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We present examples of domains that do not admit any complete Kähler metric with bisectional curvature bounded between prescribed two negative constants by a modification of a method of P. Yang.

1. Introduction

Yang [1976] showed that the polydiscs of complex dimension at least two do not admit any complete Kähler metrics with their holomorphic bisectional curvature bounded between two negative constants. He also pointed out that the same argument applies to the bounded symmetric domains with rank higher than one; these domains therefore do not admit such metrics with the same curvature condition. On the other hand the Poincaré–Bergman metric of the unit ball is a complete Kähler metric with its bisectional curvature bounded between two negative constants. Thus the following question [Yau 1982] seems natural:

> *Which complex manifolds admit a complete Kähler metric with bisectional curvature bounded between two negative constants?*

Yang's original interest was linked with a question on the holomorphic universal covering manifold of a compact Kähler manifold with negative curvature, it was conjectured that it should be biholomorphic to a bounded domain. Since such a universal cover was shown to be a Stein manifold by Greene and Wu [1971;1979] during that period, Yang's work was a natural one in the sense that he investigated the curvature of possible Kähler metrics on bounded symmetric domains. On the other hand, the question posed above seems, in its own right, to contain sufficient significance to deserve further study. Incidentally, the method developed in [Yang 1976], since it is the almost unique one known to this day, also seems worth deeper investigation.

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Indeed, Seshadri and Zheng [2008] investigated this line of thought and showed that Yang's method works not only for domains but also for product manifolds. It occurred to us that it is meaningful to investigate whether there are manifolds which do not admit complete Kähler metrics with negative bisectional curvature that are neither biholomorphic to product nor homogeneous manifolds. In fact the main purpose of this paper is to present a modification of Yang's method and applications, and the following theorems:

Theorem 1.1. *There exists a domain* Ω *in* $\mathbb{C}^2$ *satisfying the following properties:*

(i) Ω *is a bounded pseudoconvex domain with smooth boundary.*

(ii) $\operatorname{Aut}\Omega$ *does not act transitively on* Ω.

(iii) Ω *is not biholomorphic to the product of complex manifolds.*

(iv) Ω *does not admit complete Kähler metric with bisectional curvature bounded between two negative constants.*

Theorem 1.2. *Let* $\Omega = \{(z, w) \in \mathbb{C}^2 : |z| < 1, |w| < r(z)\}$, *where* $r : D \to \mathbb{R}$ *is a smooth positive function on* D *such that* $\frac{\partial^2}{\partial z \partial \bar{z}} \frac{1}{r(z)^2} < 0$. *Then* Ω *cannot admit any complete Kähler metric with bisectional curvature bounded between two negative constants.*

The rest of the paper is organized as follows: Section 2 introduces some basic facts and terminology. In Section 3, our modification of Yang's proof is presented with a proof of Theorem 1.1. In Section 4, we give yet another modification of Yang's method and the proof of Theorem 1.2.

2. Preliminary and fundamental facts

First, we introduce some facts and terminology. Let (M, J, h) be a Kähler manifold M of dimension n with a Kähler metric h and a complex structure J. The curvature tensor R on (M, J, h) is given by

$$R_{i\bar{j}k\bar{l}} = \frac{\partial^2 h_{i\bar{j}}}{\partial z_k \partial \bar{z}_l} - \sum_{\alpha,\beta=1}^{n} h^{\alpha\bar{\beta}} \frac{\partial h_{i\bar{\beta}}}{\partial z_k} \frac{\partial h_{\alpha\bar{j}}}{\partial \bar{z}_l}$$

in local coordinates $(z_1, \dots, z_n)$. The bisectional curvature $B(X, Y)$ for X, Y in T_pM at $p \in M$ is given by

$$B(X, Y) = \frac{R(X, JX, Y, JY)}{h(X, X)h(Y, Y)}.$$

In terms of local coordinates,

$$B(X, Y) = -\frac{\sum_{i,j,k,l=1}^{n} R_{i\bar{j}k\bar{l}} X_i \overline{X}_j Y_k \overline{Y}_l}{\sum_{i,j=1}^{n} h_{i\bar{j}} X_i \overline{X}_j \sum_{i,j=1}^{n} h_{i\bar{j}} Y_i \overline{Y}_j}$$

where

$$X = \mathrm{Re}\sum_{j=1}^{n} X_j \frac{\partial}{\partial z_j} \quad \text{and} \quad Y = \mathrm{Re}\sum_{j=1}^{n} Y_j \frac{\partial}{\partial z_j}.$$

The following well known theorems will play important roles in our exposition:

Theorem 2.1 (generalization of Schwarz's lemma [Yau 1978]). *Let (M, g) be a complete Kähler manifold with Ricci curvature bounded below by a constant $-k$ and let (N, h) be a Hermitian manifold with bisectional curvature bounded above by a negative constant $-K$. Then every holomorphic mapping $f : M \to N$ satisfies*

$$f^* h \le \frac{k}{K} g.$$

Theorem 2.2 (generalized maximum principle [Omori 1967; Yau 1975]). *Let M be a complete Riemannian manifold with Ricci curvature bounded below. Then for every C^2 function $f : M \to \mathbb{R}$ that is bounded from above, there exists a sequence $\{p_k\}_{k=0}^{\infty}$ in M such that*

$$\lim_{k\to\infty} |\mathrm{grad} f(p_k)| = 0, \quad \limsup_{k\to\infty} \triangle f(p_k) \le 0 \quad \text{and} \quad \lim_{k\to\infty} f(p_k) = \sup_M f.$$

For $D_r = \{z \in \mathbb{C} : |z| < r\}$, denote by

$$g_r = \frac{r^2\, dz\, d\bar{z}}{(r^2 - |z|^2)^2}$$

the Poincaré metric on D_r with curvature -4. Let D be the unit disc and g the Poincaré metric on D. Denote by D^n the n-dimensional polydisc. Let grad_h and $\triangle_h$ be the gradient and the Laplacian with respect to the Riemannian metric h.

3. A modified proof of the theorem of Paul Yang

We begin this section by proving Yang's theorem again, using a modification of his method; our modification lies in that we do not use integrals.

Theorem 3.1 [Yang 1976]. *The polydisc D^n $(n \ge 2)$ and bounded symmetric domains with rank ≥ 2 do not admit any complete Kähler metrics with its bisectional curvature bounded between two negative constants.*

Proof. For simplicity, we will prove the result for D^2. However, the same proof can be applied to higher-dimensional polydiscs and bounded symmetric domains with rank ≥ 2. Suppose that there exists a complete Kähler metric h on D^2 with $-d \le B(X, Y) \le -c < 0$ for any X, Y, where c, d are positive constants. Fix $z \in D$ and define $i_z : D \to D^2$ by $i_z(w) = (z, w)$. By Theorem 2.1, we have $i_z^* h \le \frac{4}{c} g$; that is,

$$h_{2\bar{2}}(z, w) \le \frac{4}{c} \frac{1}{(1 - |w|^2)^2}.$$

Define $F : D \to \mathbb{R}$ by $F(z) = h_{2\bar{2}}(z, 0)$. Then F is a smooth, positive, bounded function. We will induce a contradiction by calculating the Laplacian of F:

$$\begin{aligned} \triangle_g F(z) &= (1-|z|^2)^2 \frac{\partial^2 F}{\partial z \partial \bar{z}}(z) \\ &= (1-|z|^2)^2 \left(R_{1\bar{1}2\bar{2}}(z,0) + \sum_{\alpha,\beta=1}^{2} h^{\alpha\bar{\beta}} \frac{\partial h_{2\bar{\beta}}}{\partial z} \frac{\partial h_{\alpha\bar{2}}}{\partial \bar{z}} \right) \\ &\geq c(1-|z|^2)^2 h_{2\bar{2}}(z,0) h_{1\bar{1}}(z,0). \end{aligned}$$

From Schwarz's lemma applied to $\pi : D^2 \to D$, $\pi(z, w) = z$, we get $\pi^* g \leq \frac{2d}{4} h$, that is, $(1-|z|^2)^2 h_{1\bar{1}}(z, w) \geq 2/d$. So

$$\triangle_g F \geq \frac{2c}{d} F.$$

Since $\triangle_g \log F = \triangle_g F/|F| - |\operatorname{grad}_g F|^2/|F|^2$ and $\log F$ is a well defined bounded function, by the almost maximum principle (Theorem 2.2) there exists a sequence $\{p_k\}_{n=1}^{\infty}$ in D such that

$$\lim_{k\to\infty} F(p_k) = \sup_D F, \quad \lim_{k\to\infty} |\operatorname{grad}_g \log F(p_k)| = 0 \quad \text{and} \quad \limsup_{k\to\infty} \triangle_g \log F(p_k) \leq 0.$$

But this contradicts $\limsup_{k\to\infty} \triangle_g \log F(p_k) \geq 2c/d$. □

Though clear, the crux of the above argument can be summarized as follows:

Proposition 3.2. *Let Ω be a domain in $\mathbb{C}^n$. Suppose that there exist an embedding*

$$\iota : D^2 \to \Omega, \quad \iota(z, w) = (z, w, 0, \ldots, 0)$$

and a projection

$$\pi : \Omega \to D, \quad \pi(z_1, z_2, \ldots, z_n) = z_1.$$

Then for any constants $c > d > 0$, there is no complete Kähler metric on Ω with

$$-c \leq \operatorname{Bisec} \leq -d.$$

Example 3.3. Let $\Omega = \{(z, w, u) \in \mathbb{C}^2 : |z| < 1, |w| < \alpha, |z + w| < \beta\}$. If $\alpha \geq 1$ and $\beta \geq 2$, then Ω does not have any complete Kähler metrics with bisectional curvature bounded between two negative constants. This Ω is a nonproduct and nonhomogeneous analytic polyhedron that is not biholomorphic to the bidisc: If Ω were a product manifold, Ω would be biholomorphic to D^2 by the Riemann mapping theorem. However, Ω cannot be biholomorphic to D^2 [Fridman 1978]. Likewise, Ω is not homogeneous; if it were, then it would be biholomorphic to D^2 [Kim 1992].

For a sequence of domains $\{\Omega_p\}_{p=1}^{\infty}$, it is possible that Ω_p does not admit any complete Kähler metrics with bisectional curvature bounded between two negative constants for all p, but that $\{\Omega_p\}$ converges to the ball as $p \to \infty$.

Corollary 3.4. *For any* $0 < r < 1$ *and positive constants* $0 < c < d$,

$$\Omega = \{(z, w) \in \mathbb{C} \times \mathbb{C}^n : |z|^2 + |w|^2 < 1, |z| < r\}$$

does not admit complete Kähler metric with

$$-d < Bisec < -c.$$

Proof. The mapping

$$(z, w) \mapsto \Big(\frac{1}{r}z, \frac{1}{\sqrt{1-r^2}}w\Big)$$

sends Ω to $\{(z, w) \in \mathbb{C} \times \mathbb{C}^n : r^2|z|^2 + (1-r^2)|w|^2 < 1, |z| < 1\}$. So an embedding and a projection like in Proposition 3.2 exist. □

Remark 3.5. In contrast, the sequence of domains $\{\Omega_m\}_{m=1}^{\infty}$, where the domain $\Omega_m = \{(z, w) \in \mathbb{C}^n : |z|^2 + |w|^{2m} < 1\}$ has a complete Kähler metric with sectional curvature bounded between two negative constants for all m, converges to the bidisc. Bland [1986] proved that the Kähler–Einstein metric on Ω_m has sectional curvature bounded between two negative constants. However we could find another complete Kähler metric explicitly which has sectional curvature bounded between two negative constants.

On $\Omega_m = \{(z, w) \in \mathbb{C}^n : |z|^2 + |w|^{2m} < 1\}$, let

$$g_m(z, w) = -\partial\bar\partial \log \rho(z, w), \quad \text{where } \rho(z, w) = (1-|z|^2)^{\frac{1}{m}} - |w|^2.$$

Then g_m is a complete Kähler metric on Ω_m. The metric $g_m(z, w)$ is given by

$$\frac{\frac{1}{m}(1-|z|^2)^{\frac{1}{m}-2}}{((1-|z|^2)^{\frac{1}{m}} - |w|^2)^2}\begin{pmatrix} (1-|z|^2)^{\frac{1}{m}} - |w|^2 + \frac{1}{m}|z|^2|w|^2 & w\bar{z}(1-|z|^2) \\ \overline{w}z(1-|z|^2) & m(1-|z|^2)^2 \end{pmatrix},$$

where

$$\det g_m = \frac{\frac{1}{m}(1-|z|^2)^{\frac{2}{m}-2}}{((1-|z|^2)^{\frac{1}{m}} - |w|^2)^3},$$

and the inverse $g_m^{-1}(z, w)$ is

$$\frac{(1-|z|^2)^{\frac{1}{m}} - |w|^2}{(1-|z|^2)^{\frac{1}{m}}}\begin{pmatrix} m(1-|z|^2)^2 & -w\bar{z}(1-|z|^2) \\ -\overline{w}z(1-|z|^2) & (1-|z|^2)^{\frac{1}{m}} - |w|^2 + \frac{1}{m}|z|^2|w|^2 \end{pmatrix}.$$

This metric g_m is invariant with respect to the automorphisms

$$f_\alpha(z, w) = \left(\frac{\alpha - z}{1 - \overline{\alpha} z}, \frac{(1-\alpha^2)^{\frac{1}{2m}}}{(1-\overline{\alpha} z)^{\frac{1}{m}}} w \right) \quad \text{for } \alpha \in D,$$

so it is enough to calculate the section curvature at $(0, w)$. By calculation,

$$\begin{aligned} 2m g_{1\bar{1}}^2 > R_{1\bar{1}1\bar{1}} &= 2 g_{1\bar{1}}^2 (|w|^2 + m(1 - |w|^2)) > 2 g_{1\bar{1}}^2, \\ R_{2\bar{2}2\bar{2}} &= 2 g_{2\bar{2}}^2, \\ R_{1\bar{1}2\bar{2}} &= R_{2\bar{2}1\bar{1}} = R_{1\bar{2}2\bar{1}} = R_{2\bar{1}1\bar{2}} = g_{1\bar{1}} g_{2\bar{2}}, \end{aligned}$$

and the other $R_{i\bar{j}k\bar{l}}$ are zero at $(0, w)$. This implies that

$$-2m < \text{ sectional curvature of } \Omega_m < -\tfrac{1}{2}.$$

However the lower bound on the sectional curvature of this metric and of the Kähler–Einstein metric depends on m and goes to $-\infty$ as $m \to \infty$. So this question is still open:

If Ω_m has a complete Kähler metric with $-d <$ (bi)sect $< -c$ for some constants $c, d > 0$ for all m and converges to Ω as $m \to \infty$, then does Ω have a complete Kähler metric with $-d <$ (bi)sect $< -c$?

Proof of Theorem 1.1. For $\epsilon > 0$, $\alpha < 1$, let

$$\text{(3-1)} \quad \Omega_\epsilon = \left\{ (z, w) \in \mathbb{C}^2 : |z|^2 + \alpha e^{-1/(|w|^2 - \epsilon)} < 1,\ |w| > 1/\sqrt{\epsilon} \right\} \cup \left\{ (z, w) \in \mathbb{C}^2 : |z| < 1,\ |w| \le 1/\sqrt{\epsilon} \right\}$$

Then Ω_ϵ is a nonhomogeneous, nonproduct, bounded pseudoconvex domain with smooth boundary. The points $(z, w) \in \partial\Omega$ with $|w|^2 > \epsilon$ are strongly pseudoconvex. By dilating $(z, w) \mapsto (z, w/\sqrt{\epsilon})$, we see that Ω_ϵ is biholomorphic to

$$\left\{ (z, w) \in \mathbb{C}^2 : |z|^2 + \alpha^2 \exp(1 - 1/(\epsilon |w|^2 - \epsilon)) < 1, |z| < 1 \right\} \cup D^2.$$

Therefore we can apply Proposition 3.2. □

4. Further modification

We now find more examples of domain that do not admit complete Kähler metric with bisectional curvature bounded between two negative constants. The domains we look for will be different from analytic polyhedra. We will achieve this by modifying Yang's method one more time.

Proof of Theorem 1.2. Suppose that there exists a complete Kähler metric h on Ω with $-d \le B(X, Y) \le -c < 0$ for any X, Y for some positive constants c, d.

Define $i_z : D_{r(z)} \to \Omega$ by $i_z(w) = (z, w)$. By Schwarz's lemma, $i_z^* h \le \frac{4}{c} g_{r(z)}$. So $h_{2\bar{2}}(z, w) \le \frac{4}{c} r(z)^2/(r(z)^2 - |w|^2)^2$. Define $F : D \to \mathbb{R}$ by

$$F(z) = r(z)^2 h_{2\bar{2}}(z, 0).$$

Then F is a smooth, positive, bounded function. We have

$$\frac{\partial F}{\partial z}(z) = \frac{\partial r^2}{\partial z}(z) h_{2\bar{2}}(z, 0) + r(z)^2 \frac{\partial h_{2\bar{2}}}{\partial z}(z, 0)$$

and

$$\begin{aligned}
\frac{\partial^2 F}{\partial z \partial \bar{z}} &= r^2 \frac{\partial^2 h_{2\bar{2}}}{\partial z \partial \bar{z}} + \frac{\partial^2 r^2}{\partial z \partial \bar{z}} h_{2\bar{2}} + 2\,\mathrm{Re}\Big(\frac{\partial r^2}{\partial z} \frac{\partial h_{2\bar{2}}}{\partial \bar{z}}\Big) \\
&= r^2 \frac{\partial^2 h_{2\bar{2}}}{\partial z \partial \bar{z}} + \frac{\partial^2 r^2}{\partial z \partial \bar{z}} h_{2\bar{2}} + 2\,\mathrm{Re}\Big(\frac{\partial r^2}{\partial z} \frac{1}{r^2}\Big(\frac{\partial F}{\partial \bar{z}} - \frac{\partial r^2}{\partial \bar{z}} h_{2\bar{2}}\Big)\Big) \\
&= r^2 \frac{\partial^2 h_{2\bar{2}}}{\partial z \partial \bar{z}} + \Big(\frac{\partial^2 r^2}{\partial z \partial \bar{z}} - \frac{2}{r^2}\Big|\frac{\partial r^2}{\partial z}\Big|^2\Big) h_{2\bar{2}} + \frac{2}{r^2}\,\mathrm{Re}\Big(\frac{\partial r^2}{\partial z} \frac{\partial F}{\partial \bar{z}}\Big) \\
&= r^2 \frac{\partial^2 h_{2\bar{2}}}{\partial z \partial \bar{z}} - r^4 \frac{\partial^2 r^{-2}}{\partial z \partial \bar{z}} h_{2\bar{2}} + \frac{2}{r^2}\,\mathrm{Re}\Big(\frac{\partial r^2}{\partial z} \frac{\partial F}{\partial \bar{z}}\Big).
\end{aligned}$$

Thus

$$\begin{aligned}
\triangle_g F &\ge c(1 - |z|^2)^2 r^2 h_{2\bar{2}} h_{1\bar{1}} - (1 - |z|^2)^2 r^2 \frac{\partial^2 r^{-2}}{\partial z \partial \bar{z}} F + \frac{2(1-|z|^2)^2}{r^2}\,\mathrm{Re}\Big(\frac{\partial r^2}{\partial z} \frac{\partial F}{\partial \bar{z}}\Big) \\
&\ge \frac{2c}{d} F - (1 - |z|^2)^2 r^2 \frac{\partial^2 r^{-2}}{\partial z \partial \bar{z}} F + \frac{2(1-|z|^2)^2}{r^2}\,\mathrm{Re}\Big(\frac{\partial r^2}{\partial z} \frac{\partial F}{\partial \bar{z}}\Big).
\end{aligned}$$

So if r satisfies $\frac{\partial^2 r^{-2}}{\partial z \partial \bar{z}} < 0$, then the desired result follows by the almost maximum principle as in the previous modification of Yang's proof. □

Corollary 4.1. *Let Ω is the domain in the Theorem 1.2. Suppose $1/r(z)^2 = \rho(|z|)$ for some positive function ρ on $[0, 1]$ and suppose*

$$t \frac{d\rho}{dt}(t)$$

is a decreasing function. Then Ω does not admit any complete Kähler metrics with bisectional curvature bounded between any two prescribed negative constants.

Proof. This follows by

$$\frac{\partial^2 r^{-2}}{\partial z \partial \bar{z}} = \frac{d^2 \rho}{dt^2} + \frac{1}{t} \frac{d\rho}{dt} = \frac{1}{t}\Big(\frac{d}{dt}\Big(t \frac{d\rho}{dt}\Big)\Big).$$ □

Typical examples of ρ and Ω satisfying the conditions in Corollary 4.1 are:

Example 4.2. Let $\rho(t) = \exp(-t)$ and

$$\Omega = \{(z, w) \in \mathbb{C}^2 : |z| < 1, |w|^2 < \exp(|z|)\}.$$

Example 4.3. Let $\rho(t) = \alpha - t^2$ and

$$\Omega = \{(z, w) \in \mathbb{C}^2 : |z| < 1, |w|^2(\alpha - |z|^2) < 1\}.$$

If we take $\alpha > 1$, then Ω is a bounded domain.

Remark 4.4. If

$$\frac{\partial^2 r^{-2}}{\partial z \partial \bar{z}} < 0 \quad \text{then} \quad \frac{\partial^2 r^2}{\partial z \partial \bar{z}} > \frac{2}{r^2}\left|\frac{\partial r^2}{\partial z}\right|^2.$$

Notice that the domains given in Theorem 1.2 are in fact pseudoconcave.

The domains in Theorem 1.2 converge to $\left\{(z, w) \in \mathbb{C}^2 : |z|^2 + \alpha \exp\left(\frac{-1}{|w|^2}\right) < 1\right\}$ as $\epsilon \to 0$, which does not admit a complete Kähler metric with $-d <$ Bisec $< -c$ for some constants $c, d > 0$:

Corollary 4.5. *Let* $\Omega = \left\{(z, w) \in \mathbb{C}^2 : |z|^2 + \alpha \exp\left(\frac{-1}{|w|^2}\right) < 1\right\}$, *for some constant* $\alpha > 1$. *For any positive constants* $c < d$, *such that* $c/d > (2 \log \alpha)^{-1}$, Ω *cannot admit a complete Kähler metric with*

$$-d \leq \text{Bisec} \leq -c.$$

Proof. Notice that

$$\Omega = \{(z, w) \in \mathbb{C}^2 : |z| < 1, |w| < r(z)\}$$

where $r(z) = 1/(\log \alpha - \log(1 - |z|^2))^{\frac{1}{2}}$. Since $\dfrac{\partial^2 r^{-2}}{\partial z \partial \bar{z}} = \dfrac{1}{(1-|z|^2)^2}$, we obtain for

$$F : D \to \mathbb{R}, \quad F(z) = r(z)^2 h_{2\bar{2}}(z, 0)$$

the inequality

$$\triangle_g F(z) \geq \frac{2c}{d} F(z) - \frac{F}{\log \alpha - \log(1-|z|^2)} - \frac{2(1-|z|^2)}{\log \alpha - \log(1-|z|^2)} \operatorname{Re}\left(\bar{z} \frac{\partial F}{\partial \bar{z}}(z)\right).$$

Since

$$0 < \frac{2(1-|z|^2)}{\log \alpha - \log(1-|z|^2)} \leq \frac{1}{\log \alpha}$$

and $\lim_{|z| \to 1} 1/(\log \alpha - \log(1 - |z|^2)) = 0$, for $c/d > (2 \log \alpha)^{-1}$, the desired result follows by the almost maximum principle. $\square$

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