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FAR-REACHING HELLENISTIC GEOGRAPHICAL KNOWLEDGE HIDDEN IN PTOLEMY'S DATA

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The paper summarizes and discusses the main theses exposed in a previous book (*L'America dimenticata*, Mondadori Università, 2013; in Italian) in light of more recent results. Specifically, the work addresses the problem of explaining the origin of the systematic error on longitudes in Ptolemy's *Geographia* and its logical relation with the reduced estimate for the dimension of the Earth given there. The thesis is sustained that, contrary to a frequently advanced conjecture, the shrinking of the dimension of the Earth is a consequence of a scale error in longitudes, which, in turn, was originated by a misidentification of the Islands of the Blessed. The location of the Islands of the Blessed according to the source of Ptolemy is identified in the Caribbean. The analysis of a passage of Pliny provides an independent and quantitative confirmation of the proposed identification, which sheds new light on possible contact among civilizations.

1. The shrinking of the Earth and the dilation of longitudes in Ptolemy's *Geographia*

It is well known that Eratosthenes, in the 3rd century BC, measured the circumference of the Earth, obtaining the value of 252,000 stadia (corresponding to 700 stadia per degree). Four centuries later Ptolemy, accepting a value suggested by Posidonius in the 1st century BC, estimated the same circumference in 180,000 stadia (500 stadia per degree). The method employed by Eratosthenes is described, in outline at least,[1] by Cleomedes (*Caelestia*, I, 7, 48–120), while we have no information about the origin of the smaller value. Almost all scholars have always believed that Eratosthenes and Ptolemy were using the same stadium, even if this

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[1] For a discussion of Eratosthenes' method see [Russo 2004, 273–276]; concerning the possible simplifications introduced by Cleomedes in his popular account, see [Russo 2013a, 71–76], which also contains an attempt to reconstruct the original computation by Eratosthenes.

has been called into question by certain historians.[2]

Much more controversial has been the actual value of the stadium considered by the two. Indeed, many scholars[3] have accepted the value of 157.5 m, deduced from a passage in Pliny,[4] while several others have maintained that the Olympic stadium (ca. 185 m) was employed.[5] Other values have also been proposed.[6] Assuming the value of 157.5 m, Eratosthenes would have made an error of less than 1%, whereas for Ptolemy and Posidonius the error is large, roughly 40%. While these error estimates rely on the determination of the value of the stadium, another error in Ptolemy's *Geographia*, which can be studied independently of the question of length units, is the systematic error in longitudes. It is well known that longitude differences were systematically dilated by Ptolemy; in particular he grossly overestimated the longitudinal amplitude of the oikoumene (the inhabited part of the world).

In [Russo 2013a] the longitudes reported by Ptolemy were used to determine the "effective" value of Ptolemy's stadium. The main result of that work was an independent validation of the substantial exactness of the value 157.5 deduced from Pliny, which implies that the estimate for the Earth's circumference by Ptolemy was

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[2]A few scholars have suggested that Ptolemy (and Posidonius before him) replaced Erastosthenes' stadium with a new stadium corresponding to about 222 meters, so that the two measures of the circumference of the Earth should coincide [Gossellin 1790; Jomard1817; Valerio 2013]. This conjecture is contradicted by the explicit statement by Strabo that Posidonius' measure of 180,000 stadia had reduced the dimension of the Earth (*Geographica*, II, ii, 2). Furthermore, if Ptolemy was using a stadium 40% longer than that of Eratosthenes, one would be hard pressed to explain why he dilated in (approximately) the same measure the numerical values of differences of longitude.

[3]See, among others, [Letronne 1851, 104–119, 212–246; Hultsch 1882, 60–63; Tannery 1893, 109–110; Dreyer 1953, 175; Miller 1919, 6–7; Oxé 1963, 269–270; Aujac 1966, 176–179; Fraser 1972, II, 599, n. 312; Stückelberger 1988, 188; Dutka 1993/94, 63–64; Meuret 1998, 163–164, Tupikova and Geus 2013, 21.]

[4]Pliny writes: "schoenus patet Eratosthenis ratione stadia XL, hoc est p. $\overline{\text{v}}$" (*Naturalis Historia*, XII, 53). This sentence, using the known value of the schoenus, gives for the stadium the value of 157.5 meters. It is true that in another passage (*N.H.*, II, 247) Pliny translates Eratosthenes' result into 31500 milia passuum (a calculation involving the use of the Olympic stadium of 185 meters), but this circumstance enhances, in my opinion, the role of the words "Eratosthenis ratione" used in the first case. Indeed, the Olympic stadium was certainly widely used and its automatic use by Pliny is not surprising, while his need to clarify, in the other case, that he is considering the particular unit introduced by Eratosthenes, gives us a precious testimony. Given the enormous influence of Eratosthenes' *Geographica*, it is not surprising that the new "stadium" introduced by him was adopted as the standard unit in geographical treatises, while the Olympic stadium remained in use for other purposes.

[5]See, among others, [Columba 1895, 63–68; Czwalina 1925, 295; Dicks 1960, 42–46; Rawlins 1981, 218; Pothecary 1995, 49–67; Berggren and Jones 2000, 14, footnote 10].

[6]Most of them are analyzed in [Tupikova and Geus 2013, 20–22] (where, however, the values outside the interval 148–180m are considered quite implausible and the extreme possibility mentioned in note 2 above is not considered worthy of mention).

affected by a very large error. That paper was based on the analysis of longitudes of a sample of 80 cities, chosen as the most renowned among those in the part of the world best- nown in Hellenistic times. Plotting Ptolemy's longitudes against the values currently accepted for them, a graph was obtained which is approximated remarkably well by means of a linear regression. The equation of the regression line is

$$y = 1.428x + 17.06 \tag{1}$$

and the coefficient of determination is $R^2 = 0.9935$. More recently, in [Shcheglov 2014] a larger sample of 245 locations (including some river's mouths and capes) was considered, yielding very similar results. Indeed, in that study the equation of the regression line is

$$y = 1.4279x + 16.425, \tag{2}$$

and the relative coefficient of determination is $R^2 = 0.9874$. This result confirms that of [Russo 2013a], and the slightly lower value of R^2 (still very close to 1) is likely linked to the inclusion in the sample of locations from lesser known regions such as Northern Europe and India. In the two papers the slope of the regression line is virtually identical,[7] and it is close to the ratio of 1.4 between Ptolemy's and Eratosthenes' values for the Earth's circumference. We know that Ptolemy generally deduced differences of longitudes from distances expressed in stadia along circles of latitude (mostly taken from Eratosthenes, who in his geographical work, instead of degrees of longitude, had reported distances between meridians along a particular parallel of latitude). It is therefore not surprising that Ptolemy's longitude differences were dilated in such a way to compensate, for such distances, for the error in the dimension of the Earth. (We will return later to the slight difference between 1.4 and the value of the regression coefficient.) Hence it is very likely that a logical link does exist between the error on the dimension of the Earth and the error on the differences of longitudes.

Some scholars have interpreted this link by proposing the following implication:

A* : *Ptolemy assumed a wrong measure for the Earth's circumference. As a consequence he systematically deduced dilated longitudes from his data involving distances along circles of latitude.*[8]

[7]The method used in [Russo 2013a] for estimating the value of the stadium was actually based on the regression coefficient, so a validation of the value of this coefficient by means of the much larger set of locations considered in [Shcheglov 2014] provides a sounder statistical basis for that estimate.

[8]This implication is maintained, in particular, in [Rawlins 1985; Rawlins 2008; Tupikova and Geus 2013; Tupikova 2013]. Strangely enough, in Shcheglov 2014 the same opinion is ascribed also to [Russo 2013a] (where, in this regard, it is only written that "the distortion operated by Ptolemy on the longitudes is not independent of the new value he had assumed for the length of the Earth's circumference") and even to [Russo 2013b], where the implication is explicitly criticized.

This implication is not, however, a necessary consequence of the link we mentioned before and in the next section we will show that actually several arguments allow us to discard it.

2. Rereading the relationship between the two errors by Ptolemy

A first argument against A* consists in observing that it leaves unsolved the problem of the origin of the error in the Earth's dimension. The value obtained by Eratosthenes was indeed well known, and it is transmitted by ten different extant sources, dating from the I century BC to the V century AD.[9] Posidonius gave two values for the Earth's circumference. For one of them (240,000 stadia), not too far from the one by Eratosthenes, we know the actual procedure by which it was obtained (Cleomedes, *Caelestia*, I, 7, 1–47). Had the other one (180,000 stadia) been a result of some measurement procedure, one could wonder about three things:

1. Why is there no source describing it?
2. How could it have produced such a large error?
3. Why should such a grossly wrong estimate, which no extant source considers worth of being described, have prevailed over the value produced by the celebrated measurement by Eratosthenes?

A further strong argument against A* was provided in [Shcheglov 2014]. Namely, if all differences of longitudes were dilated because of an error in the assumed dimension of the Earth, we should have more or less the same dilation in different regions (at least for the better known ones). In contrast, splitting his sample in nine subsets corresponding to different geographic areas, Shcheglov verified that linear regressions performed on the different subsets give substantially different values for the slope, in every case with the coefficient R^2 very close to 1.

Those arguments are largely sufficient, in my opinion, to reject implication A*.

3. How did Ptolemy actually work?

Ptolemy, in his *Geographia*, states that the oikoumene is spread over 180° of longitude, from the westernmost locations (four of the six Islands of the Blessed, the other two being one degree further east) to the easternmost ones (some towns in

[9]Strabo, *Geographica*, II, v, 7, 34; Geminus, *Introduction to the Phenomena*, XVI, 6; Macrobius, *Commentarii in Somnium Scipionis*, I, xx, 20; Vitruvius, *De Architectura*, I, vi, 9; Pliny the Elder, *Naturalis Historia*, II, 247; Censorinus, *De Die Natali*, xiii, 5; Theon of Smyrna, *De Utilitate Mathematicae*, 124, 10–12 (ed. Hiller); Heron of Alexandria, *Dioptra*, xxxv, 302, 10–17 (ed. Schöne); Martianus Capella, *De Nuptiis Philologiae et Mercurii*, VI, 596; and the already mentioned Cleomedes (*Caelestia*, I, 7, 48–120). This last author is the only one giving a round value of 250,000.

China). He claims (*Geographia*, I, 12) to have determined the amplitude of the oikoumene in pieces, by considering a path from the meridian of the Islands of the Blessed to the Metropolis of the Seres (the capital of China). The path is formed from twelve portions, whose relative lengths are derived by heavily modifying, with very rough arguments,[10] data given by Marinus of Tyre. The values obtained after this procedure are finally translated into differences of longitude assuming the aforementioned estimate for the Earth's circumference: 500 stadia per degree. The result is $177°\,15'$. Since the easternmost locations known to Ptolemy are just $2°\,45'$ to the east of the Metropolis of the Seres, he eventually gets a total of 180°. Clearly Ptolemy is fiddling with the numbers: the likelihood is negligible that he'd have reached this round value by accident after arbitrarily modifying twelve terms in a sum.

At the same time, Ptolemy himself, at the beginning of his *Geographia*, explains his actual method, when he points out the difference between the subject of his work and chorography:

> The goal of chorography is an impression of a part, as when one makes an image of just an ear or an eye; but [the goal] of geography is a general view, analogous to making a portrait of the whole head. That is, whenever a portrait is to be made, one has to fit in the main parts [of the head] in a determined pattern and an order of priority. Furthermore the [surfaces] that are going to hold the drawings ought to be of a suitable size for the spacing of the visual rays at an appropriate distance, whether the drawing be of whole or part, so that everything will be grasped by the sense [of sight].[11]

From the metaphor it is clear that Ptolemy, having to represent the whole oikoumene, wants first to fix the positions of some key locations, and in particular its global dimension, and only afterwards to add the coordinates of all the remaining localities.

One may wonder in what sense the positions of these key locations were fixed — whether by means of their angular coordinates or their relative distances. In this regard, it can be recalled that Ptolemy himself explains that the astronomical method,

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[10] For instance, dividing a value by two to account for tortuous roads.

[11] Ἔχεται δὲ τὸ μὲν χωρογραφικὸν τέλος τῆς ἐπὶ μέρους προσβολῆς, ὡς ἂν εἴ τις οὖς μόνον ἢ ὀφθαλμὸν μιμοῖτο, τὸ δὲ γεωγραφικὸν τῆς καθόλου θεωρίας κατὰ τὸ ἀνάλογον τοῖς ὅλην τὴν κεφαλὴν ὑπογραφομένοις. Πάσαις γὰρ ταῖς ὑποτεθειμέναις εἰκόσι τῶν πρώτων μερῶν ἀναγκαίως καὶ προηγουμένως ἐφαρμοζομένων, καὶ ἔτι τῶν δεξομένων τὰς γραφὰς συμμέτρων ὀφειλόντων εἶναι ταῖς ἐξ ἀποχῆς αὐτάρχως τῶν ὄψεων διαστάσεσιν, ἐάν τε τέλειον ᾗ τὸ γραφόμενον ἐάν τ'ἐπι` μέρους, ἵν'ἅπαν αισθητῶς παραλαμβάνηται [...] (Ptolemy, *Geographia*, I, 1, 2–3; translation adapted from [Berggren and Jones 2000]).

from which only angular measures can be derived, is to be preferred to the surveying which could provide distances in stadia:

> The surveying component is that which indicates the relative positions of localities solely through measurement of distances; the astronomical component [is that which does the same] by means of the phenomenon [obtained] from astronomical sighting and shadow-casting instruments. Astronomical observation is a self-sufficient thing and less subject to error, while surveying is cruder and incomplete without [astronomical observation].[12]

The order in which the operations were performed is indeed explicitly indicated in Ptolemy's *Geographia*, in the title of chapter I, 4:

> That it is necessary to give a priority to the [astronomical] phenomena over [data] from records of travel.[13]

Shortly afterward he points out:

> It would therefore also be reasonable for one who intended to practice geography following these [principles] to give priority in his map to the [features] that have been obtained through the more accurate observations, as foundations, so to speak, but to fit [the features] that come from the other [kinds of data] to these, until their positions with respect to each other and to the first [features] stand as much as possible in agreement with those reports that are less subject to error.[14]

From these passages, we can deduce that Ptolemy first fixed a few longitudes known by astronomical methods (in particular those of some extreme points determining the amplitude in degrees of the oikoumene), and then he interpolated the longitudes of the intermediate locations by using known distances along circles of latitude. Therefore, the first error among the two mentioned in the previous section has necessarily to be the one on longitudes. Hence, as already said, A* has to be discarded. Since, on the other hand, Ptolemy's scale error on longitudes, combined

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[12] γεωμετρικὸν μὲν τὸ διὰ ψιλῆς τῆς ἀναμετρήσεως τῶν διαστάσεων τὰς πρὸς ἀλλήλους θέσεις τῶν τόπων ἐμφανίζον, μετεωροσκοπικὸν δὲ τὸ διὰ τῶν φαινομένων ἀπὸ τῶν ἀστρολάβων καὶ σκιοθήρων ὀργάνων· τοῦτο μὲν, ὡς αὐτοτελές τι καὶ ἀδιστακτότερον, ἐκεῖνο δὲ, ὡς ὁλοσχερέστερον καὶ τούτου προσδεόμενον. (Ptolemy, *Geographia*, I, 1, 2; translation by Berggren and Jones.)

[13] Ὅτι δεῖ τὰ ἐκ τῶν φαινομένων προϋποτίθεσθαι τῶν ἐκ τῆς περιοδικῆς ἱστορίας (Ptolemy, *Geographia*, I, 4, T; translation by Berggren and Jones).

[14] εὔλογον ἂν εἴη καὶ τὸν τούτοις ἀκολούθως γεωγραφήσοντα τὰ μὲν διὰ τῶν ἀκριβεστέρων τηρήσεων εἰλημμένα προϋποτίθεσθαι τῇ καταγραφῇ καθάπερ θεμελίους, τὰ δ' ἀπὸ τῶν ἄλλων ἐφαρμόζειν τούτοις, ἕως ἂν αἱ πρὸς ἄλληλα θέσεις αὐτῶν μετὰ τῶν πρὸς τὰ πρῶτα τηρῶσιν ὡς ἔνι μάλιστα συμφώνως τὰς ἀδιστακτοτέρας τῶν παραδόσεων. (Ptolemy, *Geographia*, I, 4, 1; adapted from the translation by Berggren and Jones.)

with a set of right distances along circles of latitude, necessarily implies his error on Earth's dimension, we are led to propose the following implication:

A** : *Ptolemy's error on the amplitude in longitude of the oikoumene was the cause of his systematic dilation of the differences of longitude and of his deduction, from the known data for the distances along circles of latitude, of an underestimation of the dimension of the Earth.*

As for the source of the scale error on longitudes, we start by observing that, since Ptolemy emphasizes the importance of astronomical data provided by Hipparchus (*Geographia*, I, 4, 1), we can conjecture that he took from him also the value of 180° for the amplitude of the oikoumene. An argument supporting this is provided once again by Ptolemy himself, in his Almagest, when he states that the oikoumene occupies approximately one fourth of the earth surface bounded by half of the equator and the halves of two opposite meridians. The amplitude of 180° is justified in the following passage:

> In the case of longitude (that is in the east-west direction) the main proof is that observations of the same eclipse (especially a lunar eclipse) by those at the extreme western and extreme eastern regions of our part of the inhabited world (which occur at the same [absolute] time) never differ by more than twelve equinoctial hours [in local time]; and the quarter [of the earth] contains a twelve-hour interval in longitude, since it is bounded by one of the two halves of the equator. [15]

The reference to the method proposed by Hipparchus for measuring differences of longitude from local times of lunar eclipses[16] gives a clue in support of our hypothesis, which is also consistent with the fact that the amplitude of 180° for the oikoumene was already transmitted, before Ptolemy and Marinus, by Strabo. Indeed, shortly after having recalled the method used by Hipparchus for determining differences of longitude (*Geographica*, I, 1, 12), Strabo observes that people living in the extreme eastern regions are in a sense the antipodes of those living at the extreme west of Iberia (*Geographica*, I, 1, 13).

If the amplitude of the oikoumene was taken from Hipparchus, and was at the origin of the wrong estimate of the Earth's circumference, it is understandable that

[15] ἐπὶ δὲ τοῦς μήκους, τουτέστιν τῆς ἀπὸ ἀνατολῶν πρὸς δυσμὰς παρόδου, διὰ τοῦ τὰς αυτὰς ἐκλείψεις, μάλιστα δὲ τὰς σεληνιακὰς, παρά τε τοῖς ἐπ' ἄκρων τῶν ἀνατολικῶν μερῶν τῆς καθ'ἡμᾶς οἰκουμένης οἰκοῦσι καὶ παρὰ τοῖς ἐπ' ἄκρων τῶν δυτικῶν κατὰ τὸν αυτὸν χρόνον θεορουμένας μὴ πλέον δώδεκα προτερεῖν ἤ ὑστερεῖν ὡρ˜ν ἰσημερινῶν αὐτοῦ κατὰ μῆκος τοῦ τεταρτημορίου δωδεκάωρον διάστημα περιέχοντος, ἐπειδήπερ ὑφ'ἑνὸς τῶν τοῦ ἰσημερινοῦ ἡμικυκλίων ἀφορίζεται. (Ptolemy, *Almagest*, II, 1, p.88 Heiberg, 10–19; translation by G. J. Toomer.)

[16] Obviously it is not possible to observe the same lunar eclipse from two opposite semi-meridians (or not without assuming unrealistic observations made from points close to a pole), but it *is* possible to obtain the total longitude between them by summing two (or more) smaller longitude differences.

this last error was already made by Posidonius, whose main source in astronomical and geographical matters was probably Hipparchus himself. [17] Furthermore, the rejection of the celebrated result by Eratosthenes becomes understandable if it was girded by a belief that one was following Hipparchus, the famous scientist whose successful criticism of Eratosthenes was well known.

Finally, we can notice that the discrepancy among the ratio 1.4 between the two estimates for the Earth's circumference and the value 1.428 of the regression coefficient concerning the longitude dilation, though small, is not negligible and hardly compatible with A*, in view of the high value of R^2. This discrepancy becomes easily understandable by adopting the implication A**. A contraction of Eratosthenes' value of 700 stadia per degree by a factor 1.428 leads indeed to a value of 490.2 stadia per degree, but it is reasonable that Ptolemy (and Posidonius before him), having to replace an estimate given by a round figure,[18] wanted to select an equally round figure, therefore choosing 500.

4. Global and local errors

Let us go back to chapter I, 4 of Ptolemy's *Geographia*:

> [...] most distances, especially the east-west ones, have come down to us in a less precise form, not through the negligence of those who devoted themselves to research, but perhaps because they had not yet understood the usefulness of more scientific methods and because they had not observed many lunar eclipses at the same time in different places (such as the one that was seen in Arbela at the fifth hour and at Carthage at the second hour), from which it would have been clear how many equinoctial time units separated the localities to the east or west.[19]

The mention of people who had "not yet" (μηδέπω) understood the usefulness of the method based on lunar eclipses for determining the longitudes implies that

[17] We recall that Hipparchus worked in the same Rhodes where Posidonius, a generation later, established his school.

[18] The aforementioned passage from Pliny about the stadium according to the "Eratosthenis ratio" suggests that in the case of Eratosthenes the round figure could have been the result of the definition of the new stadium as a convenient submultiple of the circumference of the Earth (252,000 is a particularly convenient number, because it is divisible by all numbers from 1 to 10, whose least common multiple is in fact 2,520).

[19] τὰ δὲ πλεῖστα τῶν διαστημάτων καὶ μάλιστα τῶν πρὸς ἀνατολὰς ἢ δυσμὰς ὁλοσχερεστέρας ἔτυχε παραδόσεως, οὐ ῥᾳθυμίᾳ τῶν ἐπιβαλόντων ταῖς ἱστορίαις, ἀλλ' ἴσως τῷ μηδέπω τὸ πρόχειρον κατειλῆφθαι τῆς μαθηματικωτέρας ἐπισκέψεως, καὶ διὰ τὸ μὴ πλείους τῶν ὑπὸ τὸν αὐτὸν χρόνον ἐν διαφόροις τόποις τετηρημένων σεληνιακῶν ἐκλείψεων, ὡς τὴν μὲν Ἀρβήλοις πέμπτης ὥρας φανεῖσαν, ἐν δὲ Καρχηδόνι δευτέρας, ἀναγραφῆς ἠξιῶσθαι, ἐξ ὧν ἐφαίνετ' ἂν πόσους ἀπέχουσιν ἀλλήλων οἱ τόποι χρόνους ἰσμερινοὺς πρὸς ἀνατολὰς ἢ δυσμάς· (Ptolemy, Geographia, I, 4, 1).

such usefulness was clear to later scholars. Ptolemy indeed had used such astronomical method in the case of Arbela and Carthage[20] and most probably, as already observed, in some other cases.

Suppose that Ptolemy had obtained all his longitudes by dividing the (wrong) amplitude of the oikoumene assumed by him in parts proportional to the known distances along circles of latitude. In this case, and in the absence of large systematical errors in distances, we should expect that Ptolemy's longitudes were well approximated everywhere by the same linear function of the actual ones, i.e., all differences of longitude are dilated almost exactly in the same proportion the oikoumene as a whole is.

But if instead, as suggested by Ptolemy himself, the interpolation procedure started after the insertion of some milestone astronomical data — which would be very unlikely to fit well with the linear relation above[21] — then the set of all the locations considered would have been broken into regional subsets such that:

(a) In each subset the longitudes, being obtained with an interpolation procedure, are very close to a linear function of the actual longitudes, so that we should get in any case very high values of the coefficients of determination R^2.

(b) The regression coefficients corresponding to different subsets should differ considerably from each other.

(c) The longitudes inserted on an astronomical basis should mark the fracture points between different subsets.

(d) The deviations of the different regression coefficients from the global dilation ratio should compensate each other in such a way that the set of all the longitudes can be well approximated by a linear regression with a coefficient equal to such dilation ratio.

To these points, one can add a further observation. The data given by Ptolemy are interconnected by a very complex chain of relations. In particular, he states (*Geographia*, I, 4, 1) that in some cases he has the information that two locations are on the same meridian. Considering this kind of interconnection in relation with the previous reasoning, it is clear that:

(e) There exists the possibility that two sets of data covering more or less the same longitude area (but coming from different latitudes), are broken in subsets in such a way that the fracture points are approximately at the same longitude for both.

This scenario, suggested by Ptolemy's exposition, describes well the features of the longitudes reported in his *Geographia*, as is apparent from the analysis made

[20] See next page.

[21] Of course we cannot exclude the possibility that Ptolemy discarded data too far from it.

in [Shcheglov 2014]. This analysis, which allowed Shcheglov to correctly discard thesis A*, is not only consistent with A**, but gives a strong argument in its support once one takes into account that, in addition to the systematic dilation, significant and well characterized errors due to the insertion of some longitudes deduced from astronomical data are to be expected. In this regard, the fact that the sample chosen by Shcheglov as a whole leads to a value of R^2 that is close to 1 (0.9874) should not be, in my opinion, dismissed as "deceptive" (as Shcheglov does), but provides a key element for the reconstruction of the overall framework.

We cannot hope to identify all the cases in which Ptolemy used astronomical data concerning longitudes, but in one significant case it is possible to reasonably conjecture the origin of a local error. Let us start by observing that the presence of local regression coefficients which largely differ from the global one is especially significant in areas that were well-known in Hellenistic times. In this regard, the data coming form the south and north coasts of the Mediterranean sea are particularly relevant. Analyzing the data shown in Figure 6 of [Shcheglov 2014], one can see that both the southern and the northern coasts of the Mediterranean Sea can be divided in two parts, eastern and western, which feature substantially different values for the regression coefficient. Specifically, the regression coefficients are 1.81 and 1.19 for the western and eastern part of the south coast; 1.77 and 1.15 for the western and eastern part of the north coast. It is also striking that the fracture points between the two different slopes for the north and south coasts[22] are situated at approximately the same longitude — in modern terms, slightly more than 10° E. A good explanation for that fact would be that, for some location at a longitude slightly more than 10° E, Ptolemy had an astronomical datum that was inconsistent with the systematic dilation of longitudes. It is then perhaps not by chance that that longitude corresponds very well to one of the endpoints (Carthage, 10°19′) of the only longitude interval for which Ptolemy explicitly states that he possessed an astronomical datum. The value given by Ptolemy for the difference in longitude between Arbela and Carthage (45°10′) in fact agrees very closely with the one corresponding to the difference of three hours mentioned in the previous passage. On the other hand, this value is dilated by a factor of approximately 1.30 with respect to the actual difference, a value which significantly differs from the global coefficient of regression.[23] If one considers that the longitude given by Ptolemy for Arbela places it almost exactly on the global regression line, this strongly suggests that Ptolemy, taking into account the astronomical datum for the

[22]See point (e) on the previous page.

[23]We notice that, as a consequence, the longitudinal distance of Carthage from the Pillars of Heracles is dilated by Ptolemy by the factor 1.74, in good agreement with the regression coefficients (1.81 and 1.77) found by Shcheglov for the western part of the Mediterranean Sea.

determination of the longitude of Carthage,[24] may have had to adjust the other data which were linked to it by relations between distances or other more complex kinds of interconnections. This hypothesis can explain:

(i) the different slopes observed for eastern and western areas;

(ii) the high values of R^2 for both subsets;

(iii) the fact that the regression coefficient for the union of the considered subsets agrees with a good approximation with the global dilation ratio;[25]

(iv) the fact that the northern and southern Mediterranean coasts each break into different slope subsets more or less at the same longitude.[26]

A further confirmation of this reconstruction comes from a comparison between the global regression coefficient (1.428) and the ratio between the actual longitudinal amplitude for the Ptolemy's oikoumene and the value of 180° accepted by him.

The westernmost locations considered by Ptolemy are the Islands of the Blessed, which he identified with the Canary Islands.[27]

In order to estimate the global dilation of the oikoumene, on the other extreme we consider the Sera Metropolis (today's Xi'an[28]), since more eastern locations

[24]The poor accuracy of this particular value is easily understandable if we take into account that it relies on non-scientific descriptions of the eclipse, dating to the fourth century BC, two centuries before that Hipparchus suggested that this kind of data could be used to determine differences of longitude.

[25]A linear regression performed over the whole set of locations indicated in [Shcheglov 2014] as the "north coast" and "south coast" of the Mediterranean gives the line $y = 1.437x + 17.00$, with $R^2 = 0.980$. The very low values of the regression coefficients found by Shcheglov for the eastern part of the Mediterranean Sea (1.19 and 1.15) are thus the right ones to compensate the overestimation of the dilation on the other side.

[26]See again point (e) above.

[27]This identification was usual in imperial times. It appears for the first time implicitly in the work of Pomponius Mela, who places the Islands of the Blessed in front of the Atlas Mountain (*De chorographia*, III, 101–102). In Pliny's *Naturalis Historia* (VI, 202–203) the identification is even clearer, since he places them in front of Mauretania and gives with a good approximation their distances from Gades (Cadiz). As for Ptolemy (who hardly dares to question knowledge widely accepted in his days), he gives names and coordinates of six "Islands of the Blessed" (Μακάρων νῆσοι) (*Geographia*, IV, 6, 34). Their identification with the Canaries is implied by their longitude, and more importantly by the names of the islands: three of Ptolemy's names are obvious Greek correspondents of latin names given by Pliny: Canaria, Junonia and Pluvialia. The slight discrepancy between the number given by Ptolemy (six) and the actual number (seven) of the major islands of the archipelago can be explained in many ways. Almost all scholars agree with this identification; see, for instance, [Stückelberger and Graßhoff 2006, 455, footnote 200]. Nevertheless, a few scholars have questioned the identification with the Canaries, on the basis of the latitude given by Ptolemy, which is very far from that of the Canaries. We shall return to this point.

[28]For the identification of Sera Metropolis with Xi'an see [Stückelberger and Graßhoff 2006, 669, note 229].

are not unanimously identified today. Since the actual longitudes of Xi'an and the Canary Islands [29] are respectively $108°54'$E and $15°$W, and the corresponding values given by Ptolemy are $177°15'$ and $0°$, the ratio between their difference of longitude given by Ptolemy and the corresponding actual value is approximately 1.43, and thus very close to the regression coefficient given in [Russo 2013a] and [Shcheglov 2014].

Of course a detailed reconstruction of the procedure followed by Ptolemy is very difficult, and probably impossible, since it should also take into account the presence of local errors due to the different level of geographical knowledge for the various regions, and the complex interconnections between his data, already alluded to. Nevertheless, as we saw, some firm points could be established, the most important of which is a general dilation of all the differences of longitude by a mean factor equal to the dilation of the whole oikoumene. The implication A** is thus confirmed.

However, we still did not tackle the point on which our first criticism to A* was based, i.e, its failure to explain the origin of the double mistake, which is left by that hypothesis in the darkest obscurity. We will devote the next section to test the proposed thesis A** against this last problem.

5. The origin of the error on longitudes by Ptolemy

We conjectured that the value of 180° for the amplitude of the oikoumene, i.e., the difference in longitude between the Islands of the Blessed and the easternmost regions, was taken from a Hellenistic source (most probably Hipparchus, who may well have intended it as a rounded value). On the other hand we know that Hellenistic scientists, and in particular Hipparchus, had accepted the value of Eratosthenes for the Earth's circumference (Strabo, *Geographica*, I, iv, 1; II, v, 7; II, v, 34), while the smaller estimate, grossly wrong, was first introduced (as far as we know) by Posidonius (Strabo, *Geographica*, II, ii, 2). Why does this value of 180°, which according to Hipparchus was consistent with the measurement by Eratosthenes, coexist with a much "smaller" Earth in (among others) Ptolemy? Since there is substantial agreement in the distances along the circles of latitude between Ptolemy's and Eratosthenes' data, it is clear that the only possibility is that there was a misinterpretation on the identification of one of the two extrema of the oikoumene. Two arguments allow us to exclude that the issue involved locations in the far East:

[29] As the modern value for the Canary Islands we take the value of 15° W, which is the best rounded value for the average longitude.

1. It is a priori easier to misinterpret the identification of oceanic islands, for which no close locations are known for very large distances, than that of a city which is reachable through a series of intermediate locations.
2. To produce an error of the order of magnitude of that made by Ptolemy, the original location of some town in China assumed by his source as the eastern extremum of the oikoumene had to be very far into the Pacific Ocean.

The only remaining possibility, therefore, is that the two errors were originated by an erroneous identification of the Islands of the Blessed.

The previous reasoning, which up to now was intended at analyzing the origin of the errors in Ptolemy's *Geographia*, can also disclose new possibilities in a more general (and possibly more relevant) issue. We are indeed led to the conclusion that the Islands of the Blessed, to which the Hellenistic source of Ptolemy (most probably Hipparchus) referred, can be found approximately on the opposite semi-meridian with respect to the more eastern locations cited in the *Geographia*. This entails the need to consider longitudes that are much farther on the west than those believed as known in Hellenistic times. We arrived at this conclusion by elimination, after having considered and discarded every possible alternative. Therefore, in my opinion, we should seriously consider the extreme consequences of the previous reasoning. As a renowned investigator used to say, when you have eliminated the impossible, whatever remains, however improbable, must be the truth.

6. The Islands of the Blessed and the Caribbean

The elimination process illustrated before conduced us to the conclusion that the first cause for the errors made by Ptolemy was the wrong identification of the Islands of the Blessed with the Canary Islands. One may wonder how such a misinterpretation of the sources was possible. It is therefore useful, in my opinion, to briefly recall some historical processes which played a key role in this connection.

Mentions of travels in the Atlantic Ocean are not negligible in ancient sources.[30] However, between the Hellenistic age and Ptolemy, a significant loss of geographic knowledge concerning this Ocean occurred, and in a short time even well known descriptions of voyages started to be considered unreliable. The report of the famous expedition made by Pytheas, for instance, in which he described the iced sea and the midnight sun, was considered trustworthy by Hellenistic scientists such as Eratosthenes and Hipparchus,[31] but was later rejected by Strabo (*Geographica*, II, iv, 1; II, iv, 2; II, iii, 5), was not copied and transmitted anymore, and finally lost.

[30] For a review of classical sources on travels in the Atlantic Ocean see [Roller 2006].

[31] See, among other passages, Hipparchus, *In Arati et Eudoxi phaenomena commentariorum libri iii*, I, 4, 1; *Strabo, Geographica*, I, 1, 9; II, 1, 12; II, 4, 2.

The error made by Ptolemy according to our reconstruction was made possible by the combination of the loss of knowledge about the Atlantic Ocean with a general weakening of science in the Roman world with respect to the Hellenistic age. [32] Geography, in particular, was transformed radically, from the mathematically founded subject it was in Hellenistic times to the purely descriptive one it became in works like those by Strabo and Polybius. An error like the misidentification of an archipelago became clearly much more probable once the use of spherical coordinates (latitude and longitude) for the identification of the locations had been abandoned, as it was in the first century BC. The oldest evidence concerning the identification of the Islands of the Blessed with the Canaries dates indeed from the first century BC, [33] and then was inherited by Ptolemy when he attempted at reconstructing quantitatively the mathematical geography for the first time after the methodological crisis had occurred. Posidonius seems the most probable candidate as the source of the misidentification. He is, to our knowledge, the oldest source giving the length of 500 stadia per degree for the Earth's circumference, which tends to suggest that the transition from the "old" to the "new" value occurred in his work.

A quantitative analysis of the data given by Ptolemy strongly supports the idea of a mistaken identification of the archipelago. Indeed:

1. The latitude he gives for the Islands is wrong by about 15°, a huge error which can be regarded as incompatible with any data coming from real measurements and can be only explained as the result of a confusion between two different archipelagos.
2. The Canaries are spread over a total longitude of about four and a half degrees, while the archipelago considered by Ptolemy covers just one degree in west-east direction; moreover, the Canary Islands cover less than two degrees in north-south direction, while Ptolemy's Islands five and a half. In conclusion, both archipelagos have a strip-like shape, but they are approximately oriented in orthogonal directions.

A decisive test in order to check the proposed thesis is at this point possible. We can indeed compute the original longitude of the Islands of the Blessed simply taking a difference of longitude of 180° with respect of Ptolemy's eastern extreme locations, and verify whether we come close to some archipelago. The longitude of the eastern bound of Ptolemy's oikoumene can be computed by the regression line (1) given on page 183, solving the equation

$$180 = 1.428x + 17.06.$$

[32] On this point see [Russo 2004, passim].

[33] See note 27 above.

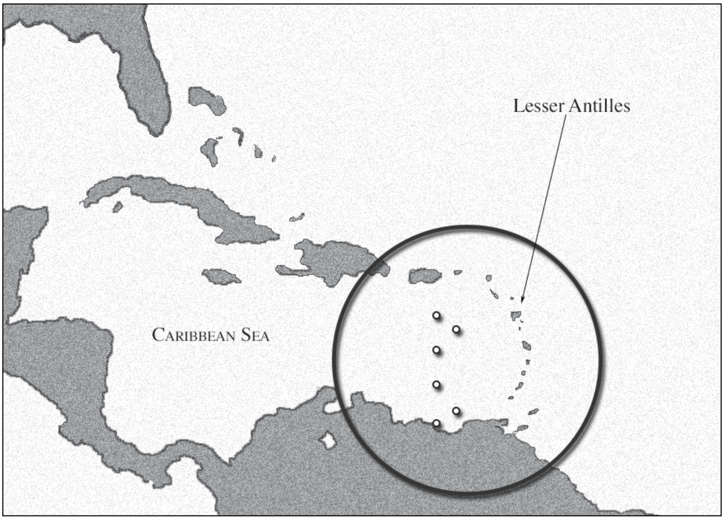

Figure 1. White dots indicate the original locations of the Islands of the Blessed as computed in the text.

In this way, we eventually get the following coordinates for the Islands:

$$65°54'\text{W}, 16°\text{N}; \quad 64°54'\text{W}, 15°15'\text{N}; \quad 65°54'\text{W}, 14°15'\text{N};$$
$$65°54'\text{W}, 12°30'\text{N}; \quad 64°54'\text{W}, 11°\text{N}; \quad 65°54'\text{W}, 10°30'\text{N}.$$

These locations are marked by circles in Figure 1.

As the reader can see, not only do we come very close to an actual archipelago (formed by the Leeward Islands and the Windward Islands, in the Lesser Antilles), but also its dimension, shape and orientation correspond well to the dots. If we compute the eastern extreme using the regression line (2), given in [Shcheglov 2014], we obtain an even slightly better match with the Leeward and Windward Islands, since all the points move east by about 0.44 degrees.

The available descriptions of the Islands of the Blessed in the sources can provide further arguments in support of this proposed (and admittedly surprising) identification. In this regard, one can observe that there is a substantial consistency between the numerous descriptions we have dating from the archaic and classical periods, and that these descriptions are hardly reconcilable with the identification with the Canaries. The issue was examined in detail in [Manfredi 1993]; in his opinion, some striking characteristics mentioned frequently in the sources (and especially the presence of lush and evergreen foliage) can exclude the possibility that they referred to the Canaries, which are generally bleak, and would rather suggest islands in the Caribbean sea [Manfredi 1993, p. 204].

In the following section a further independent and quantitative confirmation of the proposed thesis will be provided.

7. A passage from Pliny

The following passage by Pliny has eluded, until now, any reasonable interpretation:

> Regarding the length and the breadth this is what I deem worthy of mention. For the whole circumference, Eratosthenes, a man highly regarded by all and surpassing others in every subtlety of learning, and especially in the present matter, gave the value of 252,000 stadia. [...] Hipparchus, a man to be admired for taking issue with him and for much more besides, then added to that number a little less than 26,000 stadia.[34]

The last statement by Pliny is contradicted by passages in Strabo to the effect that Hipparchus accepted the value given by Eratosthenes for the Earth's circumference (*Geographica*, I, iv, 1; II, v, 7; II, v, 34). The key to understanding the passage above, in my opinion, lies in its first words. Just before that passage Pliny's topic was the dimensions of the oikoumene. Since it makes no sense to talk of the length and the breadth of a spherical Earth, it is therefore very plausible that Pliny's source was still discussing the oikoumene when talking about the 26,000 stadia, and that Pliny misunderstood the reference as being about the earth's circumference because that was Eratosthenes' most celebrated measurement. The length and breadth of the oikoumene were typically discussed in geographical works, and we know from Strabo (*Geographica*, I, iv, 5) that Eratosthenes did calculate the length of the oikoumene along the parallel of Athens, getting the result of 77,800 stadia, of which 5,000 to the west of the Pillars of Heracles. Pliny's passage can therefore be explained if we conjecture that Hipparchus extended the oikoumene in longitude by adding just under 26,000 stadia to the value given by Eratosthenes. Since this extension could hardly have concerned the Pacific Ocean, we must conclude that the western boundary of the oikoumene according to Hipparchus was at 31,000 (26,000 + 5000) stadia from the Pillars of Heracles along the parallel of Athens. Since the cosine of the latitude of Athens is about 0.788, a degree of longitude along this parallel, according to Eratosthenes, corresponds to about $700 \times 0.788 = 552$ stadia, which yields a location approximately $56°10'$ the west of Gibraltar, corresponding to a longitude of $61°31'$W. The corresponding semi-meridian is shown in

[34] De longitude ac latitude haec sunt, quae digna memoratu putem. Universum autem circuitum Eratosthenes, in omnium quidem literarum subtilitate, et in hac utique praeter ceteros sollers, quem cunctis probari video, CCLII milium stadiorium prodidit, [...] Hipparchus, et in coarguendo eo, et in reliqua omni diligentia mirus, adicit stadiorum paulo minus XXVI milia (Pliny, *Naturalis Historia*, II, 247).

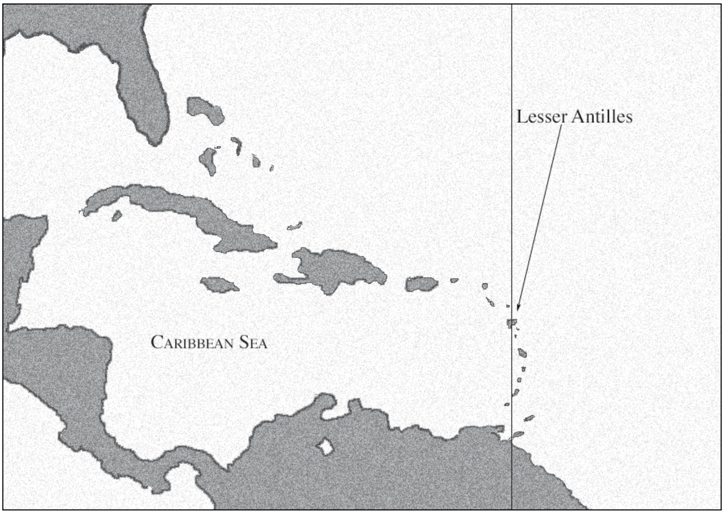

Figure 2. The westernmost longitude of the oikoumene according to Hipparchus, as recovered from Pliny's passage.

Figure 2, and represents a striking confirmation of the thesis that Hipparchus knew the coordinates of the Leeward and Windward Islands, and also of the correctness of our reconstruction of the meaning of the passage by Pliny. Moreover, it indirectly supports the idea that the source of Ptolemy on the Islands of the Blessed was Hipparchus. We notice that it is not surprising that the match between theoretical previsions and actual geographical data is much better in Figure 2 than in Figure 1, since the reconstruction underlying Figure 1 had a statistical basis and relied on the value of 180° for the amplitude of the oikoumene, which was possibly rounded off, while in the case of Figure 2 the method only uses one quantitative datum taken from the sources.

Of course, the addition of 26,000 stadia by Hipparchus should be considered not as a correction to the value given by Eratosthenes, but rather an update due to new geographical discoveries.

8. Conclusions

We want here to summarize the conclusions of the present paper and the arguments that can be considered in their support.

The main thesis can be expressed as follows:

> *Some source of Ptolemy's Geographia knew with remarkable precision the position of some locations in the Caribbean Sea, i.e. the Leeward and Windward Islands. These islands were the locations originally intended*

as the "Islands of the Blessed". Their misidentification with the Canaries first occurred in the first century BC and was then accepted by Ptolemy.

This thesis, which is plausible in view of the loss of knowledge concerning the Atlantic Ocean and the lapse into disuse of spherical coordinates between Hipparchus and Ptolemy, is justified by abductive inference based on its ability to explain a number of facts for which so far no explanation had been offered:

1. The thesis is implied by a procedure performed by Ptolemy which is suggested by his own words and in turn explains both (i) the very high values for the determination coefficient (and the virtually identical regression coefficients) found in [Russo 2013a] and [Shcheglov 2014], and (ii) the significant differences between regression coefficients relative to different regions shown in [Shcheglov 2014].

2. It explains why the archaic and classical descriptions of the Islands of the Blessed match better with Caribbean islands rather than the Canaries (as discussed in [Manfredi 1993]).

3. It explains why Ptolemy's coordinates for the Canary Islands describe an elongated archipelago whose orientation is basically orthogonal to the actual one — in other words, as wrong as it could be).

4. It explains the huge error of 15° made by Ptolemy in the latitude of the Canary Islands.

5. It provides a simple, and quantitatively accurate, explanation of the systematic dilation of differences of longitudes operated by Ptolemy.

6. It explains as a simple consequence of the previous point the new estimate in the measure of the Earth accepted by Posidonius and Ptolemy — even more so because the ratio between the old and the new estimates is close to, but not identical with, the dilation coefficient.

7. It explains the striking match between the shape and the position of the Leeward and Windward Islands and the locations indicated by Ptolemy as the "Islands of the Blessed", once their original coordinates are reconstructed by means of the statistical approach herein considered.

8. It provides a simple (and quantitatively accurate) explanation for the passage of Pliny discussed above.

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