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We investigate the rationality problem for purely monomial actions of finite groups. We solve it affirmatively in the following case: K is a field with char $K \neq 2$ and G is a subgroup of GL(n; $\mathbb{Z}$) isomorphic to $(C_2)^n$, where $n > 0$. Then the fixed field of $K(x_1, \dots x_n)$ under the purely monomial action of G is rational over K.

1. Introduction

We investigate the rationality problem for purely monomial actions of finite groups, solving it affirmatively for groups of a special kind (Theorem 1.2). The main point of this paper is not only our main theorem itself, but also the method of proof.

The problem is formulated as follows. Let K be a field, n a natural number, and $K(x_1, \dots, x_n)$ the rational function field in n variables $x_1, \dots, x_n$ over K. Let G be a finite subgroup of GL(n; $\mathbb{Z}$). We define a G-action on $K(x_1, \dots, x_n)$ by

$$\sigma(x_j) = \prod_{i=1}^{n} x_i^{a_{i,j}} \quad \text{for } 1 \leq j \leq n, \quad \text{if} \quad \sigma = [a_{i,j}]_{1 \leq i,j \leq n} \in G.$$

We call σ the purely monomial action of G.

Problem 1.1. Let G be a finite subgroup of GL(n; $\mathbb{Z}$). Let $K(x_1, \dots, x_n)^G$ be the fixed field of $K(x_1, \dots x_n)$ under the purely monomial action of G. Then is $K(x_1, \dots, x_n)^G$ rational (i.e., purely transcendental) over K?

The main result of this paper is the following statement, in which $(C_2)^n$ denotes the direct product of n copies of the cyclic group of order two.

Theorem 1.2 (Main result). *Let K be a field of* char $K \neq 2$, *let n be a natural number, and let G be a subgroup of* GL(n; $\mathbb{Z}$) *isomorphic to $(C_2)^n$. The fixed field $K(x_1, \dots, x_n)^G$ under the purely monomial action of G is rational over K.*

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Next, we review some known cases of Problem 1.1 and compare our result with them. If $n = 2$ or 3, Problem 1.1 has been solved affirmatively for any finite subgroup of $\mathrm{GL}(n;\mathbb{Z})$ and any field K in [Hajja 1983, 1987; Hajja and Kang 1992, 1994; Hoshi and Rikuna 2008]. (See also [Kang and Prokhorov 2010; Yamasaki 2010; Hoshi et al. 2011; Hoshi and Kang 2010] for the rationality problem for general twisted monomial group actions.)

If $n \geq 4$, then Problem 1.1 is still open and there are some examples of negative answers to Problem 1.1. For example, $\mathbb{Q}(x_1, x_2, x_3, x_4)^G$ is not rational over $\mathbb{Q}$ if

$$G = \left\langle \begin{bmatrix} & & & -1 \\ 1 & & & \\ & 1 & & \\ & & 1 & \end{bmatrix} \right\rangle;$$

see [Hajja 1990]. In the aforementioned papers that treated the cases $n = 2, 3$, the problem is solved by complicated case-by-case calculations using the known list of $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes for each n. The result of Problem 1.1 depends only on $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes since a $\mathrm{GL}(n;\mathbb{Z})$-conjugation corresponds to a change of the basis $\{x_1, \dots, x_n\}$ of $K(x_1, \dots, x_n)$.

As far as the author knows, the complete list of $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes is known for $n \leq 6$ (see [Brown et al. 1978] for $n \leq 4$). However, it seems to be almost impossible to obtain the whole result of Problem 1.1 for higher n in the same manner as for $n \leq 3$ because there are as many as 710 conjugacy classes when $n = 4$ and the number grows rapidly with the degree n. So it is quite natural to aim to obtain general results independent of degree n.

To make the problem tractable, we restrict ourselves to groups of exponent two. Problem 1.1 has an affirmative answer for any degree n and any group isomorphic to $(C_2)^1$; see [Hajja 1981, Theorem 1.3]. Theorem 1.2 is a step in our attempt to obtain similar results for other groups.

Next, we explain our method to prove Theorem 1.2. Our method is divided into two steps. The first one is to consider how we can describe, with n general, all $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes that are isomorphic to $(C_2)^n$. The number of $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes that are isomorphic to $(C_2)^n$ for each n is as follows:[1]

n	1	2	3	4	5	6	7	8
no.	1	2	4	8	16	36	80	194

(The two classes of $n = 2$ are given by G_1 and G_2 in [Hajja 1987], and the four classes of $n = 3$ are given by $W_3(187)$, $W_4(187)$, $W_5(187)$ and $W_6(187)$ in [Hajja and Kang 1992].)

However, what we need in order to prove Theorem 1.2 is a description for general n. The main point of the first step of our proof is to use the idea of this

[1]This table was obtained by Markus Kirschmer using the program "sublattices" in the computer algebra system MAGMA.

algorithm with n general, and obtain the necessary information. We will explain the method generally in Section 2A and consider the cases of $(C_2)^n$ in Section 3. The second step is to prove rationality for the purely monomial actions corresponding to the $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes obtained in the first step. This is not easy, since the $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes are described in a general form.

We therefore introduce a *conversion method*, implicit in [Hoshi et al. 2011], which allows us to prove rationality successfully. We will explain this method in Section 2B and use it to prove Theorem 1.2 in Section 3. The aforementioned two methods seem to be effective in many cases other than Theorem 1.2 in studying the rationality problem for purely monomial actions, but we will discuss them on another occasion since the situation would become more complicated.

Remarks. (1) Purely monomial actions are also widely known as *multiplicative actions*. As a background reference on this general topic, refer to [Lorenz 2005]. Our Problem 1.1 is discussed as Problem 12 on page 159 therein.

(2) A related question asks if the rationality problem for multiplicative actions of finite subgroups $G \subset \mathrm{GL}(n;\mathbb{Z})$ depends only on the $\mathrm{GL}(n;\mathbb{Q})$-conjugacy class of G and not on the actual $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes in the $\mathbb{Q}$-class. Our Theorem 1.2 depends only on the $\mathbb{Q}$-class, but the author does not know whether this is generally true.

(3) Our Theorem 1.2 requires the assumption $\operatorname{char} K \neq 2$, since our conversion method in our proof does not work if $\operatorname{char} K = 2$. There are some individual cases where we can prove rationality easily for a fixed n even if $\operatorname{char} K = 2$, but definite results for general n are not known as far as the author knows.

2. Methods

In this section, we explain the two methods used in the proof of Theorem 1.2 in Section 3. The first is a method to determine how a given $\mathrm{GL}(n;\mathbb{Q})$-conjugacy class of finite subgroups of $\mathrm{GL}(n;\mathbb{Q})$ splits into $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes. As mentioned, we will use this method to determine all $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes isomorphic to $(C_2)^n$ with n general. The second method converts Problem 1.1 corresponding to a given $\mathrm{GL}(n;\mathbb{Z})$-conjugacy class into the problem corresponding to another $\mathrm{GL}(n;\mathbb{Z})$-conjugacy class. We will use this method to prove rationality with respect to all the $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes together.

2A. ***Splitting a $\mathbb{Q}$-class into $\mathbb{Z}$-classes.*** In this subsection, we consider generally how a given $\mathrm{GL}(n;\mathbb{Q})$-conjugacy class of finite subgroups of $\mathrm{GL}(n;\mathbb{Q})$ splits into $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes. For example, we put

$$G := \left\langle \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \right\rangle.$$

Then it is known that the subgroups of $\mathrm{GL}(3;\mathbb{Z})$ that are not $\mathrm{GL}(3;\mathbb{Z})$-conjugate but are $\mathrm{GL}(3;\mathbb{Q})$-conjugate to G are given, up to $\mathrm{GL}(3;\mathbb{Z})$-conjugation, by G, $P_1^{-1}GP_1$, $P_2^{-1}GP_2$, and $P_3^{-1}GP_3$, where

$$P_1 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}.$$

How can we find such P's that give all $\mathrm{GL}(n;\mathbb{Z})$-conjugacy of a given group? For this purpose, we use some fundamental facts (Lemmas 2.1 and 2.2 below) from crystallography [Opgenorth et al. 1998].

We denote by $\mathbb{Q}^n$ the vector space consisting of row vectors of degree n. We call the $\mathbb{Z}$-module consisting of the $\mathbb{Z}$-linear combinations of a $\mathbb{Q}$-basis of $\mathbb{Q}^n$ a full $\mathbb{Z}$-lattice, and we denote the set of all full $\mathbb{Z}$-lattices in $\mathbb{Q}^n$ by $\mathfrak{L}_n$. Then $\mathrm{GL}(n;\mathbb{Q})$ acts on $\mathfrak{L}_n$ by $g \cdot L := \{gl \mid l \in L\}$ for any $g \in \mathrm{GL}(n;\mathbb{Q})$ and $L \in \mathfrak{L}_n$. The following lemma means that any finite subgroup of $\mathrm{GL}(n;\mathbb{Q})$ is $\mathrm{GL}(n;\mathbb{Q})$-conjugate to a subgroup of $\mathrm{GL}(n;\mathbb{Z})$.

Lemma 2.1. *Let G be a finite subgroup of* $\mathrm{GL}(n;\mathbb{Q})$. *Then*

$$\mathrm{Fix}_G(\mathfrak{L}_n) := \{L \in \mathfrak{L}_n \mid g \cdot L = L \ \textit{for all } g \in G\}$$

is not empty.

We denote the normalizer of G in $\mathrm{GL}(n;\mathbb{Q})$ by $N_{\mathrm{GL}(n;\mathbb{Q})}(G)$. Then we can obtain (at least theoretically) all $\mathrm{GL}(n;\mathbb{Z})$-conjugacy classes contained in a given $\mathrm{GL}(n;\mathbb{Q})$-conjugacy class by the following lemma.

Lemma 2.2. *Let G be a finite subgroup of* $\mathrm{GL}(n;\mathbb{Q})$. *The set of orbits*

$$N_{\mathrm{GL}(n;\mathbb{Q})}(G) \backslash \mathrm{Fix}_G(\mathfrak{L}_n)$$

is in bijection with the set of all $\mathrm{GL}(n;\mathbb{Z})$*-conjugacy classes contained in the* $\mathrm{GL}(n;\mathbb{Q})$*-conjugacy class of G.*

However, in practice, the determination of $N_{\mathrm{GL}(n;\mathbb{Q})}(G) \backslash \mathrm{Fix}_G(\mathfrak{L}_n)$ is much more difficult than expected. [2] So we restrict ourselves to obtaining the minimum information necessary for the application to Problem 1.1. For our purpose, we do not necessarily require the complete classification.

Let G be a finite subgroup of $\mathrm{GL}(n;\mathbb{Z})$. We take $P \in \mathrm{GL}(n;\mathbb{Q})$ such that $P^{-1}GP$ is a subgroup of $\mathrm{GL}(n;\mathbb{Z})$, and we assume that $P^{-1}GP$ is not $\mathrm{GL}(n;\mathbb{Z})$-conjugate to G. In Section 3, we will consider transforming P into its simplest form. We can multiply P from the right by an element of $\mathrm{GL}(n;\mathbb{Z})$ and from the left by an element of $N_{\mathrm{GL}(n;\mathbb{Q})}(G)$. The following standard normal form is suitable for our study.

[2] Prof. Kirschmer informed the author that this set can be determined for a given G if $n \leq 9$ by the program “sublattices” in the computer algebra system MAGMA.

Definition. A square matrix $H = [h_{i,j}]$ with integer entries is in *Hermite normal form* if the following conditions are satisfied:

(1) $h_{i,j} = 0$ for $j > i$,

(2) $h_{i,i} > 0$ for all i, and

(3) $0 \le h_{i,j} < h_{i,i}$ for $i > j$.

Lemma 2.3 (e.g., [Cohen 1993]). *Let A be a square matrix of degree n with integer entries. Then there exists a unique matrix H in Hermite normal form $H = AU$ with $U \in \mathrm{GL}(n; \mathbb{Z})$.*

2B. ***Conversion method of a corresponding $\mathbb{Z}$-class.*** Our conversion method can be formulated as follows. Let G and G' be subgroups of $\mathrm{GL}(n; \mathbb{Z})$ that are conjugate in $\mathrm{GL}(n; \mathbb{Q})$; so $G' = P^{-1}GP$ for some matrix $P = [p_{i,j}] \in \mathrm{GL}(n; \mathbb{Q})$ with integer entries. Put

$$y_j := \prod_{i=1}^{n} {x_i}^{p_{i,j}} = P(x_j) \in F := K(x_1, \dots, x_n) \quad \text{for } j = 1, \dots, n.$$

Then the subfield $F' := K(y_1, \dots, y_n) \subseteq F$ is K-isomorphic to F via the map $F \to F'$ given by $f \mapsto f' := P(f)$. Moreover, the action of G' on F translates into the action of G on F': writing $\sigma' = P^{-1}\sigma P$ for $\sigma \in G$, we have

$$(\sigma'(f))' = \sigma(f')$$

for all $f \in F$. In particular, the invariant fields $F^{G'}$ and F'^{G} are K-isomorphic. The following result is now clear.

Theorem 2.4. *Assume that $K(y_1, \dots, y_n)$ above can be given as the fixed field $K(x_1, \dots, x_n)^S$ under the set S of some actions. The fixed field $K(x_1, \dots, x_n)^{G'}$ under the purely monomial action of G' is rational over K if and only if the fixed field $K(x_1, \dots, x_n)^{\langle G, S\rangle}$ under the purely monomial action of G and the actions of S is rational over K.*

We will prove Theorem 1.2 using Theorem 2.4 simultaneously for all $\mathrm{GL}(n; \mathbb{Z})$-conjugacy classes isomorphic to $(C_2)^n$.

3. Proof of Theorem 1.2

Any subgroup of $\mathrm{GL}(n; \mathbb{Z})$ that is isomorphic to $C_2 \times \cdots \times C_2$ (the direct product of n copies of C_2) is $\mathrm{GL}(n; \mathbb{Q})$-conjugate to the group

$$G := \left\langle \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}, \dots, \begin{bmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & -1 \end{bmatrix} \right\rangle.$$

We take a matrix $P = [a_{i,j}]_{1\le i,j\le n} \in \mathrm{GL}(n;\mathbb{Q})$ such that $P^{-1}GP \subset \mathrm{GL}(n;\mathbb{Z})$ and suppose that G is not $\mathrm{GL}(n;\mathbb{Z})$-conjugate to $P^{-1}GP$. We need to transform P into its simplest form. We can multiply P by an element of $\mathrm{GL}(n;\mathbb{Z})$ from the right and by an element of $N_{\mathrm{GL}(n;\mathbb{Q})}(G)$ from the left. The following lemma does not give the complete classification of $\mathrm{GL}(n;\mathbb{Z})$-conjugacy, but it is sufficient for our purpose.

Lemma 3.1. *We can transform P into a matrix of the form*

$$\begin{bmatrix} \mathbf{1}_m & \mathbf{0} \\ A & 2\cdot\mathbf{1}_{n-m} \end{bmatrix}$$

with some integer m (≥ 1) and some matrix A of size $(n-m)\times m$ whose components are 0 *or* 1.

Proof. By multiplying P by a scalar, we can assume $P \in M(n;\mathbb{Z})$. As the normalizer $N_{\mathrm{GL}(n;\mathbb{Q})}(G)$ contains all diagonal matrices, we can assume that $a_{i,1},\dots,a_{i,n}$ do not have a common divisor for each i.

We denote by g_i $(1\le i\le n)$ the diagonal matrix of degree n whose (i,i)-component is -1 and whose other diagonal components are 1. We see from a direct calculation that the (j,k)-component of $P^{-1}g_iP$ is

$$\begin{cases} 1-\dfrac{2\Delta_{i,j}a_{i,k}}{\det P} & \text{if } j=k, \\ -\dfrac{2\Delta_{i,j}a_{i,k}}{\det P} & \text{if } j\ne k, \end{cases}$$

where $\Delta_{i,j}$ is the (i,j)-cofactor of P. Since $P^{-1}g_iP \in \mathrm{GL}(n;\mathbb{Z})$, we see that

$$(*) \qquad \frac{2\Delta_{i,j}a_{i,k}}{\det P} \in \mathbb{Z} \quad \text{for all } 1\le i,j,k\le n.$$

Let p be an odd prime number. We suppose $p^l \mid \det P$ with an integer l (≥ 1). Then $p^l \mid \Delta_{i,j}a_{i,k}$ for all $1\le i,j,k\le n$. Since $a_{i,1},\dots,a_{i,n}$ do not have a common divisor, we see that $p^l \mid \Delta_{i,j}$ for all i and j. Comparing the determinants of both sides of

$$\det P\cdot P^{-1} = \begin{bmatrix} \Delta_{1,1} & \cdots & \Delta_{n,1} \\ \vdots & & \vdots \\ \Delta_{1,n} & \cdots & \Delta_{n,n} \end{bmatrix},$$

we see that $(\det P)^{n-1}$ is divisible by $(p^l)^n$. So, $p^{l+1} \mid \det P$, and this means that $\det P$ is divisible by an arbitrarily large power of p. Hence $\det P$ is not divisible by any odd prime number. If $\det P = 1$, this contradicts the assumption that $P^{-1}GP$ is not $\mathrm{GL}(n;\mathbb{Z})$-conjugate to G. Hence $\det P = 2^t$ with an integer t (≥ 1). Again from the same argument about (*) as above, $\Delta_{i,j}$ is divisible by 2^{t-1}, and hence $(\det P)^{n-1}$ is divisible by $(2^{t-1})^n$. Hence $t\le n$.

We transform P to its Hermite normal form by multiplying it by an element of $\mathrm{GL}(n;\mathbb{Z})$ from the right. Then its diagonal components are some 2-powers. We assume that the (i,i)-component is 2^l with $l \geq 2$. Then $2\Delta_{i,i} = 2^{t-l+1} \lneq 2^t$. Again from (*), we see that $a_{i,1},\ldots,a_{i,n}$ are all divisible by 2, but this contradicts the assumption that $a_{i,1},\ldots,a_{i,n}$ do not have a common divisor. Thus the diagonal components of P are 1 or 2. Since the normalizer $N_{\mathrm{GL}(n;\mathbb{Q})}(G)$ contains all permutation matrices, P can be transformed into the desired form. □

Proof of Theorem 1.2. We use the conversion method explained in Section 2B. We use P of the form in Lemma 3.1. We denote the (i,j)-component of A by $a_{i,j}$. We define K-automorphisms $\sigma_1,\ldots,\sigma_{n-m},\tau_1,\ldots,\tau_n$ by

$$\sigma_i : x_j \mapsto \left\{\begin{array}{rl} (-1)^{a_{i,j}} x_j & \text{if } 1 \leq j \leq m, \\ -x_j & \text{if } j = m+i, \\ -x_j & \text{otherwise} \end{array}\right\} \quad \text{for } i = 1,\ldots,n-m;$$

$$\tau_l : x_j \mapsto \left\{\begin{array}{rl} 1/x_j & \text{if } j = l, \\ x_j & \text{otherwise} \end{array}\right\} \quad \text{for } l = 1,\ldots,n.$$

Then the fixed field $K(x_1,\ldots,x_n)^{\langle\sigma_1,\ldots,\sigma_{n-m}\rangle}$ is $K(y_1,\ldots,y_n)$, where

$$y_j = \begin{cases} x_j \prod_{k=1}^{n-m} x_{m+k}{}^{a_{k,j}} & \text{if } 1 \leq j \leq m, \\ x_j{}^2 & \text{if } m+1 \leq j \leq n, \end{cases}$$

and the purely monomial action of $P^{-1}GP$ on $K(x_1,\ldots,x_n)$ coincides with that of $\langle\tau_1,\ldots,\tau_n\rangle$ on $K(y_1,\ldots,y_n)$. Hence the fixed field $K(x_1,\ldots,x_n)^{P^{-1}GP}$ is rational over K if and only if the fixed field $K(x_1,\ldots,x_n)^{\langle\sigma_1,\ldots,\sigma_{n-m},\tau_1,\ldots,\tau_n\rangle}$ is rational over K. We put $z_j := x_j + (1/x_j)$ for $j = 1,\ldots,n$. Then

$$\begin{aligned} K(x_1,\ldots,x_n)^{\langle\sigma_1,\ldots,\sigma_{n-m},\tau_1,\ldots,\tau_n\rangle} &= \bigl(K(x_1,\ldots,x_n)^{\langle\tau_1,\ldots,\tau_n\rangle}\bigr)^{\langle\sigma_1,\ldots,\sigma_{n-m}\rangle} \\ &= K(z_1,\ldots,z_n)^{\langle\sigma_1,\ldots,\sigma_{n-m}\rangle}. \end{aligned}$$

Since $\langle\sigma_1,\ldots,\sigma_{n-m}\rangle$ acts on $K(z_1,\ldots,z_n)$ in the same way as on $K(x_1,\ldots,x_n)$, we see that $K(z_1,\ldots,z_n)^{\langle\sigma_1,\ldots,\sigma_{n-m}\rangle}$ is rational over K. Theorem 1.2 has been proved. □

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