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**CLASSIFICATION OF EMBEDDED PROJECTIVE MANIFOLDS  
SWEEPED OUT BY RATIONAL HOMOGENEOUS VARIETIES  
OF CODIMENSION ONE**

KIWAMU WATANABE

# CLASSIFICATION OF EMBEDDED PROJECTIVE MANIFOLDS SWEEPED OUT BY RATIONAL HOMOGENEOUS VARIETIES OF CODIMENSION ONE

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**We give a classification of embedded smooth projective varieties swept out by rational homogeneous varieties whose Picard number and codimension are one.**

## 1. Introduction

A central problem in the theory of polarized varieties is to classify smooth projective varieties admitting special varieties  $A$  as ample divisors. In [Watanabe 2008], we investigated this problem in the case where  $A$  is a homogeneous variety. On the other hand, related to the classification problem of polarized varieties, several authors have studied the structure of embedded projective varieties swept out by special varieties [Beltrametti and Ionescu 2008; Muñoz and Solá Conde 2009; Sato 1997; Watanabe 2010]. Inspired by these results, we give a classification of embedded smooth projective varieties swept out by rational homogeneous varieties whose Picard number and codimension are one. Our main result is:

**Theorem 1.1.** *Let  $X \subset \mathbb{P}^N$  be a complex smooth projective variety of dimension  $n \geq 3$  and  $A$  an  $(n - 1)$ -dimensional rational homogeneous variety with  $\text{Pic}(A) \cong \mathbb{Z}[\mathbb{O}_A(1)]$ . Assume that  $X$  satisfies either of the following properties.*

- (a) *Through a general point  $x \in X$ , there is a subvariety  $Z_x \subset X$  such that  $(Z_x, \mathbb{O}_{Z_x}(1))$  is isomorphic to  $(A, \mathbb{O}_A(1))$ .*
- (b) *There is a subvariety  $Z \subset X$  such that  $(Z, \mathbb{O}_Z(1))$  is isomorphic to  $(A, \mathbb{O}_A(1))$  and the normal bundle  $N_{Z/X}$  is nef.*

*Then we have one of the following:*

- (i)  *$X$  is a projective space  $\mathbb{P}^n$ ;*

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- (ii)  $X$  is a quadric hypersurface  $Q^n$ ;
- (iii)  $X$  is the Grassmannian of lines  $G(1, \mathbb{P}^m)$ ;
- (iv)  $X$  is an  $E_6$  variety  $E_6(\omega_1)$ , where  $E_6(\omega_1) \subset \mathbb{P}^{26}$  is the projectivization of the highest weight vector orbit in the 27-dimensional irreducible representation of a simple algebraic group of Dynkin type  $E_6$ ; or
- (v)  $X$  admits an extremal contraction of a ray  $\varphi : X \rightarrow C$  to a smooth curve whose general fibers are projectively equivalent to  $(A, \mathcal{O}_A(1))$ .

In cases (i)–(iv), the corresponding rational homogeneous variety  $A$  is one of those in [Theorem 2.2](#).

We outline the proof of [Theorem 1.1](#). A significant step is to show the existence of a covering family  $\mathcal{H}$  of lines on  $X$  induced from lines on rational homogeneous varieties of codimension one ([Claim 3.2](#)). Then we see that the rationally connected fibration associated to  $\mathcal{H}$  is an extremal contraction of the ray  $\mathbb{R}_{\geq 0}[\mathcal{H}]$ . By applying results from [[Watanabe 2008](#)], we obtain our theorem. In this paper, we work over the field of complex numbers.

## 2. Preliminaries

We denote a simple linear algebraic group of Dynkin type  $G$  simply by  $G$ , and for a dominant integral weight  $\omega$  of  $G$ , the minimal closed orbit of  $G$  in  $\mathbb{P}(V_\omega)$  by  $G(\omega)$ , where  $V_\omega$  is the irreducible representation space of  $G$  with highest weight  $\omega$ . For example,  $E_6(\omega_1)$  is the minimal closed orbit of an algebraic group of type  $E_6$  in  $\mathbb{P}(V_{\omega_1})$ , where  $\omega_1$  is the first fundamental dominant weight in the standard notation of Bourbaki [[1968](#)]. For any rational homogeneous variety  $X$  of Picard number one, there exists a simple linear algebraic group  $G$  and a dominant integral weight  $\omega$  of  $G$  such that  $X$  is isomorphic to  $G(\omega)$ . A rational homogeneous variety  $A$  is a Fano variety, that is, the anticanonical divisor of  $A$  is ample. If the Picard number of  $A$  is one, we have  $\text{Pic}(A) \cong \mathbb{Z}[\mathcal{O}_A(1)]$ , where  $\mathcal{O}_A(1)$  is a very ample line bundle on  $A$ . We recall two results on rational homogeneous varieties.

**Theorem 2.1** [[Hwang and Mok 2005](#), Main Theorem; [1998](#), 5.2]. *Let  $A$  be a rational homogeneous variety of Picard number one. Let  $\rho : X \rightarrow Z$  be a smooth proper morphism between two varieties. Suppose for some point  $y$  on  $Z$ , the fiber  $X_y$  is isomorphic to  $A$ . Then, for any point  $z$  on  $Z$ , the fiber  $X_z$  is isomorphic to  $A$ .*

**Theorem 2.2** [[Watanabe 2008](#)]. *Let  $X$  be a smooth projective variety and  $A$  a rational homogeneous variety of Picard number one. If  $A$  is an ample divisor on  $X$ ,  $(X, A)$  is isomorphic to  $(\mathbb{P}^n, \mathbb{P}^{n-1})$ ,  $(\mathbb{P}^n, Q^{n-1})$ ,  $(Q^n, Q^{n-1})$ ,  $(G(2, \mathbb{C}^{2l}), C_l(\omega_2))$  or  $(E_6(\omega_1), F_4(\omega_4))$ .*

For a numerical polynomial  $P(t) \in \mathbb{Q}[t]$ , write  $\text{Hilb}_{P(t)}(X)$  for the Hilbert scheme of  $X$  relative to  $P(t)$ . More generally, for an  $m$ -tuple of numerical polynomials  $\mathbb{P}(t) := (P_1(t), \dots, P_m(t))$ , denote by  $\text{FH}_{\mathbb{P}(t)}(X)$  the flag Hilbert scheme of  $X$  relative to  $\mathbb{P}(t)$  [Sernesi 2006, Section 4.5]. For the Hilbert polynomial of a line  $P_1(t)$ , an irreducible component of  $\text{Hilb}_{P_1(t)}(X)$  is called a *family of lines* on  $X$ . Let  $\text{Univ}(X)$  be the universal family of  $\text{Hilb}(X)$  with the associated morphisms  $\pi : \text{Univ}(X) \rightarrow \text{Hilb}(X)$  and  $\iota : \text{Univ}(X) \rightarrow X$ . For a subset  $V$  of  $\text{Hilb}(X)$ ,  $\iota(\pi^{-1}(V))$  is denoted by  $\text{Locus}(V) \subset X$ . A *covering family of lines*  $\mathcal{K}$  means an irreducible component of  $F_1(X)$  satisfying  $\text{Locus}(\mathcal{K}) = X$ . For a covering family of lines, we have the following fibration.

**Theorem 2.3** [Campana 1992; Kollár et al. 1992]. *Let  $X \subset \mathbb{P}^N$  be a smooth projective variety and  $\mathcal{K}$  a covering family of lines. Then there exists an open subset  $X^0 \subset X$  and a proper morphism  $\varphi : X^0 \rightarrow Y^0$  with connected fibers onto a normal variety, such that any two points on the fiber of  $\varphi$  can be joined by a connected chain of finite  $\mathcal{K}$ -lines.*

We shall call the morphism  $\varphi$  a *rationally connected fibration with respect to  $\mathcal{K}$* .

**Theorem 2.4** [Bonavero et al. 2007, Theorem 2]. *Under the conditions and notation of Theorem 2.3, assume that  $3 \geq \dim Y^0$ . Then  $\mathbb{R}_{\geq 0}[\mathcal{K}]$  is extremal in the sense of Mori theory and the associated contraction yields a rationally connected fibration with respect to  $\mathcal{K}$ .*

### 3. Proof of Theorem 1.1

For a subset  $V \subset X$ , denote the closure by  $\overline{V}$ . Let  $P_1(t), P_2(t)$  be the Hilbert polynomials of a line  $(A, \mathcal{O}_A(1))$  and set  $\mathbb{P}(t) := (P_1(t), P_2(t))$ . We denote the natural projections by

$$(1) \quad p_i : \text{FH}_{\mathbb{P}(t)}(X) \rightarrow \text{Hilb}_{P_i(t)}(X), \text{ where } i = 1, 2.$$

Let  $\mathcal{H}$  be the open subscheme of  $\text{Hilb}_{P_2(t)}(X)$  parametrizing smooth subvarieties of  $X$  with Hilbert polynomial  $P_2(t)$ . Now we work under the assumption that  $X$  satisfies (a) or (b) in Theorem 1.1.

**Claim 3.1.** *In both cases (a) and (b), there exists a curve  $C \subset \mathcal{H}$  that contains a point  $o$  corresponding to a subvariety isomorphic to  $(A, \mathcal{O}_A(1))$ .*

*Proof.* If the assumption (a) holds, there exists an irreducible component  $\mathcal{H}_0$  of  $\mathcal{H}$  that contains  $o := [Z_x]$  for some  $x \in X$  and satisfies  $\overline{\text{Locus}(\mathcal{H}_0)} = X$ . Then we can take a curve  $C \subset \mathcal{H}_0$  that contains  $o$ . If the assumption (b) holds, we see that  $h^1(N_{Z/X}) = 0$  and  $h^0(N_{Z/X}) \geq 1$ . Since there is no obstruction in the deformation of  $Z$  in  $X$ , it turns out that  $\mathcal{H}$  is smooth at  $o := [Z]$  and  $\dim_{[Z]} \mathcal{H} \geq 1$ . Then we can also take a curve  $C \subset \mathcal{H}_0$  that contains  $o$ .  $\square$

From now on, we shall not use the assumptions (a) and (b) except through the property proved in Claim 3.1.  $\overline{\text{Locus}(C)} = X$ . Denote by  $\mathcal{H}$  an irreducible component of  $\mathcal{H}$  that contains  $C$ . For the universal family  $\pi : \mathcal{U}_0 \rightarrow \mathcal{H}_0$  and the normalization  $\nu : \tilde{C} \rightarrow C \subset \mathcal{H}_0$ , we denote  $\tilde{C} \times_{\mathcal{H}_0} \mathcal{U}_0$  by  $\mathcal{U}_{\tilde{C}}$  and a natural projection by  $\tilde{\pi} : \mathcal{U}_{\tilde{C}} \rightarrow \tilde{C}$ . Let  $(\mathcal{U}_{\tilde{C}})_{\text{red}}$  be the reduced scheme associated to  $\mathcal{U}_{\tilde{C}}$  and  $\Pi : (\mathcal{U}_{\tilde{C}})_{\text{red}} \rightarrow \tilde{C}$  the composition of  $\tilde{\pi}$  and  $(\mathcal{U}_{\tilde{C}})_{\text{red}} \rightarrow \mathcal{U}_{\tilde{C}}$ . Then we have the following diagram:

$$\begin{array}{ccccc}
 (\mathcal{U}_{\tilde{C}})_{\text{red}} & \longrightarrow & \mathcal{U}_{\tilde{C}} & \longrightarrow & \mathcal{U}_0 \\
 \searrow \Pi & & \downarrow \tilde{\pi} & & \downarrow \pi \\
 & & \tilde{C} & \xrightarrow{\nu} & \mathcal{H}_0
 \end{array}$$

Now we have an isomorphism between scheme theoretic fibers

$$\tilde{\pi}^{-1}(p) \cong \pi^{-1}(\nu(p))$$

for any closed point  $p \in \tilde{C}$ . In particular,  $\tilde{\pi}^{-1}(p)$  is a smooth projective variety and  $\tilde{\pi}^{-1}(\tilde{o}) \cong A$  for a point  $\tilde{o} \in \tilde{C}$  corresponding to  $o \in C$ . Moreover, a natural morphism  $\Pi^{-1}(p) \rightarrow \tilde{\pi}^{-1}(p)$  is a homeomorphic closed immersion for any closed point  $p \in \tilde{C}$ . Since  $\tilde{\pi}^{-1}(p)$  is reduced, we see that  $\Pi^{-1}(p) \cong \tilde{\pi}^{-1}(p)$ . Thus we conclude that  $\Pi$  is a proper flat morphism whose fibers on closed points are smooth projective varieties, that is, a proper smooth morphism. Because  $\Pi$  admits a central fiber  $\tilde{\Pi}^{-1}(\tilde{o}) \cong A$ , it follows that every fiber  $\tilde{\Pi}^{-1}(\tilde{p})$  is isomorphic to  $A$  from Theorem 2.1. Hence it turns out that every fiber of  $\pi$  over a closed point in  $C$  is isomorphic to  $A$ . Let consider a constructible subset  $p_1(p_2^{-1}(C)) \subset \text{Hilb}_{p_1(t)}(X)$ . Since  $C$  parametrizes subvarieties isomorphic to  $(A, \mathcal{O}_A(1))$  which is covered by lines, we see that

$$\overline{\text{Locus}(p_1(p_2^{-1}(C)))} = X.$$

**Claim 3.2.** *There exists a covering family of lines  $\mathcal{K}$  on  $X$  satisfying the following property: Through a general point  $x \in X$ , there is a subvariety  $S_x \subset X$  such that  $(S_x, \mathcal{O}_{S_x}(1)) \cong (A, \mathcal{O}_A(1))$  and any line lying in  $S_x$  is a member of  $\mathcal{K}$ .*

*Proof.* Take an irreducible component  $\mathcal{K}^0$  of  $p_1(p_2^{-1}(C))$  such that  $\overline{\text{Locus}(\mathcal{K}^0)} = X$ . Through a general point  $x$  on  $X$ , there is a line  $[l_x]$  in  $\mathcal{K}^0$  that is not contained in any irreducible component of  $p_1(p_2^{-1}(C))$  except  $\mathcal{K}^0$ . There is also a subvariety  $[S_x]$  in  $C$  containing  $l_x$ . Because  $p_1(p_2^{-1}([S_x]))$  is the Hilbert scheme of lines on  $S_x$ , it is irreducible [Landsberg and Manivel 2003, Theorem 4.3; Strickland 2002, Theorem 1]). Therefore  $p_1(p_2^{-1}([S_x]))$  is contained in an irreducible component of  $p_1(p_2^{-1}(C))$ . Since  $p_1(p_2^{-1}([S_x]))$  contains  $[l_x]$ , this implies that  $p_1(p_2^{-1}([S_x]))$  is contained in  $\mathcal{K}^0$ . Thus we put  $\mathcal{K}$  as an irreducible component of  $\text{Hilb}_{p_1(t)}(X)$  containing  $\mathcal{K}^0$ . □

Two points on  $S_x \cong A$  can be joined by a connected chain of lines in  $\mathcal{H}$ . This implies that the relative dimension of the rationally connected fibration  $\varphi : X \dashrightarrow Y$  with respect to  $\mathcal{H}$  is at least  $n - 1$ . According to [Theorem 2.4](#),  $\mathbb{R}_{\geq 0}[\mathcal{H}]$  spans an extremal ray of  $NE(X)$  and  $\varphi$  is its extremal contraction. In particular,  $\varphi$  is a morphism that contracts  $S_x$  to a point. If  $\dim Y = 0$ , we see that the Picard number of  $X$  is one. This implies that  $S_x$  is a very ample divisor on  $X$ . From [Theorem 2.2](#),  $X$  is  $\mathbb{P}^n$ ,  $Q^n$ ,  $G(1, \mathbb{P}^m)$  or  $E_6(\omega_1)$ . If  $\dim Y = 1$ , then  $Y$  is a smooth curve  $C$  and a general fiber of  $\varphi$  coincides with  $S_x$ . Therefore  $\varphi$  is an  $A$ -fibration on a smooth curve  $C$ . Hence [Theorem 1.1](#) holds.

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