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HENRY ABEL DYE

V. S. VARADARAJAN

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HENRY ABEL DYE

HENRY ABEL DYE

(1926–1986)

The untimely and unexpected passing away of Henry Dye on November 26, 1986 was a great blow to his family, friends, and the large circle of mathematicians who had always regarded him as a major figure in the development of Functional Analysis in the Pacific Basin. It became clear almost immediately following his death that the Pacific Journal of Mathematics should bring out an issue dedicated to his memory and consisting of research articles by leading mathematicians on subjects closely related to his interests and accomplishments. The Board of Governors of the Pacific Journal agreed unanimously and enthusiastically with this sentiment and authorized the publication of such a Memorial issue. The response to our solicitations for articles has exceeded all our projections with the result that we are bringing out a full volume (2 issues).

Henry Abel Dye was born on February 14, 1926, in Dunkirk, New York. He got his Ph.D. in Mathematics from the University of Chicago in 1950. After spending a few years with the California Institute of Technology, The University of Southern California, and the University of Iowa, he joined the University of California at Los Angeles in 1960 as a Professor, and he remained there until his death. During the 50's and early 60's he was a visible and influential personality in the mathematical scene of Southern California. As he became older, his advice and services were increasingly sought after, and he held, with great distinction, a number of important positions in the University, including the Chairmanship of the Department of Mathematics at UCLA (1975–1978), and membership of the Council on Academic Personnel (1982–1985). He was a Senior Visiting Fellow of the Science Research Council of the United Kingdom at Warwick, England, in 1981.

As was the case with many functional analysts of his generation, Henry Dye's scientific work had its origins in the epoch making theory of operator algebras created by Murray and von Neumann. But he was no slavish imitator; he brought to everything he did a certain originality and insight that were characteristically his own. He was one of the first to study in depth many important aspects of noncom-

mutative integration, and his noncommutative version of the Radon-Nikodym Theorem (Trans. Amer. Math. Soc. **72** (1952), pp. 243–280) is certainly one of his more beautiful achievements. Later on he explored the relationship between operator algebras and ergodic theory in two seminal articles (Amer. Jour. of Math. **81** (1959), pp. 119–159; **85** (1963), pp. 551–576) that were far ahead of their time. His uncompromisingly high standards did not allow him to publish much; however not many mathematicians have exerted as much influence through their work as he did, publishing only as much as he did.

It is clearly appropriate to discuss the two ergodic theory papers of Dye in some detail. One of the most striking early results of Murray and von Neumann implied that all finite measure preserving actions of countable abelian groups give rise to isomorphic operator algebras. It was Henry Dye who discovered that this isomorphism had a more fundamental and geometric origin, and was actually induced by an isomorphism of the underlying measure spaces that carried orbits to orbits. More precisely, in the first paper he proved that if S and T are measure preserving ergodic transformations of probability spaces X and Y , there is a measure preserving isomorphism of X with Y that takes the S -orbits into T -orbits and vice versa. In the second paper he proved that if G is a countable abelian group acting in a measure preserving and ergodic manner on a probability space, there is such an action of \mathbf{Z} on the same space that has the same orbits. It took more than ten years before these results of Dye and his notion of *orbit* (we may say *Dye*) *equivalence* of group actions were properly understood and generalized. Among the many later developments are the following: (1) Krieger's classification of non singular (rather than measure preserving) ergodic \mathbf{Z} -actions upto Dye equivalence; this played an important role in the Connes-Haagerup classification of hyperfinite factors. Moreover Krieger's work exhibited to ergodic theorists the real power and scope of Dye's ideas which provided the first systematic tools for understanding nonsingular transformations. (2) Zimmer's rigidity result asserting that for certain groups, $\mathrm{SL}(3, \mathbf{Z})$ for example, Dye equivalence implies conjugacy upto an automorphism of the group, (3) the Connes-Feldman-Ornstein-Weiss result that any ergodic action of a countable amenable group by nonsingular transformations is Dye equivalent to such a \mathbf{Z} -action. There are not many papers in ergodic theory that have had the impact that these papers of Dye had.

In spite of increasing demands on his time Henry kept up with his interests. His high standards did not allow him to come to terms with the inevitable slowing down of the creative process that happens to

all. I do think however that the attention his ergodic theoretic work received was very gratifying to him.

I cannot do better than end this short appreciation with the words that Klaus Schmidt spoke on the occasion of the Memorial service at UCLA. "He was a man of great humour and generosity, and he never allowed the high standards he set for himself as a person and mathematician to interfere with his genuine warmth and kindness towards others. With himself he was much less kind: although I cannot claim to have known him all that well I always had the feeling that he drove himself quite relentlessly, and that he could never forgive himself for not having been able to prove some of the later and quite spectacular results in the subject. On those rare occasions, when he allowed himself to relax completely, he was wonderful company. I still remember with great fondness Genie's and Henry's visit to Warwick and Henry's epic battles with the British plumbing system, or a wonderful day in 1983 when Genie and Henry showed me the Californian poppies in the desert. Henry was a nice man, and I shall miss both him and his mathematics."

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