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EXTENSIONS BETWEEN IRREDUCIBLE REPRESENTATIONS OF A P-ADIC $GL(n)$

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To the memory of Olga Taussky-Todd

Let H be the group of points of a connected reductive group over a local non archimedean field F. Let ω be a character of the center of H. Let $\mathcal{C} := \mathrm{Mod}_\omega H$ be the category of complex representations of H which are smooth (the stabilizer of a vector is an open subgroup of H), with central character ω. It is known that $\mathcal{C}$ has enough injectives and projectives, and we can define $\mathrm{Ext}^i_{\mathcal{C}}(V, V')$ for two representations $V, V' \in \mathcal{C}$, using a projective resolution $(P^i)_{i\geq 0}$ of V, or an injective resolution $(I^i)_{i\geq 0}$ of V'. The cohomology of the complex $\mathrm{Hom}_{\mathcal{C}}(P^i, V')$ and of the complex $\mathrm{Hom}_{\mathcal{C}}(V, I^i)$ are the same, and are equal to $\mathrm{Ext}^i_{\mathcal{C}}(V, V')$ by definition.

Question. Let $V, V' \in \mathcal{C}$ irreducible, with V essentially square integrable (essentially because of the center), and V' essentially tempered. Is is true that

$$\mathrm{Ext}^i_{\mathcal{C}}(V, V') = \mathrm{Ext}^i_{\mathcal{C}}(V', V) = 0$$

for all integers $i > 0$?

This question is motivated by the orthogonal decomposition of the Schwartz algebra of H given by the Plancherel formula ([**Sil**, Th.3, page 4679] for example). I tried to prove without success that the answer was yes, some years ago while writing [**Vig1**]. The answer (yes) is an exercise for $GL(n, F)$ for any integer $n > 1$.

It can be worth to publish it.

Let $H = G := GL(n, F)$. Let $V \in \mathcal{C}$ irreducible essentially square integrable. We can describe all the irreducible $V' \in \mathcal{C}$ such that $\mathrm{Ext}^i_{\mathcal{C}}(V', V) \neq 0$ for at least one integer $i \geq 0$. For such a V', there is a unique i such that $\mathrm{Ext}^i_{\mathcal{C}}(V', V) \simeq \mathbf{C}$, and is zero otherwise. If $V' \not\simeq V$, then V' does not have a Whittaker model. An irreducible essentially tempered representation has a Whittaker model. For all irreducible tempered representation V' not isomorphic to V, we get $\mathrm{Ext}^*_{\mathcal{C}}(V', V) = 0$. Using duality, we get $\mathrm{Ext}^*_{\mathcal{C}}(V, V') = 0$.

The computation of $\mathrm{Ext}^*_{\mathcal{C}}(V', V)$ for V irreducible essentially square integrable and V' irreducible, is a corollary of the classification of square integrable representations by Zelevinski, the theory of simple types by Bushnell and Kutzko, the Zelevinski involution by Aubert, Schneider and Stuhler, the computation of $\mathrm{Ext}^*_{\mathcal{C}}(1, V')$ by Casselman.

We give a very short proof of $\mathrm{Ext}^*_{\mathcal{C}}(V, V') = 0$ for $V,\ V' \in \mathcal{C}$, irreducible tempered and not isomorphic, suggested by Waldspurger. The group G has the particularity to have at most one irreducible tempered representation with a given infinitesimal character (i.e. cuspidal support), and $\mathrm{Ext}^*_{\mathcal{C}}(V, V') = 0$ for two irreducible representations V, V' of G having different infinitesimal characters. This second fact is very general, and uses the interpretation by Yoneda of $\mathrm{Ext}^n_{\mathcal{C}}(V, V')$ by n-extensions, as in the real case.

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1. We set $G := GL(n, F)$ and $\mathcal{C} = \mathrm{Mod}\, G$ (we do not fix the central character). From Bernstein [**Z**, 9.3], any $V \in \mathcal{C}$ irreducible essentially square integrable is a *Steinberg representation* $St_k(\rho)$ where ρ is an irreducible cuspidal representation of $GL(r, F)$ for some integer $r > 0$, and $rk = n$. The Steinberg representation $St_k(\rho)$ is the unique irreducible subquotient with a Whittaker model in the natural representation of G in the space of locally constant functions $f : G \to \otimes^k \rho$ such that $f(mug) = \otimes^k \rho(m) f(g)$ for any $g \in G$ and any element mu $(m \in M,\ u \in U)$, in a parabolic subgroup of G with Levi component M isomorphic to $GL(r, F)^k$, and unipotent radical U. When $r = 1$ and $\rho = 1$ is the trivial character of F^*, $St_n(1) = St$ is the usual Steinberg representation.

A *block* in the abelian category $\mathcal{C}$ is an indecomposable abelian subcategory which is a direct factor. There are no non trivial homomorphisms between two different blocks. The blocks are classified by the semi-simple types of Bushnell-Kutzko [**BK2**, **BK3**], and also by the irreducible cuspidal representations of Levi subgroups modulo G-conjugation, and twist by unramified characters [**BD**].

The *semi-simple type* of a block is a distinguished irreducible representation σ of a distinguished open compact subgroup K of G, such that the functor

$$F_\sigma : V \to \mathrm{Hom}_G(\mathrm{ind}_{G,K}\, \sigma, V)$$

is *an equivalence of categories between the block and the category of right* $\mathrm{End}_G\, \mathrm{ind}_{G,K}\, \sigma$*-modules.*

Let I be an Iwahori subgroup (unique modulo G-conjugation). A representation $V \in \mathcal{C}$ generated by the I-invariant vectors V^I, is called *unipotent*. The unipotent representations form a block, of semisimple type the trivial representation of I. Set $F_I = \mathrm{Hom}_G(\mathrm{ind}_{G,I}\, 1, -)$.

Let (e, f, d), $efd = r$, be the *invariants* of ρ [**Vig1**, III.5]. Let q be the order of the residual field of F. Let F' be any local non archimedean field, with residual field of order $q' = q^{fd}$. We set $G' = GL(k, F')$ and $\mathcal{C}' = \mathrm{Mod}\, G'$. Denote I' an Iwahori subgroup of G'.

Bushnell and Kutzko [**BK1**, 7.6.18] have shown that there is a natural algebra isomorphism [**BK1**, 7.6.18, 7.6.21]

$$i : \mathrm{End}_{G'} \mathrm{ind}_{G',I'} 1 \to \mathrm{End}_G \mathrm{ind}_{G,K} \sigma.$$

We get a functor Φ which is an equivalence of categories, from the *unipotent block* in $\mathcal{C}'$ to the block in $\mathcal{C}$ containing $St_k(\rho)$ such that

$$i^* \circ F_\sigma \circ \Phi' = \mathrm{Hom}_{G'}(\mathrm{ind}_{G',I'} 1, -).$$

For any Levi subgroup M' of G', there is a similar functor Φ' which is an equivalence from the unipotent block of M' to a block in a Levi subgroup M of G. This is compatible with the normalized parabolic induction $i_{G',M'}$ and $i_{G,M}$, or restriction $r_{M',G'}$ and $r_{M,G}$, along $Q' = M'Q'_o$ and $Q = MQ_o$, where Q'_o and Q_o are suitable Borel subgroups of G' and G:

$$\Phi \circ i_{G',M'} = i_{G,M} \circ \Phi', \qquad \Phi' \circ r_{M',G'} = r_{M,G} \circ \Phi.$$

This is a consequence of [**BK1**, 7.6.21].

Proposition. *The functor Φ sends an essentially square integrable (resp. unitary, having a Whittaker model, essentially tempered) irreducible unipotent representation of G' to an essentially square integrable (resp. unitary, having a Whittaker model, essentially tempered) irreducible representation of G.*

For essentially square integrable see [**BK1**, 7.7]. For unitary see [**BK1**, 7.6.25]. The irreducible representations of G with a Whittaker model are induced from essentially square integrable representations of Levi subgroups [**Z**, 9.11]. The assertion for the Whittaker model follows from this and the compatibility of Φ', Φ with the induction. The tempered irreducible representations of G are induced from square integrable representations [**Sil**, 4.5.11]. Hence the assertion for essentially tempered representations.

2. We want to prove a vanishing result for Ext^1, between characters of affine Hecke algebras, directly and in an elementary way. In fact, the best method to compute Ext^* between modules for affine Hecke algebras, is to use the dictionnary with representations. This paragraph could be skipped.

The Hecke algebra $\mathrm{End}_G \mathrm{ind}_{G,I} 1$ is naturally isomorphic to the *affine Hecke algebra* $H_{\mathbf{C}}(n,q)$ *of type* A_{n-1} *and parameter* q [**BK1**, 5.6.6].

The Hecke **C**-algebra $H^o_{\mathbf{C}}(n,x)$ of type A_{n-1} with parameter $x \in \mathbf{C}^*$, $x \neq 0,1$, is the **C**-algebra generated by $(s_1,\dots,s_{n-1})$ with the relations
$(s_i+1)(s_i-x)=0\ (1\le i\le n-1)$,
$s_is_j=s_js_i\ (1\le i,j\le n-1,\ |j-i|\neq 1)$
$s_is_{i+1}s_i=s_{i+1}s_is_{i+1}\ (1\le i\le n-2)$.

The affine Hecke **C**-algebra $H_{\mathbf{C}}(n,x)$ of type A_{n-1} with parameter x is generated by $H^o_{\mathbf{C}}(n,x)$ and t with
$tt^{-1}=t^{-1}t=1,\ ts_i=s_{i-1}t\ (1<i<n),\ t^2s_1=s_{n-1}t^2$.
Note that this description [**BK**, 5.4, page 177] is not the Bernstein description.

The finite algebra $H^o_{\mathbf{C}}(n,x)$ is isomorphic to the group algebra $\mathbf{C}[S_n]$ of the symmetric group S_n and has two characters. For the character sign, the image of all the s_i is -1. For the trivial character, the image of all the s_i is x. The two characters extend to characters of $H_{\mathbf{C}}(n,x)$, the image of t beeing an arbitrary non zero complex element. The $H_{\mathbf{C}}(k,q)$-module $F(St)$ is a sign character of $H_{\mathbf{C}}(n,q)$.

The center of G is naturally identified with F^* diagonally enbedded in G. The center of $H_{\mathbf{C}}(n,x)$ contains t^n. The central character of St is trivial. The category of unipotent representations of G with *trivial central character* is isomorphic by the functor F_I defined in (1) to the category $\mathrm{Mod}\, H_{\mathbf{C}}(n,q)_1$ of right modules of the quotient $H_{\mathbf{C}}(n,q)_1$ of $H_{\mathbf{C}}(n,q)$ by the two-sided ideal generated by t^n-1 [**Vig2**, I.3.14].

Lemma 2.1. *Let* $\chi,\chi' \in \mathcal{C} := \mathrm{Mod}\, H_{\mathbf{C}}(n,q)_1$ *be two characters. Then* $\mathrm{Ext}^1_{\mathcal{C}}(\chi,\chi')=0$.

Indeed the algebras $H^o_{\mathbf{C}}(n,q)$ and $\mathbf{C}[t], t^n=1$, are semisimple (but the quotient $H_{\mathbf{C}}(n,q)_1$ is not semisimple). If $V\in\mathcal{C}$ is an extension of χ by χ, then $hv=\chi(h)v$ for all $h\in H^o_{\mathbf{C}}(n,q),\ v\in V$, and t acting semisimply, $V\simeq\chi\oplus\chi$.

There is another proof when $n=2$ in [**DPrasad**, p. 175, proof of the Lemma 7]. Note that if we were not fixing the center, we could have extensions. I do not know how to compute directly Ext^i when $i>1$.

When V is an extension of two different characters $\chi'\neq\chi$ in $\mathrm{Mod}\, H_{\mathbf{C}}(n,q)_1$ or in $\mathrm{Mod}\, H_{\mathbf{C}}(n,q)$, one sees that $V\simeq\chi\oplus\chi'$ by restriction to the commuting algebras $H^o_{\mathbf{C}}(n,q)$ and $\mathbf{C}[t]$.

2.2. From (2.1), $\mathrm{Ext}^1(St,St)=0$ in $\mathrm{Mod}_1 G$. Any irreducible square integrable unipotent representation V of G is the twist $\mathrm{St}\otimes\chi_x$ of St by an unramified character of G

$$\chi_x(g) = x^{\mathrm{val}\det g}, \quad g \in G,$$

for some $x \in \mathbf{C}^*$, where val : $(F)^* \to \mathbf{Z}$ is the valuation of F, sending an uniformizing parameter to 1. The central character of $\mathrm{St} \otimes \chi_x$ is the character χ_{kx} of F^*. It is trivial if and only if $\mathrm{St} \otimes \chi_x \simeq \mathrm{St}$.

The twist by a character χ of G does not change the value of Ext^*. If $V, V' \in \mathcal{C} := \mathrm{Mod}_\omega G$, then $V \otimes \chi, V' \otimes \chi \in \mathcal{C}_\chi := \mathrm{Mod}_{\omega\omega(\chi)} G$ where $\omega(\chi)$ is the restriction of χ to the center of G. We have:

$$\mathrm{Ext}^*_{\mathcal{C}}(V, V') \simeq \mathrm{Ext}^*_{\mathcal{C}_\chi}(V \otimes \chi, V' \otimes \chi).$$

Using the functor F_I of (1), we get:

Proposition. *Let V, V' irreducible and essentially square integrable in the category $\mathcal{C} := \mathrm{Mod}_\omega G$. Then*

$$\mathrm{Ext}^1_{\mathcal{C}}(V, V') = 0.$$

There is another proof due to Silberger of this result, valid for a general reductive group [**Sil2**]. To compute some $\mathrm{Ext}^i_{\mathcal{C}}(V, V')$ when $i > 1$, we use the results of Casselman [**Cas**].

3. Let H as in the introduction. Let $\mathcal{C} := \mathrm{Mod}_1 H$ be the category of representations of H with trivial character. Denote by $St_Q \in \mathcal{C}$ the *Steinberg representation* defined by a parabolic subgroup Q of H [**BW**, 4.6, page 308]. If τ_Q is the natural representation of H on the complex space of locally constant left Q-invariant functions $H \to \mathbf{C}$, then St_Q is the quotient of τ_Q by the subrepresentation generated by the natural images of $\tau_{Q'}$ in τ_Q, for all parabolic subgroups Q' of H which contain Q. We have $St_H = 1$. When $Q = Q_o$ is minimal, then $St_{Q_o} = St$ is the usual Steinberg representation. The representations St_Q are irreducible and not isomorphic.

The *parabolic rank* of Q is the rank of a maximal split torus in the center of a Levi component of Q. We denote

$$m_Q = \text{parabolic rank of } \; Q - \text{parabolic rank of } \; H.$$

This an integer ≥ 0.

Theorem ([**BW**, 5.1, Th.4.12, page 313])**.** *Let $V \in \mathcal{C} := \mathrm{Mod}_1 H$ irreducible such that $\mathrm{Ext}^*(1, V) \neq 0$. Then there exists a parabolic subgroup Q*

of H such that $V \simeq St_Q$. Moreover $\mathrm{Ext}^m_{\mathcal{C}}(1, St_Q) \simeq \mathbf{C}$ *if* $m = m_Q$, *and is zero otherwise.*

Remark. Suppose $H = G := GL(n, F)$, and $\mathcal{C} := \mathrm{Mod}_1 G$.

We have $\mathrm{Ext}^o_{\mathcal{C}}(1, 1) \simeq \mathbf{C}$ and $\mathrm{Ext}^m_{\mathcal{C}}(1, 1) = 0$ for any integer $m \geq 1$.

The representation $\tau_Q \in \mathcal{C}$ has a unique irreducible subquotient with a Whittaker model, this unique subquotient is isomorphic to St [**Z**, 9.7]. In particular, when $Q \neq Q_o$ the representation St_Q does not have a Whittaker model. Hence $\mathrm{Ext}^*_{\mathcal{C}}(1, V) = 0$ for any irreducible representation $V \neq \mathrm{St}$ with a Whittaker model.

4. Zelevinski involution. Let G as in (1). The Zelevinski involution τ in $\mathrm{Mod}\, G$ has the following properties :

a) τ respects the property of beeing irreducible [**A**, 2.3, 2.9].

b) τ exchanges the trivial and the usual Steinberg representation [**Z**, 9.2].

c) $\tau(- \otimes \chi) = \tau(-) \otimes \chi$ commutes with the twist by a character χ of G [**Z**, 9.1].

d) τ respects the cuspidal support [**Z**, 9.1].

e) τ is an exact contravariant functor and respects the cuspidal support [**SS**, 3.1], hence respects the representations with a given central character.

Set $\mathcal{C} := \mathrm{Mod}\, G$ or $\mathcal{C} := \mathrm{Mod}_\omega G$, where ω is a character of the center of G. By e) we have for any $V, V' \in \mathcal{C}$

$$\mathrm{Ext}^*_{\mathcal{C}}(V, V') \simeq \mathrm{Ext}^*_{\mathcal{C}}(\tau(V'), \tau(V)).$$

With the notations of (3), the representation $\tau(St_Q)$ is not isomorphic to St when $Q \neq G$ by b), and is a subquotient of τ_{Q_o} by d). Hence $\tau(St_Q)$ *does not have a Whittaker model when* $Q \neq G$, in particular is not essentially tempered. We deduce from (3):

Theorem. *Let $V, V' \in \mathcal{C} := \mathrm{Mod}_w G$, irreducible, such that $V \simeq St \otimes \chi$ is unipotent and essentially square integrable as in* 2), *and* $\mathrm{Ext}^*_{\mathcal{C}}(V', V) \neq 0$. *Then there exists a parabolic subgroup Q of G' such that $V' \simeq \tau(St'_Q) \otimes \chi$. For $V' = \tau(St_Q) \otimes \chi$, we have* $\mathrm{Ext}^m_{\mathcal{C}}(V', V) \simeq \mathbf{C}^*$ *if $m = m_Q$ as in* 3), *and zero otherwise.*

In particular, if V is a unipotent Steinberg representation, and if $V' \not\simeq V$ is essentially tempered, then

$$\mathrm{Ext}^o_{\mathcal{C}}(V, V) \simeq \mathbf{C}, \quad \mathrm{Ext}^i_{\mathcal{C}}(V, V) = \mathrm{Ext}^i_{\mathcal{C}}(V', V) = 0$$

for all integers $i > 0$. We will prove also

$$\mathrm{Ext}^i_{\mathcal{C}}(V, V') = 0 \tag{4.1}$$

using duality as follows.

5. Duality. Let (H, ω) as in the introduction. The contragredient $V \to V^*$ is a contravariant exact functor in $\operatorname{Mod} H$, which sends a projective representation to an injective representation [**Vig2**, I.4.18]. A representation V is called *admissible* when $V^{**} \simeq V$. When V is admissible, and $(P_i) \to V$ is a projective resolution of V, then $V^* \to (P_i^*)$ is an injective resolution of V^*, and $\operatorname{Hom}(P_i, W) \simeq \operatorname{Hom}(W^*, P_i^*)$ canonically [**Vig2**, I4.13]. If $V \in \operatorname{Mod}_\omega H$, then $V^* \in \operatorname{Mod}_{\omega^{-1}} H$. Set $\mathcal{C} := \mathcal{C}^* := \operatorname{Mod} H$ or $\mathcal{C} := \operatorname{Mod}_\omega H$, $\mathcal{C}^* := \operatorname{Mod}_{\omega^{-1}} H$.

Proposition. *Let $V, W \in \mathcal{C}$ admissible of contragredient $V^*, W^* \in \mathcal{C}^*$, one has* $\operatorname{Ext}^*_{\mathcal{C}}(V, W) \simeq \operatorname{Ext}^*_{\mathcal{C}^*}(W^*, V^*)$.

The contragredient respects the property of being essentially square integrable and of being essentially tempered. We deduce (4.1). Hence the answer to the question in the introduction is yes, for $G = GL(n, F)$. There is another proof, suggested by Waldspurger, using that the essentially tempered irreducible representations of G have different cuspidal support. This comes from the classification of Zelevinki [**Z**], which shows that tempered irreducible representations are not degenerate (1), and that not degenerate irreducible representations have different cuspidal support.

6. Let $(H, w), \mathcal{C}$ as in (5). There is a natural equivalence between the two bifunctors on $\mathcal{C}$,

$$\operatorname{Ext}^n_{\mathcal{C}}(A, B) \quad \text{and} \quad \operatorname{Yext}^n_{\mathcal{C}}(A, B)$$

given by the Yoneda n-extensions of A by B modulo an equivalence relation $\equiv$. The proofs are the same than in the category of (left) modules for a ring [**M**, III.6.4, III.8.2].

An n-extension X of A by B is an exact sequence starting at B and ending at A,

$$X \ : \ 0 \to B \to X_n \to \ldots \to X_1 \to A \to 0.$$

A morphism $\gamma : X \to Y$ between two n-extensions starting with β and ending with α is a commutative diagram

$$\begin{array}{llllllll} X: 0 \to & B & \to X_n \to \ldots & \to X_1 \to & A & \to 0 & \\ \downarrow \gamma & \downarrow \beta & \downarrow & \downarrow & \downarrow \alpha & & . \\ Y: 0 \to & D & \to Y_n \to \ldots & \to Y_1 \to & C & \to 0 & \end{array}$$

The equivalence relation $\equiv$ in the set of n-extensions of A by B, is generated by the relation: There exists a morphism $\gamma : X \to Y$ starting and ending with the identity.

An n-extension X ending at A can be spliced with an m-extension Y starting at A, to give an $n+m$-extension $X \circ Y$ starting like X, ending like Y. If $\alpha : A' \to A$, one defines by pull-back an extension $X\alpha$ starting like X, ending at A'. If Z is an m-extension starting by A', one defines by push-out an m-extension αZ starting at A, ending like Z. By definition of the equivalence relation, one has

$$X\alpha \circ Z \equiv X \circ \alpha Z.$$

A morphism $\gamma : X \to Y$ starting with β and ending with α gives an equivalence [**M**, III.5.1]

$$\beta X \equiv Y\alpha.$$

An element z of the center of $\mathcal{C}$ defines an endomorphism of X. If z acts on A and on B by multiplication by two different scalars $z_a \neq z_B \in R$, we deduce that the image of X in $\mathrm{Yext}^n(A,B) \simeq \mathrm{Ext}^n(A,B)$ is 0.

For $A, B \in \mathcal{C}$ irreducible of different cuspidal support, there is an element z in the center of $\mathcal{C}$ which acts by the identity on A and is zero on B'. This comes from the description of the center by Bernstein [**BD**]. We get the following theorem.

Theorem 6.1. *Let $V, V' \in \mathcal{C}$ irreducible of different cuspidal support. Then* $\mathrm{Ext}^*_{\mathcal{C}}(V, V') = 0$.

Corollary 6.2. *Suppose that $H = GL(n,F)$. Let $V, V' \in \mathcal{C}$ irreducible not degenerate, and $V \neq V'$. Then* $\mathrm{Ext}^*_{\mathcal{C}}(V, V') = 0$.

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