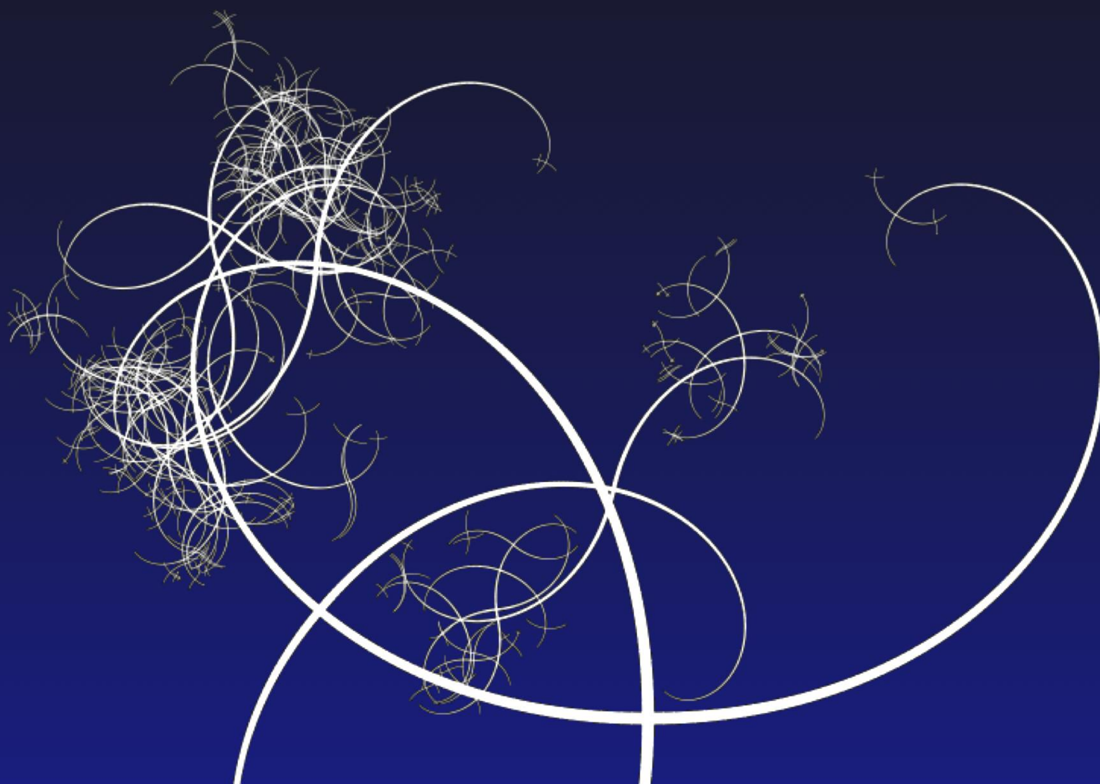

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DATING HYPATIA'S BIRTH: A PROBABILISTIC MODEL

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We propose a probabilistic approach as a dating methodology for events like the birth of a historical figure. The method is then applied to the controversial birth date of the Alexandrian scientist Hypatia, proving to be surprisingly effective.

1. Introduction

Although in historical investigation it may appear meaningless to do experiments on the basis of a preexisting theory — and in particular, it does not make sense to prove theorems of history — it can make perfect sense to use forms of reasoning typical of the exact sciences as an aid to increase the degree of reliability of a particular statement regarding a historical event. This paper deals with the problem of dating the birth of a historical figure when the only information available about it is indirect — for example, a set of testimonies, or scattered statements, about various aspects of his/her life. The strategy is then based on the construction of a probability distribution for the birth date out of each testimony and subsequently combining the distributions so obtained in a sensible way. One might raise several objections to this program. According to Charles Sanders Peirce [1901], a probability "is the known ratio of frequency of a specific event to a generic event", but a birth is neither a specific event nor a generic event but an "individual event". Nevertheless, probabilistic reasoning is used quite often in situations dealing with events that can be classified as "individual". In probabilistic forecasting, one tries to summarize what is known about future events with the assignment of a probability to each of a number of different outcomes that are often events of this kind. For instance, in sport betting, a summary of bettors' opinions about the likely outcome of a race

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is produced in order to set bookmakers' pay-off rates. By the way, this type of observation lies at the basis of the theoretical formulation of the subjective approach in probability theory [de Finetti 1931]. Although we do not endorse de Finetti's approach in all its implications, we embrace its severe criticism of the exclusive use of the frequentist interpretation in the application of probability theory to concrete problems. In particular, we feel entitled to look at an "individual" event of the historical past with a spirit similar to that with which one bets on a future outcome (this is a well known issue in the philosophy of probability; see, e.g., [Dubucs 1993]). Plainly, as the information about an event like the birth of an historical figure is first extracted by material drawn from various literary sources and then treated with mathematical tools, both our approach and goal are interdisciplinary in their essence.

2. A probabilistic method for combining testimonies

Let $X = [x_-, x_+] \subset \mathbb{Z}$ be the time interval that includes all possible birth dates of a given subject (*terminus ad quem*). X can be regarded as a set of mutually exclusive statements about a singular phenomenon (the birth of a given subject in a given year), only one of which is true, and can be made a probability space $(X, \mathcal{F}, P_0)$, with $\mathcal{F}$ the σ-algebra made of the $2^{|X|}$ events of interest and P_0 the uniform probability measure on $\mathcal{F}$ (reference measure): $P_0(A) = |A|/|X|$ (where $|A|$ denotes the number of elements of A). In the context of decision theory, the assignment of this probability space can be regarded as the expression of a basic state of knowledge, in the absence of any information that can be used to discriminate among the possible statements on the given phenomenon, namely a situation in which Laplace's *principle of indifference* can be legitimately applied.

Now suppose we have k testimonies T_i, $i = 1, \dots, k$, which in first approximation we may assume independent of each other, each providing some kind of information about the life of the subject, and which can be translated into a probability distribution p_i on $\mathcal{F}$ so that $p_i(x)$ is the probability that the subject is born in the year $x \in X$ based on the information given by the testimony T_i, assumed true, along with supplementary information such as, e.g., life tables for the historical period considered. The precise criteria for the construction of these probability distributions depends on the kind of information carried by each testimony and will be discussed case by case in the next section. Of course, we shall also take into account the possibility that some testimonies are false, thereby not producing any additional information. We model this possibility by assuming that the corresponding distributions equal the reference measure P_0.

The problem that we want to discuss in this section is the following: how can one combine the distributions p_i in such a way to get a single probability distribution Q

that somehow optimizes the available information? To address this question, let us observe that from the k testimonies taken together, each one with the possibility to be true or false, one gets $N = 2^k$ combinations, corresponding to as many binary words $\sigma_s = \sigma_s(1) \cdots \sigma_s(k) \in \{0, 1\}^k$, which can be ordered lexicographically according to $s = \sum_{i=1}^{k} \sigma_s(i) \cdot 2^{i-1} \in \{0, 1, \dots, N-1\}$, and given by

$$P_s(\,\cdot\,) = \frac{\prod_{i=1}^{k} p_i^{\sigma_s(i)}(\,\cdot\,)}{\sum_{x \in X} \prod_{i=1}^{k} p_i^{\sigma_s(i)}(x)}, \qquad p_i^{\sigma_s(i)} = \begin{cases} p_i, & \sigma_s(i) = 1, \\ P_0, & \sigma_s(i) = 0. \end{cases} \tag{2-1}$$

In particular, one readily verifies that P_0 is but the reference uniform measure.

Now, if Ω denotes the class of probability distributions $Q : X \to [0, 1]$, we look for a *pooling operator* $T : \Omega^N \to \Omega$ that combines the distributions P_s by weighing them in a sensible way. The simplest candidate has the general form of a linear combination

$$T(P_0, \dots, P_{N-1}) = \sum_{s=0}^{N-1} w_s P_s, \qquad w_s \ge 0, \; \sum_{s=0}^{N-1} w_s = 1, \tag{2-2}$$

which, as we shall see, can also be obtained by minimizing some information-theoretic function.

Remark 2.1. The issue we are discussing here has been the object of a vast amount of literature regarding the normative aspects of the formation of aggregate opinions in several contexts (see, e.g., [Genest and Zidek 1986] and references therein). In particular, it has been shown by McConway [1981] that, if one requires the existence of a function $F : [0, 1]^N \to [0, 1]$ such that

$$T(P_0, \dots, P_{N-1})(A) = F(P_0(A), \dots, P_{N-1}(A)) \quad \text{for all } A \in \mathscr{F} \tag{2-3}$$

with $P_s(A) = \sum_{x \in A} P_s(x)$, then whenever $|X| \ge 3$, F must necessarily have the form of a linear combination as in (2-2). The above condition implies in particular that the value of the combined distribution on coordinates depends only on the corresponding values on the coordinates of the distributions P_s, namely that the pooling operator commutes with marginalization.

However, some drawbacks of the linear pooling operator have also been highlighted. For example, it does not "preserve independence" in general: if $|X| \ge 5$, it is not true that $P_s(A \cap B) = P_s(A) P_s(B)$, $s = 0, \dots, N-1$, entails

$$T(P_0, \dots, P_{N-1})(A \cap B) = T(P_0, \dots, P_{N-1})(A) T(P_0, \dots, P_{N-1})(B)$$

unless $w_s = 1$ for some s and 0 for all others [Lehrer and Wagner 1983; Genest and Wagner 1987].

(Another form of the pooling operator considered in the literature to overcome the difficulties associated with the use of (2-2) is the log-linear combination

$$T(P_0, \dots, P_{N-1}) = C \prod_{s=0}^{N-1} P_s^{w_s}, \quad w_s \geq 0, \sum_{s=0}^{N-1} w_s = 1, \tag{2-4}$$

where C is a normalizing constant [Genest and Zidek 1986; Abbas 2009].)

On the other hand, in our context, the independence preservation property does not seem so desirable: the final distribution $T(P_0, \dots, P_{N-1})$ relies on a set of information much wider than that associated with the single distributions P_s, and one can easily imagine how the alleged independence between two events can disappear as the information about them increases.

2.1. ***Optimization.*** The linear combination (2-2) can also be viewed as the marginal distribution[1] of $x \in X$ under the hypothesis that one of the distributions $P_0, \dots, P_{N-1}$ is the "true" one (without knowing which) [Genest and McConway 1990]. In this perspective, (2-2) can be obtained by minimizing the expected loss of information due to the need to compromise, namely a function of the form

$$I(w, Q) = \sum_{s=0}^{N-1} w_s \, D(P_s \parallel Q) \geq 0, \tag{2-5}$$

where

$$D(P \parallel Q) = \sum_{x \in X} P(x) \log\left(\frac{P(x)}{Q(x)}\right) \tag{2-6}$$

is the *Kullback–Leibler divergence* [1951], representing the information loss using the measure Q instead of P. Note that the concavity of the logarithm and the Jensen inequality yield

$$-\sum_x P(x) \log \frac{P(x)}{Q(x)} \leq \log \sum_x P(x) \frac{Q(x)}{P(x)} = 0$$

and therefore

$$D(P \parallel Q) \geq 0 \quad \text{and} \quad D(P \parallel Q) = 0 \iff Q \equiv P. \tag{2-7}$$

We have the following result.

Lemma 2.2. *Given a probability vector* $w = (w_0, w_1, \dots, w_{N-1})$,

$$\arg\min_{Q \in \Omega} I(w, Q) = Q_w \equiv \sum_s w_s P_s. \tag{2-8}$$

Moreover,

$$I(w, Q_w) = H\left(\sum_s w_s P_s\right) - \sum_s w_s H(P_s), \tag{2-9}$$

where $H(Q) = -\sum_{x \in X} Q(x) \log Q(x)$ *is the entropy of* $Q \in \Omega$.

[1] In the sense that a marginal probability can be obtained by averaging conditional probabilities.

Proof. Equation (2-8) can be obtained using the method of Lagrange multipliers. An alternative argument makes use of the easily derived "parallelogram rule":

$$\sum_s w_s D(P_s \parallel Q) = \sum_s w_s D(P_s \parallel Q_w) + D(Q_w \parallel Q) \quad \text{for all } Q \in \Omega. \tag{2-10}$$

From (2-7), we thus get $I(w, Q_w) \le I(w, Q)$ for all $Q \in \Omega$. The uniqueness of the minimum follows from the convexity of $D(P \parallel Q)$ with respect to Q. Finally, checking (2-9) is a simple exercise. □

Remark 2.3. It is worth mentioning that, if we took $\sum_s w_s\, D(Q \parallel P_s)$ (instead of $\sum_s w_s\, D(P_s \parallel Q)$) as the function to be minimized (still varying Q with w fixed), then instead of the "arithmetic mean" (2-2), the "optimal" distribution would have been the "geometric mean" (2-4) (see also [Abbas 2009]).

2.2. *Allocating the weights.* We have seen that for each probability vector w in the N-dimensional simplex $\{w_s \ge 0 : \sum_{s=0}^{N-1} w_s = 1\}$ the distribution $Q_w = \sum_s w_s P_s$ is the "optimal" one. We are now left with the problem of determining a sensible choice for w. This cannot be achieved by using the same criterion, in that by (2-7) $\inf_w I(w, Q_w) = 0$ and the minimum is realized whenever $w_s = 1$ for some s and 0 for all others.

A suitable expression for the weights w_s can be obtained by observing that the term $\sum_{x\in X} \prod_{i=1}^k p_i^{\sigma_s(i)}(x)$ is proportional to the probability of the event (in the product space $X^{[1,k]}$) that the birth dates of k different subjects, with the i-th birth date distributed according to $p_i^{\sigma_s(i)}$, coincide, and thus, it furnishes a measure of the degree of compatibility of the distributions p_i involved in the product associated with the word σ_s.

It thus appears natural to consider the weights

$$w_s = \frac{\sum_{x\in X} \prod_{i=1}^k p_i^{\sigma_s(i)}(x)}{\sum_{s=0}^{N-1} \sum_{x\in X} \prod_{i=1}^k p_i^{\sigma_s(i)}(x)}, \tag{2-11}$$

which, once inserted in (2-2), yield the expression

$$T(P_0, \dots, P_{N-1})(\,\cdot\,) = \frac{\sum_{s=0}^{N-1} \prod_{i=1}^k p_i^{\sigma_s(i)}(\,\cdot\,)}{\sum_{x\in X} \sum_{s=0}^{N-1} \prod_{i=1}^k p_i^{\sigma_s(i)}(x)}. \tag{2-12}$$

Remark 2.4. There are at least $k+1$ strictly positive coefficients w_s. They correspond to the words $\sigma_s^{(i)}$ with $\sigma_s^{(i)}(i) = 1$ for some $i \in \{1, \dots, k\}$ and $\sigma_s^{(i)}(j) = 0$ for $j \ne i$, plus one to the word 0^k, that is, to the distributions $P_{s^{(i)}} \equiv p_i$, $i \in \{0, 1, \dots, k\}$, where $p_0 \equiv P_0$.

2.3. *Weights as likelihoods.* A somewhat complementary argument to justify the choice (2-11) for the coefficients w_s can be formulated in the language of probabilistic inference, showing that they can be interpreted as (normalized) *average* *likelihoods* associated with the various combinations corresponding to the words σ_s. More precisely, with each pair of "hypotheses" of the form

$$D_i^e = \begin{cases} \{T_i \text{ true}\}, & e = 1, \\ \{T_i \text{ false}\}, & e = 0, \end{cases}$$

we associate its *likelihood, given the event that the birth date is* $x \in X$, with the expression[2]

$$V(D_i^e \mid x) = \frac{P(x \mid D_i^e)}{P(x)} = \begin{cases} p_i(x)/p_0(x), & e = 1, \\ 1, & e = 0, \end{cases} \tag{2-13}$$

with $i \in \{1, \dots, k\}$ and $p_0 \equiv P_0$. In this way, the posterior probability $P(D_i^e \mid x)$ (the probability of D_i^e in light of the event that the subject was born in the year $x \in X$) is given by the product of $V(D_i^e \mid x)$ with the prior probability $P(D_i^e)$, according to Bayes's formula.

If we now consider two pairs of "hypotheses" $D_i^{e_i}$ and $D_j^{e_j}$, which we assume conditionally independent (without being necessarily independent), that is,

$$P(D_i^{e_i}, D_j^{e_j} \mid x) = P(D_i^{e_i} \mid x) P(D_j^{e_j} \mid x), \quad e_i, e_j \in \{0, 1\},$$

then we find

$$P(D_i^{e_i}, D_j^{e_j} \mid x) = \frac{P(x \mid D_i^{e_i}, D_j^{e_j})}{P(x)} = \frac{P(D_i^{e_i}, D_j^{e_j} \mid x)}{P(D_i^{e_i}, D_j^{e_j})} = \frac{P(D_i^{e_i} \mid x) P(D_j^{e_j} \mid x)}{P(D_i^{e_i}, D_j^{e_j})}$$
$$= \frac{P(D_i^{e_i}) P(D_j^{e_j})}{P(D_i^{e_i}, D_j^{e_j})} \cdot V(D_i^{e_i} \mid x) V(D_j^{e_j} \mid x).$$

More generally, given k testimonies T_i, to each of which there corresponds the pair of events D_i^e, and given a word $\sigma_s \in \{0, 1\}^k$, if we assume the conditional independence of the events $(D_1^{\sigma_s(1)}, \dots, D_k^{\sigma_s(k)})$, we get

$$V(D_1^{\sigma_s(1)}, \dots, D_k^{\sigma_s(k)} \mid x) = \rho_s \prod_{i=1}^{k} V(D_i^{\sigma_s(i)} \mid x) \tag{2-14}$$

where

$$\rho_s = \frac{\prod_{i=1}^{k} P(D_i^{\sigma_s(i)})}{P(D_1^{\sigma_s(1)}, \dots, D_k^{\sigma_s(k)})}. \tag{2-15}$$

If, in addition, there is grounds to assume unconditional independence, i.e., $\rho_s = 1$, then (2-14) simply reduces to the product rule. Under this assumption, we can

[2]Here the symbol P denotes either the reference measure P_0 or any probability measure on X compatible with it.

evaluate the *average likelihood* of the set of information $(D_1^{\sigma_s(1)}, \ldots, D_k^{\sigma_s(k)})$ with the expression

$$V_s = \frac{1}{|X|} \sum_{x \in X} V(D_1^{\sigma_s(1)}, \ldots, D_k^{\sigma_s(k)} \mid x) = |X|^{k-1} \sum_{x \in X} \prod_{i=1}^{k} p_i^{\sigma_s(i)}(x). \qquad (2\text{-}16)$$

Comparing with (2-11), we see that

$$w_s = \frac{V_s}{\sum_{s=0}^{N-1} V_s}. \qquad (2\text{-}17)$$

In other words, within the hypotheses made so far, the allocation of the coefficients (2-11) corresponds to assigning to each distribution P_s a weight proportional to the average likelihood of the set of information from which it is constructed.

3. Application to Hypatia

This method is now applied to a particular dating process, the one of Hypatia's birth. This choice stems from the desire to study a case both easy to handle and potentially useful in its results. The problem of dating Hypatia's birth is indeed open, in that there are different possible resolutions of the constraints imposed by the available data. According to the reconstruction given by Deakin [2007, p. 51], "Hypatia's birth has been placed as early as 350 and as late as 375. Most authors settle for 'around 370'". There are not many testimonies (historical records) concerning the birth of the Alexandrian scientist (far more are about her infamous death), but they have the desirable feature of being independent of one another, as will be apparent in the sequel, so that the scheme discussed in the previous section can be directly applied. The hope is to obtain something that is qualitatively significant when compared to the preexisting proposals, based on a qualitative discussion of the sources, and quantitatively unambiguous. A probability distribution for the year of Hypatia's birth is extracted from each testimony, the specific reasoning being briefly discussed in each case. Eventually all distributions are combined according to the criteria outlined in the previous section.

3.1. ***Hypatia was at her peak between 395 and 408.*** Under the entry Ὑπατία, the Suda (a Byzantine lexicon) informs us that she flourished under the emperor Arcadius (ἤκμασεν ἐπί τῆς βασιλείας Ἀρκαδίου).[3]

It is well established that Arcadius, the first ruler of the Byzantine Empire, reigned from 395 to 408. Guessing an age or age interval based on the Greek ἤκμασεν, however, is less straightforward. The word is related to ἀκμή, 'peak',

[3] Υ166. See http://www.stoa.org/sol-bin/search.pl?field=adlerhw_gr&searchstr=upsilon,166.

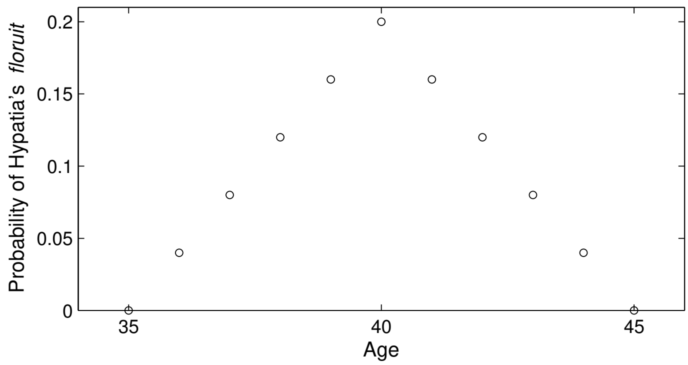

Figure 1. The probability distribution $f(x)$ assumed associated with one's peak years.

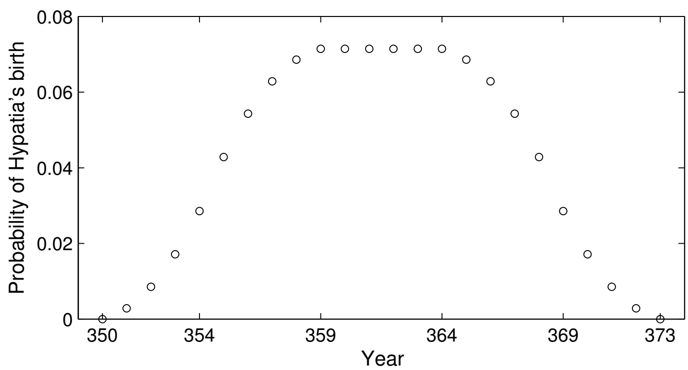

Figure 2. The probability distribution $\Upsilon_f(\xi)$ for Hypatia's birth based on her peak years.

and we follow the rule of thumb, going back to Antiquity, that it refers to the period of one's life around 40 years of age. Specifically, we adopt the probability distribution $f(x)$ in Figure 1 to model how old Hypatia would have been at her "peak" in Arcadius' reign.

Figure 2 shows $\Upsilon_f(\xi)$, the probability distribution for the year of Hypatia's birth deduced from this historical datum; it is obtained by averaging fourteen copies of the triangular $f(x)$, each centered around one of the years from 355 through 368 — the beginning and end points of Arcadius's empire, shifted back by the 40 years corresponding to the peak of $f(x)$.

3.2. ***Hypatia was intellectually active in 415.*** The sources ascribe Hypatia's martyrdom at the hands of a mob of Christian fanatics to the envy that many felt on account of her extraordinary intelligence, freedom of thought, and political influence, being a woman. Her entry in the Suda, already mentioned, states:

> Τοῦτο δὲ πέπονθε διὰ φθόνον καὶ τὴν ὑπερβάλλουσαν σοφίαν, καὶ μάλιστα εἰς τὰ περὶ ἀστρονομίαν.[4]

Socrates Scholasticus, in his Εκκλησιαστική Ιστορία, reports:

> On account of the self-possession and ease of manner, which she had acquired in consequence of the cultivation of her mind, she not infrequently appeared in public in presence of the magistrates. Neither did she feel abashed in coming to an assembly of men. For all men on account of her extraordinary dignity and virtue admired her the more. Yet even she fell a victim to the political jealousy which at that time prevailed. For as she had frequent interviews with Orestes, it was calumniously reported among the Christian populace, that it was she who prevented Orestes from being reconciled to the bishop.[5]

Because of these and similar testimonies, it seems reasonable to mark 415 as a year of intellectual activity in Hypatia's life.

To get from this information a probability distribution for the year of birth, it is necessary to have the probability distribution of being intellectually active at a given age. This can be calculated given the probability of being alive at any given age and of being active at any given age (if alive), by simple multiplication.

To derive the first of these probability distributions we have used data from a 1974 mortality table for Italian males,[6] clipping off ages under 18 since the subject was known to be intellectually active. The resulting probability distribution, $a(x)$, is shown in Figure 3.

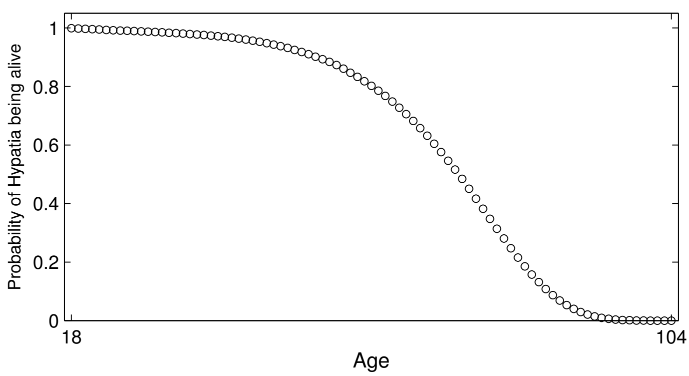

Figure 3. The probability distribution $a(x)$ for an adult to reach a given age. The life expectancy comes to 71.8 years.

[4]She suffered this [violent death] because of the envy for her extraordinary wisdom, especially in the field of astronomy.

[5]Book VII, Chapter 15; translation from [Socrates Scholasticus, p. 160].

[6]All data are taken from http://www.mortality.org.

Name	Dates of birth and death	Lifespan
Accius, Lucius	170–circa 86 BC	∼84
Adrianus (Hadrianus) of Tyre	circa AD 113–193	∼80
Aelian (Claudius Aelianus)	AD 165/170–230/235	∼65
Aeschines	circa 397–circa 322 BC	∼65
Aeschylus	524/525–456/455 BC	∼70
Agathocles (2) (of Cyzicus)	circa 275/265–circa 200/190 BC	∼75
Alexander of Tralles	AD 525–605	80
Alexis	circa 375–circa 275 BC	∼100
Ammianus Marcellinus	circa AD 330–395	∼65
Anaxagoras	probably 500–428 BC	∼72
Anaximenes (2) of Lampsacus	circa 380–320 BC	∼60
Andocides	circa 440–circa 390 BC	∼50
Androtion	circa 410–340 BC	∼70
Antiphon	circa 480–411 BC	∼69
Apollonius of Citium	circa 90–15 BC?	∼75
Arcesilaus	316/315–242/241 BC	∼74
Aristarchus of Samothrace	circa 216–144 BC	∼72
Aristophanes of Byzantium	circa 257–180 BC	∼77
Aristotle	384–322 BC	62
Arius	circa AD 260–336	∼76
Arrian (Lucius Flavius Arrianus)	circa AD 86–160	∼74
Aspasius	circa AD 100–150	∼50
Athanasius	circa AD 295–373	∼78
Atticus	circa AD 150–200	∼50
Augustine, Saint	AD 354–430	76
Bacchius of Tanagra	probably 275–200 BC	∼75
Bacchylides	circa 520–450 BC	∼70
Basil of Caesarea	circa AD 330–379	∼49
Bion of Borysthenes	circa 335–circa 245 BC	∼90
Carneades	214/213–129/128 BC	∼85
Cassius (1)	31 BC–AD 37	68
Cassius Longinus	circa AD 213–273	∼60
Cato (Censorius)	234–149 BC	85
Chrysippus of Soli	circa 280–207 BC	∼73
Chrysostom, John	circa AD 354–407	∼53
Cinesias	circa 450–390 BC	∼60
Claudius Atticus Herodes (2) Tiberius	circa AD 101–177	∼76
Cleanthes of Assos	331–232 BC	99
Clitomachus	187/186–110/119 BC	∼77
Colotes (RE 1) of Lampsacus	circa 325–260 BC	∼65
Cornelius (RE 157) Fronto, Marcus	circa AD 95–circa 166	∼71
Crantor of Soli in Cilicia	circa 335–275 BC	∼60
Crates (2)	circa 368/365–288/285 BC	∼80
Demades	circa 380–319 BC	∼61
Demochares	circa 360–275 BC	∼85
Democritus (of Abdera)	circa 460–370 BC	∼90
Demosthenes (2)	384–322 BC	62
Dinarchus	circa 360–circa 290 BC	∼70
Dio Cocceianus	circa 40/50–110/120 BC	∼70
Diodorus (3) of Agyrium, Sicily	circa 90–30 BC	∼60
Diogenes (3) (of Babylon)	circa 240–152 BC	∼88

Diogenes (2) the Cynic	circa 412/403–circa 324/321 BC	~85
Duris	circa 340–circa 260 BC	~80
Empedocles	circa 492–432 BC	~60
Ennius, Quintus	239–169 BC	70
Ennodius, Magnus Felix	AD 473/474–521	~48
Ephorus of Cyme	circa 405–330 BC	~75
Epicurus	341–270 BC	71
Epiphanius	circa AD 315–403	~88
Erasistratus	circa 315–240 BC	~75
Eratosthenes of Cyrene	circa 285–194 BC	~91
Eubulus (1)	circa 405–circa 335 BC	~70
Euclides (1) of Megara	circa 450–380 BC	~70
Euripides	probably 480s–407/406 BC	~78
Eusebius of Caesarea	circa AD 260–339	~79
Evagrius Scholasticus	circa AD 535–circa 600	~65
Favorinus	circa AD 85–155	~70
Fenestella	52 BC–AD 19 or 35 BC–AD 36	71
Galen of Pergamum	AD 129–216	87
Gorgias (1) of Leontini	circa 485–circa 380 BC	~105
Gregory (2) of Nazianzus	AD 329–389	60
Gregory (3) of Nyssa	circa AD 330–395	~65
Gregory (4) Thaumaturgus	circa AD 213–circa 275	~62
Hecataeus (2) of Abdera	circa 360–290 BC	~70
Hegesippus (1)	circa 390–circa 325 BC	~65
Hellanicus (1) of Lesbos	circa 480–395 BC	~85
Hellanicus (2)	circa 230/220–160/150 BC	~70
Herophilus of Chalcedon	circa 330–260 BC	~70
Hieronymus (2) of Rhodes	circa 290–230 BC	~60
Himerius	circa AD 310–circa 390	~80
Horace (Quintus Horatius Flaccus)	65–8 BC	57
Idomeneus (2)	circa 325–circa 270 BC	~55
Irenaeus	circa AD 130–circa 202	~72
Isaeus (1)	circa 420–340s BC	~75
Isocrates	436–338 BC	98
Ister	circa 250–200 BC	~50
Jerome (Eusebius Hieronymus)	circa AD 347–420	~73
Laberius, Decimus	circa 106–43 BC	~63
Libanius	AD 314–circa 393	~63
Livius Andronicus, Lucius	circa 280/270–200 BC	~75
Livy (Titus Livius)	59 BC–AD 17 or 64 BC–AD 12	76
Lucilius (1) Gaius	probably 180–102/101 BC	~75
Lucretius (Titus Lucretius Carus)	circa 94–55/51 BC	~41
Lyco	circa 300/298–226/224 BC	~74
Lycurgus (3)	circa 390–circa 325/324 BC	~65
Lydus	AD 490–circa 560	~70
Lysias	459/458–circa 380 BC or circa 445–circa 380 BC	~72
Malalas	circa AD 480–circa 570	~90
Mantias	circa 165–85 BC	~80
Megasthenes	circa 350–290 BC	~60

Table 1. Life spans of the first 100 "ancient intellectuals" in *The Oxford Classical Dictionary*. The average, 71.7 years, is taken as typical.

Figure 4. The probability distribution $a_a(x)$ for being active at a given age, if alive.

The choice made for this distribution might appear questionable on two grounds: Is it appropriate to use modern data in studying an Alexandrian scholar of the fourth century AD? And assuming this is so, is the particular mortality table chosen adequate?

Our chief justification for keeping this choice of $a(x)$ is that its most important feature for our purposes, the life expectancy, is in excellent agreement with a control value calculated for this purpose: the average lifespan of the first one hundred (in alphabetical order) "well dated" intellectuals found in *The Oxford Classical Dictionary* [Hornblower et al. 2012][7] (see Table 1). This suggests that using $a(x)$ as an approximation for the mortality distribution of the population of interest is consistent with the available quantitative evidence.

To model the probability $a_a(x)$ of being intellectually active at a given age if alive at that age we make some reasonable, if somewhat arbitrary, assumptions reflected in the graph in Figure 4.

Combining the two distributions $a(x)$ and $a_a(x)$ as explained, the probability of being active at any given age is calculated and — knowing that Hypatia was so in 415 — the probability distribution $\Upsilon_a(\xi)$ for the year of Hypatia's birth deduced from this historical datum is obtained in a straightforward manner (see Figure 5).

3.3. ***Hypatia reached old age.*** In his Χρονογραφία, John Malalas tells us that our subject was an old woman when she died:

> Κατʹ ἐκεῖνον δὲ τὸν καιρὸν παρρησίαν λαβόντες ὑπὸ τοῦ ἐπισκόπου οἱ Ἀλεξανδρεῖς ἔκαυσαν φρυγάνοις αὐθεντήσαντες Ὑπατίαν τὴν περι-

[7]The cutoff at 100 gives a convenient sample size large enough to be representative. Using all "ancient intellectuals" as the control population and not only those who lived in the third and fourth centuries AD is necessary in order to obtain a statistically significant sample.

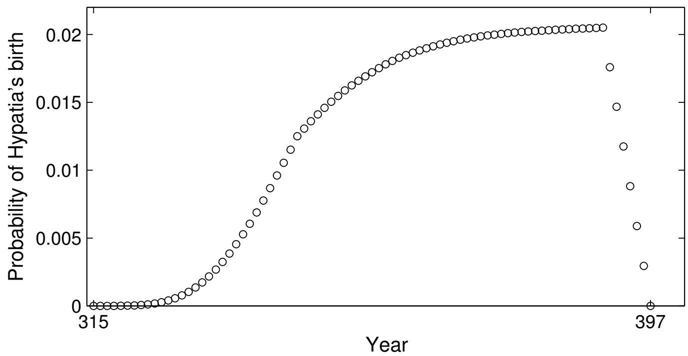

Figure 5. The probability distribution $\Upsilon_a(\xi)$ for Hypatia's birth based on her being active when she died.

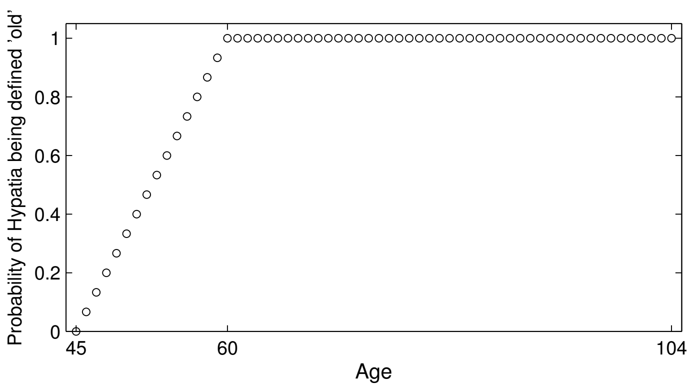

Figure 6. The probability distribution $o(x)$ for being regarded as an old woman.

βόητον φιλόσοφον, περὶ ἧς μεγάλα ἐφέρετο ἦν δὲ παλαιὰ γυνή.[8]

In light of the average lifespan of ancient intellectuals (Table 1), even a conservative interpretation of "old woman" would preclude an age much below 50.[9] Hence we model the probability distribution of someone being "old woman" by the function $o(x)$ shown in Figure 6. The resulting probability distribution, $\Upsilon_o(\xi)$, for the year of Hypatia's birth based on this datum is then easily obtained; see Figure 7.

[8]At that time the Alexandrians, given free rein by their bishop, seized and burnt on a pyre of brushwood Hypatia the famous philosopher, who had a great reputation and who was an old woman [Malalas, XIV.12].

[9]This agrees with the authoritative opinion of many historians; thus Maria Dzielska [1995]: "John Malalas argues persuasively that at the time of her ghastly death Hypatia was an elderly woman — not twenty-five years old (as Kingsley wants), nor even forty-five, as popularly assumed. Following Malalas, some scholars, including Wolf, correctly argue that Hypatia was born around 355 and was about sixty when she died".

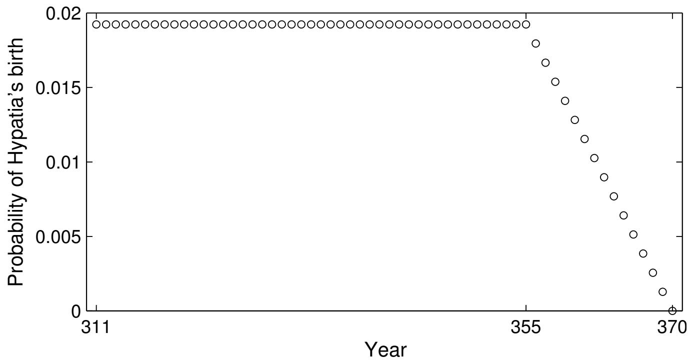

Figure 7. The probability distribution $\Upsilon_o(\xi)$ for Hypatia's birth given that she reached old age.

3.4. ***Hypatia, daughter of Theon.*** Theon of Alexandria, best known for allowing the transmission of Euclid's *Elements* to the present day, was Hypatia's father. By knowing his birth year, one might think of deducing a probability distribution for the year of Hypatia's birth; sadly, this is unknown as well. Therefore, it is necessary to calculate a probability distribution for the year of Theon's birth first. To this end, two recorded facts are useful:

- Theon was intellectually active between 364 and 377.[10]
- Hypatia overhauled the third book of Theon's *Commentary on the Almagest* (Theon refers to this in the *Commentary* itself).

This second datum makes it unlikely that Hypatia was born in Theon's old age; it also make it less probable that he stopped being intellectually active at a young age, since he was still active while his daughter made her contribution to his work. To quantify this reasoning, we define notation for the relevant events:

- F_i, Theon becomes a father at age i.
- $A_i^{T/I}$, Theon/Hypatia is intellectually active at the age of i.
- C, Theon is able to collaborate with Hypatia (both are intellectually active).
- $B_k^{T/I}$, Theon/Hypatia begins being intellectually active at age k.
- $S_k^{T/I}$, Theon/Hypatia stops being intellectually active at age k.

The probability of Theon becoming a father at various ages is described approximately by the model distribution $F(x)$ shown in Figure 8.

[10]In the *Little Commentary on Ptolemy's Handy Tables*, Theon mentioned some astronomical observations that can be dated with certainty: the two solar eclipses of June 15th and November 26th, 364 and an astral conjunction in 377. It is reasonable to assume that he was also active in the interval between those two years.

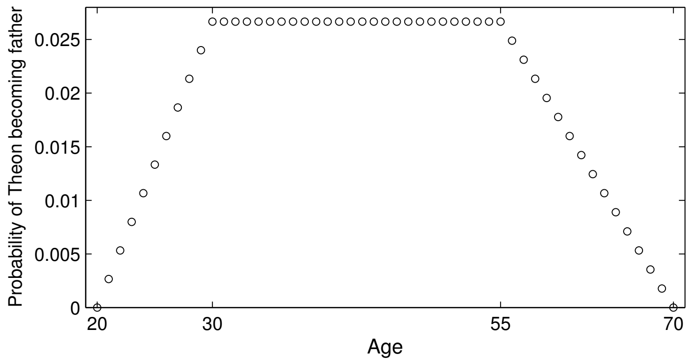

Figure 8. The probability distribution $F(x)$ for Theon's age at the time of Hypatia's birth.

The probability of a subject (Theon or Hypatia) beginning their intellectual activity at a given age is described approximately by the model distribution $B(x)$ shown in Figure 9.

The probability distribution $S(x)$ for the subject ending her intellectual activity at a given age is taken to be, up to age 70, just the probability of dying (derived from the distribution $a(x)$ of Figure 3), while after that it is the probabily of dying conditioned to that of being active, as obtained in Section 3.2. See Figure 10.

The probability of event C is therefore

$$P(C) = \sum_i \sum_k P(A^T_{i+k} \cap F_i \cap B^I_k).$$

By the definition of conditional probability,

$$\sum_i \sum_k P(A^T_{i+k} \cap F_i \cap B^I_k) = \sum_i \sum_k P(A^T_{i+k} \cap I^I_k \mid F_i) \cdot P(F_i),$$

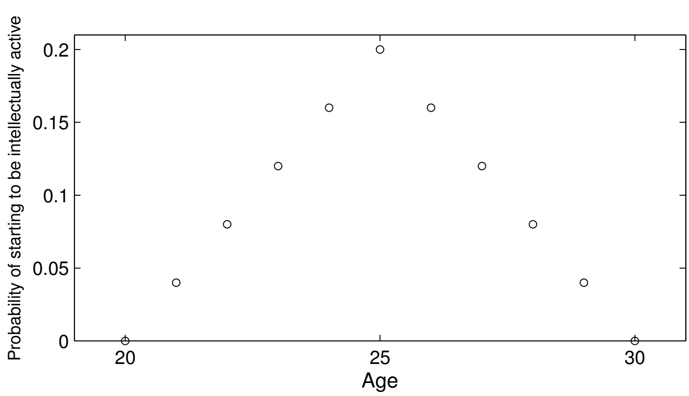

Figure 9. The probability distribution $B(x)$ for the starting point of one's intellectual career.

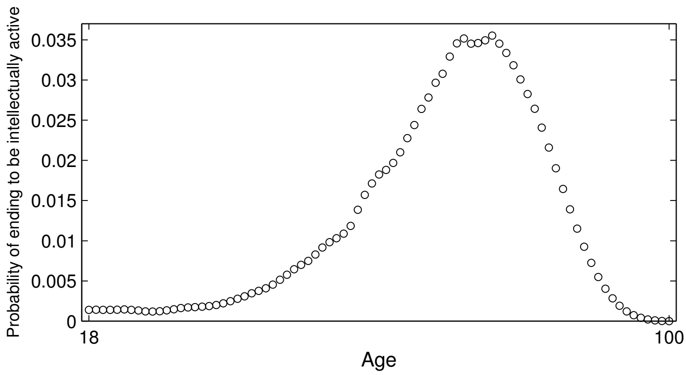

Figure 10. The probability distribution $S(x)$ for the endpoint of one's intellectual career.

and since the beginning of the active life of Hypatia does not depend on her father's activity, the following simplification can be made:

$$\sum_i \sum_k P(A^T_{i+k} \cap B^I_k \mid F_i) \cdot P(F_i) = \sum_i \sum_k P(B^I_k) \cdot P(A^T_{i+k} \mid F_i) \cdot P(F_i).$$

Without committing a large error, it is possible to confuse the probability of being active at age $i+k$ having had a daughter at age i, $P(A^T_{i+k} \mid F_i)$, with the one of being active at age $i+k$ having been alive at age i (V_i),[11] $P(A^T_{i+k} \mid V_i)$:

$$P(A^T_{i+k} \mid F_i) \approx P(A^T_{i+k} \mid V_i) = \frac{P(A^T_{i+k})}{P(V_i)}.$$

In the end, the following equation can be written:

$$P(C) = \sum_i \sum_k P(B^I_k) \cdot \frac{P(A^T_{i+k})}{P(V_i)} \cdot P(F_i).$$

Based on the idea previously introduced, the next step is to calculate $P(F_i \mid C)$ and $P(S^T_k \mid C)$ (and so $P(A^T_i \mid C) = 1 - \sum_k P(S^T_k \mid C)$):

$$P(F_i \mid C) = \frac{P(F_i \cap C)}{P(C)} = \frac{\sum_k P(B^I_k) \cdot (P(A^T_{i+k})/P(V_i)) \cdot P(F_i)}{\sum_i \sum_k P(B^I_k) \cdot (P(A^T_{i+k})/P(V_i)) \cdot P(F_i)},$$

$$P(S^T_k \mid C) = \frac{P(S_k \cap C)}{P(C)} = \frac{\sum_{i,j:i+j \le k} P(S^T_k) \cdot P(F_i) \cdot P(B^I_j)}{\sum_i \sum_j P(B^I_j) \cdot (P(A^T_{i+j})/P(V_i)) \cdot P(F_i)}.$$

$A^T_C(x)$ is the probability distribution of Theon being active at a given age, conditioned to the C event; see Figure 11.

[11] V_i is obtained from the above-mentioned 1974 Italian male mortality data set.

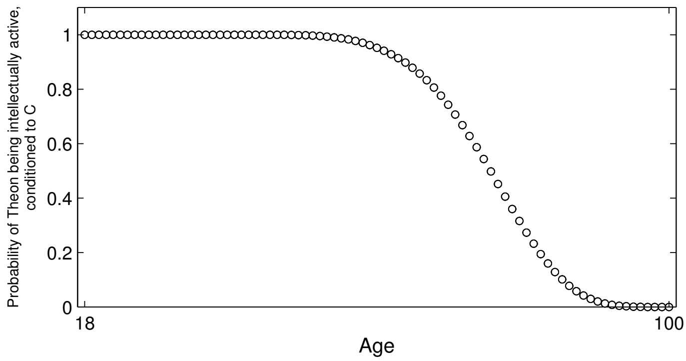

Figure 11. The probability distribution $A_C^T(x)$ for Theon being active at a given age, given that his and Hypatia's periods of activity overlap.

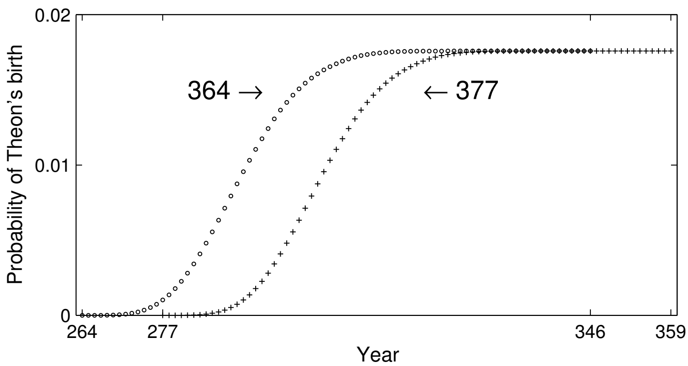

Figure 12. The probability distributions $364(\xi)$ and $377(\xi)$.

Keeping in mind the two years in which Theon was surely active (364 and 377), two distributions $364(\xi)$ and $377(\xi)$ for Theon's year of birth are deduced as previously shown in Section 3.2 (see Figure 12). Then, following the procedure introduced in Section 2, a single distribution $\Theta(\xi)$ is obtained (see Figure 13).

Finally, in order to calculate $\Upsilon_d(\xi)$, the probability distribution for the year of Hypatia's birth based on her being Theon's daughter, the probability of the various events "the age difference between father and daughter is i years" conditioned on event C must be known. This is indeed the above-calculated $P(F_i \mid C)$, now written as the function $F_C(x)$ (see Figure 14) so that $\Upsilon_d(\xi)$ is straightforward to calculate:[12]

$$\Upsilon_d(\xi) = \sum_x \Theta(\xi) \cdot F_C(\xi - \xi).$$

(See Figure 15.)

[12]The sum is taken over the whole domain of $\Theta(\xi)$.

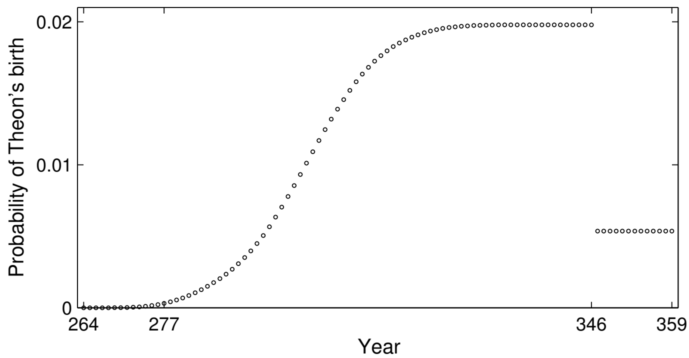

Figure 13. The probability distribution $\Theta(\xi)$ for Theon's birth.

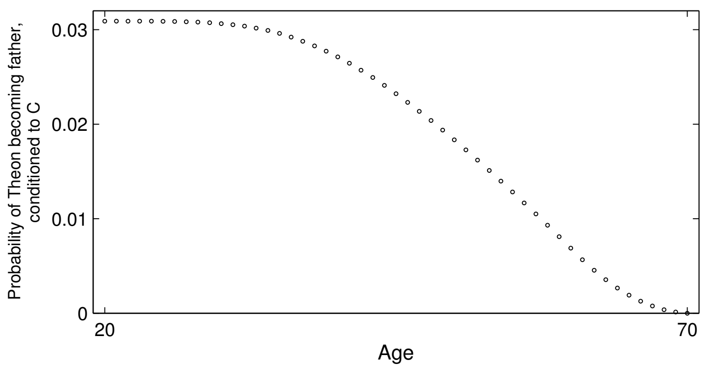

Figure 14. The probability distribution $F_C(x)$ for the difference in age between father and daughter, given that their periods of activity overlap.

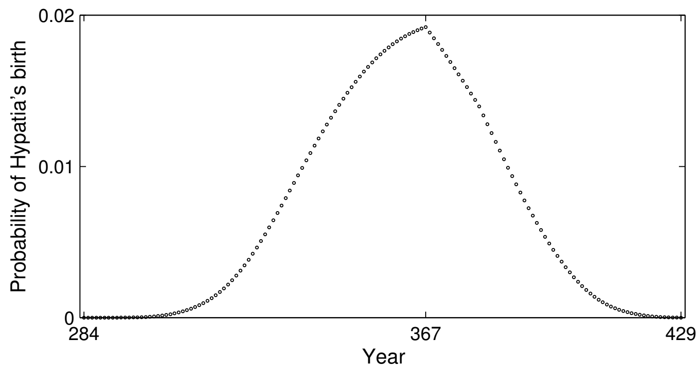

Figure 15. The probability distribution $\Upsilon_d(\xi)$ for Hypatia's birth based on her relationship to Theon.

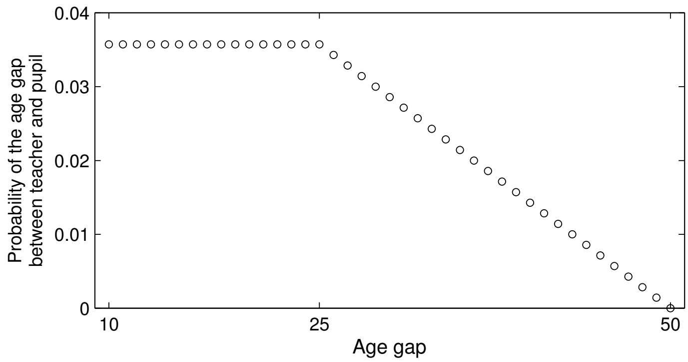

Figure 16. The probability distribution $T(x)$ for the age gap between teacher and disciple.

3.5. ***Hypatia, teacher of Synesius.*** Synesius of Cyrene, neo-Platonic philosopher and bishop of Ptolemais, was a disciple of Hypatia, as shown by a close correspondence between the two.

For instance, from his deathbed, Synesius wrote:

> Τῇ φιλοσόφῳ.
> Κλινοπετὴς ὑπηγόρευσα τὴν ἐπιστολήν, ἣν ὑγιαίνουσα κομίσαιο, μῆτερ καὶ ἀδελφὴ καὶ διδάσκαλε καὶ διὰ πάντων τούτων εὐεργετικὴ καὶ πᾶν ὅ τι τίμιον καὶ πρᾶγμα καὶ ὄνομα.[13]

The distribution $T(x)$ is introduced as a model to describe the probability of a difference of x years of age between teacher and pupil (see Figure 16).

$\Upsilon_t(\xi)$, the probability distribution for the year of Hypatia's birth deduced from this historical datum, is obtained in a straightforward manner by taking 370 as the year of birth of Synesius[14] (see Figure 17).

3.6. ***Combined distribution.*** Combining the five probability distributions deduced above for the year of Hypatia's birth, one final distribution, $\Upsilon(\xi)$, can be obtained following the rules introduced in Section 2. This final distribution $\Upsilon(\xi)$ can be compared to the distribution given by the simple arithmetic mean of the various distributions resulting from every possible combination of testimonies being considered true at the same time, $\Upsilon_A(\xi)$ (see Figure 18).

Therefore, the most probable year for the birth of Hypatia is 355 ($\sim$14.5%) with a total probability of the interval [350, 360] of about 90%.

---

[13]To the Philosopher. I am dictating this letter to you from my bed, but may you receive it in good health, mother, sister, teacher, and withal benefactress, and whatsoever is honored in name and deed [Synesius of Cyrene, Incipit of Letter 16].

[14]See, for example, [Hornblower et al. 2012].

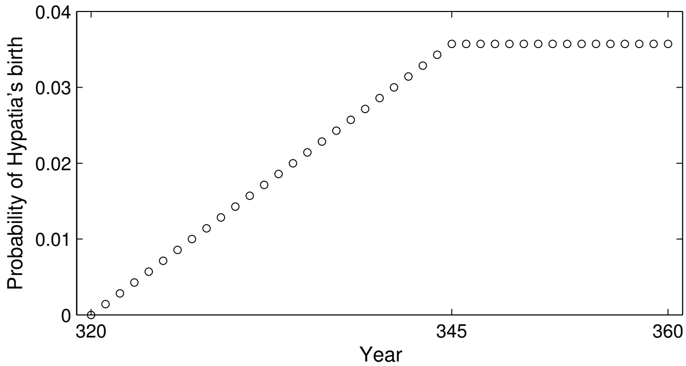

Figure 17. The probability distribution $\Upsilon_t(\xi)$ for Hypatia's age based on her having been a teacher of Synesius.

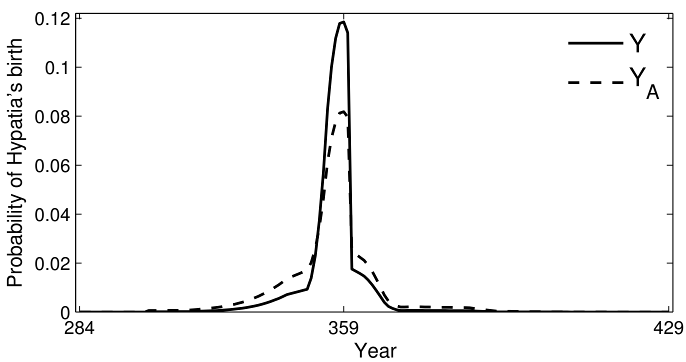

Figure 18. The final probability distribution $\Upsilon(\xi)$ calculated for the birth of Hypatia using the method in Section 2 and an average distribution $\Upsilon_A(\xi)$ based on the same historical data.

4. Conclusions

The probabilistic dating model proposed in this work, structured in three steps, could be summarized by making use of a culinary analogy. The first step is represented by the collection of enough raw ingredients (testimonies) to be refined or "cooked" in the second step (turned into probability distributions) and — finally, in the third step — put together following a recipe (provided in Section 2) so that they blend well (as a single probability distribution).

Its application to the case of Hypatia proved to be satisfactory in that the final probability distribution shows a marked peak, making it possible to give a date with good precision. The result so obtained contradicts the prevalent opinion (cf. page 25) but is in agreement with the minority view held by some highly-regarded scholars working on the issue. We have already mentioned the authoritative opinion of Maria Dzielska, who deems that Hypatia died at about age 60, having been, consequently, born around the year 355. A similar opinion is expressed in [Deakin 2007, p. 52].

Future applications appear to be far-reaching as the method could serve not only in cases strictly analogous to the one presented here but also in dating any event provided with a sufficient number of testimonies able to be turned into probability distributions.

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