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PREFACE

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SPECIAL ISSUE ON

THE MATHEMATICAL THEORY OF

CAPILLARITY

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PREFACE

The present issue of the Pacific Journal consists of invited research articles on mathematical problems of capillarity.

A *capillary surface* is the interface separating two fluids that lie adjacent to each other and do not mix. In conjunction with boundary conditions imposed by rigid "supporting walls", such interfaces can exhibit remarkable geometric properties and seemingly strange behavior, occasionally confounding intuition. The earliest known writing on the topic, due to Aristoteles, comains basic misconceptions that apparently went unchallenged for almost 2000 years, when Galileo addressed them in his *Discorsi*. Quantitative progress had to await the later discovery of the Calculus. The characterization of rise height in a circular cylindrical glass "capillary tube" dipped into a reservoir of liquid became a major scientific challenge of the eighteenth century, and was not achieved during that period. Initial breakthroughs came in 1805 and 1806 with insights of Thomas Young and Pierre-Simon de Laplace. Young professed to scorn the mathematical method but nevertheless introduced the mathematical concept of mean curvature that now underlies the entire theory. The framework for the theory achieved a clear definitive form with the 1830 paper of Gauss, who gained conceptual advantage by basing his study on an energy principle, in preference to the force balance conceived by his predecessors. Even so, the Gauss framework still leaves room for more inclusive discussion, as is pointed out in the initial article of the present volume.

During almost a century and a half following the Gauss paper interest for the topic declined, although the physical foundations continued to be studied on a molecular level by van der Waals and by his successors. With regard to global macroscopic problems, those studies led to no changes in the equations or boundary conditions, which present nonlinearities that initially defied analysis. Achievements during that time period were limited to some isolated striking insights due to Kelvin, Rayleigh and a few others, and some of the explicit unsolved problems of the time served as an impetus toward development of modern numerical methods. For the equations that apply in a gravity field, only a single nontrivial closed form solution has as yet been discovered, and classical linearizing procedures have provided little substantive information.

Inspired perhaps by the needs of space technology and of medicine, and utilizing new insights appearing in geometric measure theory, an explosion of activity has occurred during the past thirty-five years, in many directions. New problems have

been attacked, new methods introduced, and discoveries of basically new nature have appeared. Already during the initial ten years of that explosion, enough substantive new material had appeared to justify an entire issue (88:2) of this journal devoted exclusively to capillarity theory and related problems. The present issue, about a quarter century later and somewhat more restrictive as to topics addressed, is intended as a sequel to that initial one. It will be apparent to those familiar with the earlier collection that some perspectives and also some participants have changed, but that the level of activity and the interest in the problems and in the methods have not lessened. Nor have the individual results and the new insights become less striking. That point is of course best made by the papers themselves, which present their own messages. Unfortunately space and time limitations have forced us to restrict the number of papers included here; the present collection should be regarded as an effort to make accessible in a single location a representative section of the (considerable) current activity, in the context of its varying methods and perspectives.

Much of the impetus for this volume developed at the First International Summer School on Capillarity, held at the Max-Planck-Institut für Mathematik in den Naturwissenschaften, in Leipzig, Germany, 2003. A number of the papers that follow had their origins in intense discussions held during that gathering, as did early scientific training for several students who have since continued to successful graduate degrees.

The reader will perceive that the fortress guarding the inner mysteries of capillarity is under heavy siege but has not yet succumbed. We trust that the materials joined together here will serve as a stimulus leading ultimately to completion of the conquest.

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