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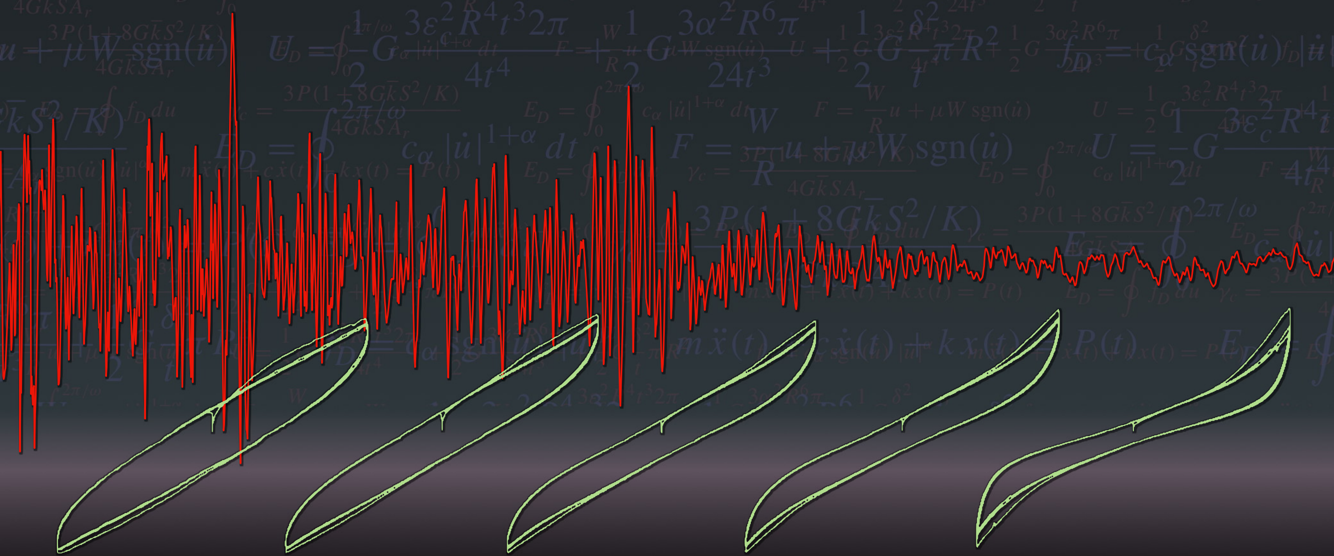

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THE USE OF TESTS ON HIGH-SHAPE-FACTOR BEARINGS TO ESTIMATE THE BULK MODULUS OF NATURAL RUBBER

JAMES M. KELLY AND JIUN-WEI LAI

The bulk modulus of elastomeric materials such as natural rubber is an extremely difficult property to measure since the bulk modulus is several orders of magnitude larger than their shear modulus, so that the material will deform only in shear if at all possible. In most applications the deformation is assumed to be a constant volume one and the material is assumed to be incompressible, but there are situations where the compressibility of the material can play an important role and where it is necessary to have an accurate estimate of the bulk modulus. It will be shown in this paper that one way to determine the bulk modulus is to use the measured vertical stiffness of bearings used as seismic isolators to estimate its value. Seismic isolators are usually made with compounds, known as high damping rubbers, that are nonlinear and have large hysteresis, which can make interpretation of the measurements difficult; but in some cases the compounds used, for example linear natural rubber, have almost no hysteresis and are very linear in shear up to very large shear strains. In this paper, test results from a particular seismic isolation project were analyzed using the theory of bearing mechanics to provide an estimate of the bulk modulus for this particular compound and to show how, if tests on bearings with other compounds are available, to interpret the data for this purpose.

1. Introduction

The bulk modulus of elastomeric materials is an extremely difficult property to measure. Elastomers such as natural rubber have a bulk modulus that is several orders of magnitude larger than their shear modulus, so the material will deform in shear only if at all possible. In most applications the deformation can be assumed to be a constant volume one and the material be assumed to be incompressible. However, there are situations where the compressibility of the material can play an important role and where it is necessary to have a more accurate estimate of the bulk modulus. An example of such a case arises in the use of multilayer elastomeric bearings as support pads for bridges or as seismic or vibration isolators for buildings. When these components are designed, the rubber is usually assumed to be incompressible and there is a fairly simple analysis procedure to predict the vertical stiffness of the bearing or isolator. However, it is somewhat surprising that, for quite modest shape factors, the bulk compressibility of the rubber can have an important role; the design formula based on the incompressible model can seriously overpredict the vertical stiffness and the buckling load of a bearing.

Accordingly, it is essential to have an accurate estimate of the bulk modulus. A quick review of the data available on this property for natural rubber in particular reveals that an accurate estimate is difficult to find. For example, the widely used handbook [Lindley 1992] provides a table of bulk modulus values based on the IRHD hardness (International Rubber Hardness Degree) that range from 1000 to 1330 MPa

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as the hardness varies from 30 to 75 IRHD. On the other hand, the reference [Fuller et al. 1988] gives values in the range from 2000 to 3500 MPa.

One way to determine the bulk modulus in a somewhat indirect way is to use the measured vertical stiffness of isolators to estimate its value. Seismic isolators are usually made with compounds known as high damping rubbers (HDR) that are nonlinear and have large hysteresis which can make interpretation of the measurements difficult but in some cases the compounds known as linear natural rubber (LNR) have almost no hysteresis and are very linear in shear up to very large shear strains.

A case in point is an isolation project in South Korea [EESK 2007] where a group of four fourteen-story residential buildings are built on isolation systems that combine lead plug rubber bearings (LRB) with low damping rubber bearings. In this project, there are two types of these LNR bearings designated as RB2 and RB3, and there are twenty bearings of each type. Each bearing of both types was tested in both horizontal shear and vertical compression before installation and it was found that the measured vertical stiffnesses were lower by 7 to 19% than the design predictions [EESK 2007]. The results of the horizontal shear tests allowed the shear modulus to be calculated and the vertical stiffness tests could provide an estimate of the bulk modulus. A complication in this case was that all the bearings had a small central hole that was not filled with rubber. At first sight it would seem that such a small hole would have no effect on the vertical stiffness but it was an unexpected result of the theory of bearings both when incompressibility was assumed and when the material was modeled as compressible that the presence of even a very small hole had a large effect on the stiffness and that the hole could not be ignored.

The test results were analyzed using the bearing mechanics theory [Gent and Meinecke 1970] to provide an estimate of the bulk modulus for this particular compound and to show how, if tests on bearings with other compounds are available, to interpret the data for this purpose.

2. Compression of pad within incompressible theory

A linear elastic theory is the most common method used to predict the compression stiffness of a thin elastomeric pad. The first analysis of the compression stiffness was done using an energy approach [Rocard 1937]; further developments were made two decades later [Gent and Lindley 1959; Gent and Meinecke 1970]. The theory given here is a simplified version of these analyses and is applicable to bearings with shape factors (S) in the approximate range $5 < S < 10$.

The analysis is an approximation based on a number of assumptions. Two kinematic assumptions are as follows [Gent and Lindley 1959]:

(i) Points on a vertical line before deformation lie on a parabola after loading applied.

(ii) Horizontal planes remain horizontal.

Consider an arbitrarily shaped pad of thickness t and locate a rectangular Cartesian coordinate system, (x, y, z), in the middle surface of the pad, as shown on the left in Figure 1. The right-hand side of the figure shows the displacements, (u, v, w), in the coordinate directions under assumptions (i) and (ii):

$$u(x, y, z) = u_0(x, y)\Big(1 - \frac{4z^2}{t^2}\Big), \quad v(x, y, z) = v_0(x, y)\Big(1 - \frac{4z^2}{t^2}\Big), \quad w(x, y, z) = w(z). \tag{2-1}$$

This displacement field equation (2-1) satisfies the constraint that the top and bottom surfaces of the pad are bonded to rigid substrates. The assumption of incompressibility produces a further constraint on

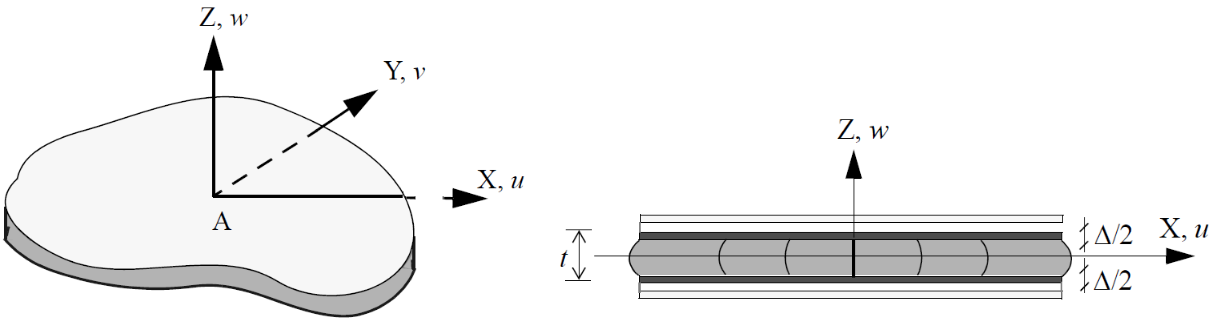

Figure 1. Constrained rubber pad and coordinate system.

the three components of strain, ϵ_{xx}, ϵ_{yy}, ϵ_{zz}, in the form

$$\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = 0. \tag{2-2}$$

Along with (2-1), this leads to

$$(u_{0,x} + v_{0,y})\Big(1 - \frac{4z^2}{t^2}\Big) + w_{,z} = 0,$$

where the commas imply partial differentiation with respect to the indicated coordinate. When integrated through the thickness this gives

$$u_{0,x} + v_{0,y} = \frac{3\Delta}{2t}, \tag{2-3}$$

where the change of thickness of the pad is Δ (assume $\Delta > 0$ in compression).

The stress state is assumed to be dominated by the internal pressure, p, such that the normal stress components, τ_{xx}, τ_{yy}, τ_{zz}, differ from $-p$ only by terms of order $(t^2/l^2)p$, i.e.,

$$\tau_{xx} \approx \tau_{yy} \approx \tau_{zz} \approx -p\Big[1 + O\Big(\frac{t^2}{l^2}\Big)\Big],$$

where l is a typical dimension of the pad. The shear stress components, τ_{xz} and τ_{yz}, which are generated by the constraints at the top and bottom of the pad, are assumed to be of order $(t/l)p$; the in-plane shear stress, τ_{xy}, is assumed to be of order $(t^2/l^2)p$.

The equilibrium equations of the stresses under these assumptions reduce to

$$\tau_{xx,x} + \tau_{xz,z} = 0, \quad \tau_{yy,y} + \tau_{yz,z} = 0. \tag{2-4}$$

Assuming that the material is linearly elastic, then shear stresses τ_{xz} and τ_{yz} are related to the shear strains, γ_{xz} and γ_{yz}, by

$$\tau_{xz} = G\gamma_{xz}, \quad \tau_{yz} = G\gamma_{yz},$$

with G being the shear modulus of the material; thus,

$$\tau_{xz} = -8Gu_0\frac{z}{t^2}, \quad \tau_{yz} = -8Gv_0\frac{z}{t^2}. \tag{2-5}$$

From the equilibrium equations (2-4), therefore,

$$\tau_{xx,x} = \frac{8Gu_0}{t^2}, \qquad \tau_{yy,y} = \frac{8Gv_0}{t^2}, \tag{2-6}$$

which when inverted to give u_0, v_0 and inserted into (2-3) gives

$$\frac{t^2}{8G}(\tau_{xx,xx} + \tau_{yy,yy}) = \frac{3\Delta}{2t}. \tag{2-7}$$

In turn, by identifying both τ_{xx} and τ_{yy} as $-p$, this reduces to

$$p_{,xx} + p_{,yy} = \nabla^2 p = -\frac{12G\Delta}{t^3} = -\frac{12G}{t^2}\epsilon_c, \tag{2-8}$$

where $\epsilon_c = \Delta/t$ is the compression strain. The boundary condition, $p = 0$, on the perimeter of the pad completes the system for $p(x, y)$.

The vertical stiffness of a rubber bearing is given by the formula

$$K_V = \frac{E_c A}{t_r},$$

where A is the area of the bearing, t_r is the total thickness of rubber in the bearing and E_c is the instantaneous compression modulus of the rubber-steel composite under the specified level of vertical load. The value of E_c for a single rubber layer is controlled by the shape factor, S, defined as

$$S = \frac{\text{load area}}{\text{force-free (bulge) area}}.$$

This is a dimensionless measure of the aspect ratio of the single layer of the elastomer. Typical shape factors for bearings of different shapes are as follows, where t is the single-layer thickness:

$$\frac{b}{t} \quad \text{for an infinite strip of width } 2b,$$

$$\frac{R}{2t} \quad \text{for an circular pad of radius } R,$$

$$\frac{a}{4t} \quad \text{for a square pad of side length } a.$$

To determine the compression modulus, E_c, one needs to solve for p and integrate over the cross section area A of the pad to determine the resultant normal load, P; where E_c is then given by

$$E_c = \frac{P}{A\epsilon_c}, \tag{2-9}$$

where A is the area of the pad.

For example, for a circular pad of radius R, as shown in Figure 2, Equation (2-8) reduces to

$$\nabla^2 p = \frac{d^2 p}{dr^2} + \frac{l}{r}\frac{dp}{dr} = -\frac{12G}{t^2}\epsilon_c, \qquad r = \sqrt{x^2 + y^2}. \tag{2-10}$$

The solution is [Kelly 1997]

$$p = A \ln r + B - \frac{3G}{t^2} r^2 \epsilon_c,$$

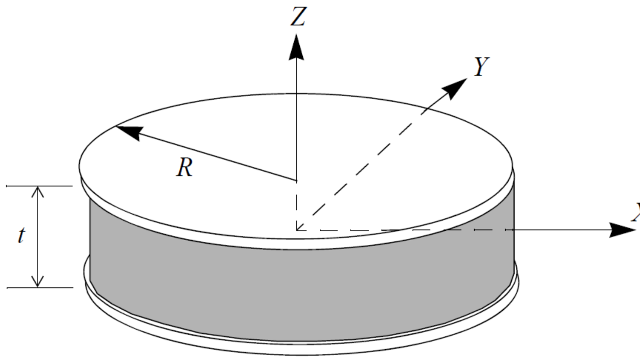

Figure 2. Coordinate system for a circular pad of radius R.

where A and B are constants of integration; because p must be bounded at $r = 0$ and $p = 0$ at $r = R$, the solution becomes

$$p = \frac{3G}{t^2}(R^2 - r^2)\epsilon_c. \tag{2-11}$$

It follows that

$$P = 2\pi \int_0^R p(r)r \,\mathrm{d}r = \frac{3G\pi R^4}{2t^2}\epsilon_c, \tag{2-12}$$

and with $S = R/2t$ and $A = \pi R^2$, we have $E_c = 6GS^2$.

2.1. ***Annular pad.*** Consider an annular pad with inside radius a, external radius b, and thickness t. The shape factor in this case is

$$S = \frac{\pi(b^2 - a^2)}{2\pi(a+b)t} = \frac{b-a}{2t}. \tag{2-13}$$

The solution of (2-10), with $p(a) = 0$ and $p(b) = 0$, is

$$p(r) = \frac{3G}{t^2}\epsilon_c\left(\frac{(b^2 - a^2)\ln\dfrac{r}{a}}{\ln\dfrac{b}{a}} - (r^2 - a^2)\right). \tag{2-14}$$

The total load, P, is given by

$$P = 2\pi \int_a^b p(r)r \,\mathrm{d}r = \frac{6\pi G}{t^2}\epsilon_c \frac{b^2 - a^2}{4}\left((b^2 + a^2) - \frac{b^2 - a^2}{\ln\dfrac{b}{a}}\right), \tag{2-15}$$

from which we have

$$E_c = \frac{P}{A\epsilon_c} = \frac{3G}{2t^2}\left((b^2 + a^2) - \frac{b^2 - a^2}{\ln\dfrac{b}{a}}\right). \tag{2-16}$$

Using the usual expression for S, we can write this in the form

$$E_c = 6GS^2\lambda, \tag{2-17}$$

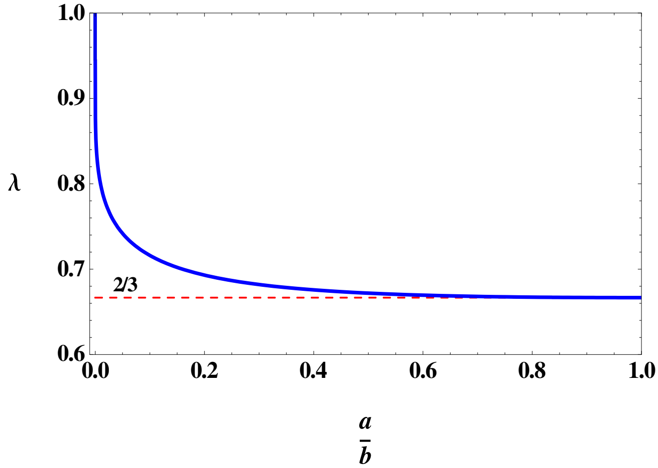

Figure 3. Reduction of compression modulus, E_c, for an annular pad [Kelly 1997].

where

$$\lambda = \frac{(b^2+a^2) - \dfrac{b^2-a^2}{\ln(b/a)}}{(b-a)^2}, \tag{2-18}$$

which, in terms of the ratio a/b, becomes

$$\lambda = \frac{1+\left(\dfrac{a}{b}\right)^2 + \dfrac{1-(a/b)^2}{\ln(a/b)}}{\left(1-\dfrac{a}{b}\right)^2}. \tag{2-19}$$

When a/b tends to 0, the value of λ tends to 1; hence, $E_c \to 6GS^2$, which is the result for the full circular pad. When a/b tends to 1, by writing $a/b = 1-\epsilon$ and letting ϵ tend to 0, we find that λ tends to $\frac{2}{3}$ and E_c to $4GS^2$, which is the result for the infinite strip. It is interesting to evaluate how rapidly the result for λ approaches $\frac{2}{3}$. To illustrate this point, we plot in Figure 3 the solution for λ versus the ratio a/b, for $0 < a/b \le 1$.

Clearly for the case when $a/b > 0.10$, the value of λ is almost two-thirds, indicating that the presence of even a small hole has a large effect on E_c, therefore, in most cases for bearings with central holes, the value of E_c should be taken as $4GS^2$ rather than $6GS^2$.

3. Compression stiffness for circular pads with large shape factors

The theory for the compression of a rubber pad given in the preceding section is based on two assumptions: first, the displacement pattern determined in (2-1); second, the normal stress components in all three directions can be approximated by the pressure, p, in the material. The equation that is solved for p is the integration through the thickness of the pad of the equation of incompressibility (2-2), leading to an equation for $p(x, y)$ of the form given in (2-8). To include the influence of bulk compressibility, we need only replace the equation of incompressibility constraint (2-2) by

$$\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = -\frac{p}{K},$$

where K is the bulk modulus. Integration through the thickness leads to an equation for $p(x, y)$ of the form

$$\nabla^2 p - \frac{12p}{t^2}\frac{G}{K} = -\frac{12G}{t^2}\epsilon_c, \tag{3-1}$$

which is solved as before, with $p = 0$ on the edge of the pad.

We now consider a circular pad with a large shape factor, an external radius, R, and thickness, t. The pressure in the pad is axisymmetric: $p = p(r)$, where $0 \le r \le R$; therefore, (3-1) becomes

$$\frac{\mathrm{d}^2 p}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}p}{\mathrm{d}r} - \lambda^2(p - K\epsilon_c) = 0, \quad \lambda^2 = \frac{12G}{Kt^2} \tag{3-2}$$

with $p = 0$ at $r = R$.

The solution involves I_0 and K_0, the modified Bessel functions of the first and second kind [Kelly 1997]. Because the solution is bounded at $r = 0$, the term in K_0 is excluded and the general solution for $p(r)$ is given by

$$p(r) = K\left(1 - \frac{I_0(\lambda r)}{I_0(\lambda R)}\right)\epsilon_c. \tag{3-3}$$

Integrating p over the area of the pad gives

$$P = K\pi R^2\left(1 - \frac{2}{\lambda R}\frac{I_1(\lambda R)}{I_0(\lambda R)}\right)\epsilon_c,$$

where I_1 is the modified Bessel function of first kind of order one. The resulting expression for the compression modulus is

$$E_c = K\left(1 - \frac{2}{\lambda R}\frac{I_1(\lambda R)}{I_0(\lambda R)}\right), \tag{3-4}$$

where

$$\lambda R = \sqrt{\frac{12GR^2}{Kt^2}} = \sqrt{\frac{48G}{K}}S$$

and the shape factor S equals $\dfrac{R}{2t}$.

3.1. ***Annular pad.*** In the case of a pad of inside radius a and outside radius b the solution of (3-2) is [Constantinou et al. 1992]

$$p(r) = \big(AI_0(\lambda r) + BK_0(\lambda r) + 1\big)K\epsilon_c, \tag{3-5}$$

and the fact that $p(a) = 0$ and $p(b) = 0$ leads to

$$A = -\frac{[K_0(\lambda b) - K_0(\lambda a)]}{I_0(\lambda a)K_0(\lambda b) - I_0(\lambda b)K_0(\lambda a)}, \quad B = \frac{[I_0(\lambda b) - I_0(\lambda a)]}{I_0(\lambda a)K_0(\lambda b) - I_0(\lambda b)K_0(\lambda a)}.$$

Integrating (3-5) with these values of A and B gives for P the value

$$2\pi K\epsilon_c\Big(\big(K_0(\lambda b)-K_0(\lambda a)\big)\big(\lambda b I_1(\lambda b)-\lambda a I_1(\lambda a)\big)+\big(I_0(\lambda b)-I_0(\lambda a)\big)\big(\lambda b K_1(\lambda b)-\lambda a K_1(\lambda a)\big)+\frac{b^2-a^2}{2}\Big),$$

from which using (2-9) leads to the result

$$E_c = K\Bigg(1-\frac{2\big(K_0(\lambda b)-K_0(\lambda a)\big)\big(\lambda b I_1(\lambda b)-\lambda a I_1(\lambda a)\big)}{\lambda^2(b^2-a^2)\big(I_0(\lambda a)K_0(\lambda b)-I_0(\lambda b)K_0(\lambda a)\big)} - \frac{2\big(I_0(\lambda b)-I_0(\lambda a)\big)\big(\lambda b K_1(\lambda b)-\lambda a K_1(\lambda a)\big)}{\lambda^2(b^2-a^2)\big(I_0(\lambda a)K_0(\lambda b)-I_0(\lambda b)K_0(\lambda a)\big)}\Bigg). \tag{3-6}$$

To use this result to determine the bulk modulus from the test data we first normalize E_c with respect to $6GS^2$, the value for a complete pad based on the radius R (here = b) when incompressibility is assumed, and note that

$$\lambda^2 R^2 = \frac{48GS^2}{K},$$

so that

$$\frac{K}{6GS^2} = \frac{8}{\lambda^2 R^2}.$$

To account for the small hole in the tested pads we set $b = R$ and $a = \epsilon R$, where ϵ is less than 0.1 in the two examples tested.

When compressibility is included, the presence of a small hole has a large effect on the compression modulus. To show this, we combine

$$y = \frac{E_c}{6GS^2} \quad \text{and} \quad x = \lambda R$$

with (3-6), obtaining the result

$$y = \frac{8}{x^2}\Big(1-\frac{2}{x}\cdot\frac{A_1+A_2}{A_3}\Big), \tag{3-7}$$

where

$$\begin{aligned} A_1 &= \big(K_0(x)-K_0(\epsilon x)\big)\big(I_1(x)-\epsilon I_1(\epsilon x)\big),\\ A_2 &= \big(K_1(x)-\epsilon K_1(\epsilon x)\big)\big(I_0(x)-I_0(\epsilon x)\big),\\ A_3 &= I_0(\epsilon x)K_0(x)-I_0(x)K_0(\epsilon x). \end{aligned}$$

Suppose we take particular values of G, K, S based on no hole and vary ϵ. It is then clear that the hole has a large effect. Let $G = 0.9375$ MPa, $K = 2000$ MPa and $S = 20$ for which the value of $x = 3$ in this case. The value of the normalized modulus y when $\epsilon = 0$ is 0.4089 and the curve of y as a function of ϵ over the range $0 \le \epsilon \le 0.1$ is shown in Figure 4. It is obvious that the slope of the curve at zero is negative infinity and this is what produces the large effect on the modulus.

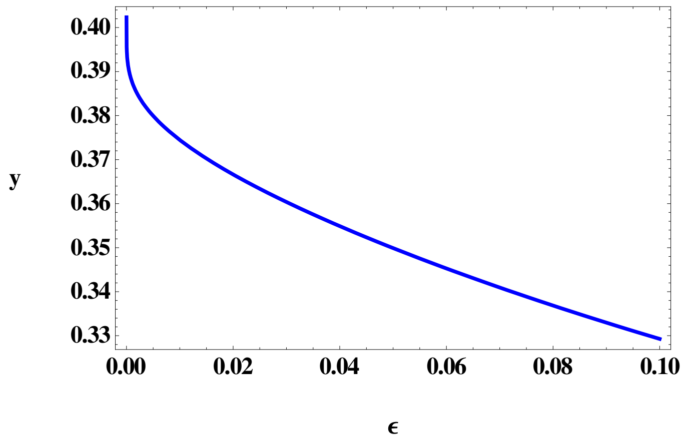

Figure 4. Reduction of normalized modulus, y, for an annular pad.

4. Influence of compressibility on buckling of bearings

It will be necessary to demonstrate that the buckling load of both types of bearings are sufficiently high in comparison to the design and test loads that there is no interaction between the axial load and the horizontal stiffness. The horizontal stiffness measurements will be used to provide the actual value of the shear modulus. Note that the presence of a small hole has a large effect on the vertical stiffness and has very little effect on the response of a bending moment. When computing the bending stiffness of a bearing, the hole can be ignored. However, the bending stiffness plays an essential role in the determination of the buckling load of a bearing. It has been shown [Kelly 1997] that bulk compressibility in the rubber has a surprisingly large effect on both the compression stiffness and bending stiffness of a bearing even for shape factors as low as 10. The buckling load P_{cr} of a bearing is given by [Kelly 1997]

$$P_{\text{cr}} = \sqrt{P_S P_E}, \tag{4-1}$$

where

$$P_S = GA \cdot \frac{h}{t_r} \quad \text{and} \quad P_E = \frac{\pi^2}{h^2} \cdot \left(\tfrac{1}{3} E_c I\right) \cdot \frac{h}{t_r}, \tag{4-2}$$

leading to

$$P_{\text{cr}} = \left(GA\frac{h}{t_r}\right)^{1/2} \left(\frac{\pi^2}{h^2}\frac{1}{3}6GS^2Ar^2\frac{h}{t_r}\right)^{1/2} = \sqrt{GA\frac{h}{t_r}\frac{\pi^2}{h^2}\frac{1}{3}6GS^2Ar^2\frac{h}{t_r}} = \sqrt{2\frac{\pi^2}{h^2}G^2S^2A^2r^2},$$

$$P_{\text{cr}} = \frac{\sqrt{2}\pi GSAr}{t_r},$$

where the radius of gyration is denoted by $r = \sqrt{I/A} = \Phi/4$, for a circular bearing with diameter, Φ. The critical pressure, $p_{\text{cr}} = P_{\text{cr}}/A$, can be expressed in terms of S and the quantity S_2, referred to as the aspect ratio or the *second shape factor*, defined by

$$S_2 = \frac{\Phi}{t_r}.$$

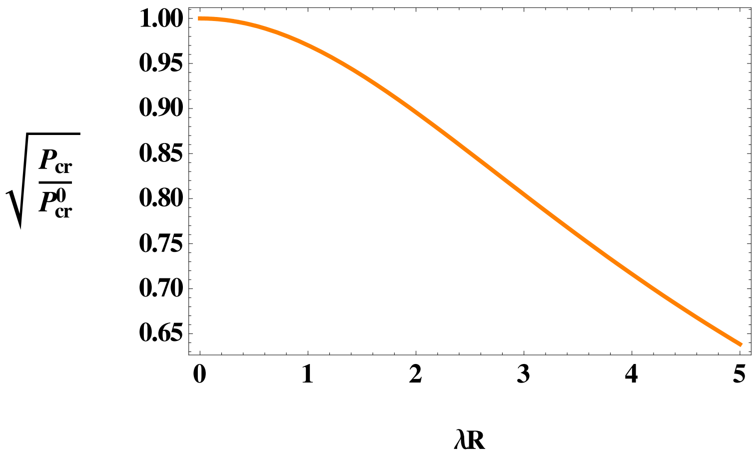

Figure 5. Effect of compressibility on buckling load of a bearing.

Thus, for a circular bearing

$$\frac{p_{\rm cr}}{G} = \frac{\pi}{2\sqrt{2}} S S_2. \tag{4-3}$$

The impression given by (4-3) is that it is possible to improve the stability of a bearing with a certain diameter and thickness of rubber by the simple process of increasing the shape factor, i.e., increasing the number of layers and reducing their thickness. However, because of the effect of bulk compressibility on the effective stiffness, the improvement is limited. The expression $\frac{1}{3}E_c I$ in (4-2) is the effective bending stiffness $(EI)_{\rm eff}$ of a pad when the material is assumed to be incompressible and we will denote it by $(EI)_{\rm inc} = 2GS^2 I$. We assume that the buckling load is given by the solution in (4-1), but substitute for $(EI)_{\rm eff}$ the expression [Kelly 1997]

$$(EI)_{\rm eff} = \frac{\pi K R^4}{4}\left(1 - \frac{4I_2(\lambda R)}{\lambda R I_1(\lambda R)}\right),$$

where $\lambda R = \sqrt{48GS^2/K}$. The resulting value of the critical load can be reduced to an expression depending only on the quantity λR, by dividing by the result for the critical load when the material is taken as incompressible, leading to

$$\left(\frac{P_{\rm cr}}{P_{\rm cr}^0}\right)^2 = \frac{24}{\lambda^2 R^2}\left(1 - \frac{4I_2(\lambda R)}{\lambda R I_1(\lambda R)}\right).$$

The reduction in terms of the varying shape factor are shown in Figure 5. The most convenient way to calculate the buckling load when bulk compressibility is included is to note that GA is unchanged, so that we have

$$P_{\rm cr} = P_{\rm inc}\sqrt{\frac{EI_{\rm eff}}{EI_{\rm inc}}},$$

where $P_{\rm cr}$ is the critical load for the compressible case and $P_{\rm inc}$ that for the incompressible case. The easiest way to estimate the buckling pressure for the two types of bearings is to calculate the buckling

property	RB2	RB3
diameter of bearing (mm)	900	850
diameter of steel plate (mm)	860	810
diameter of hole (mm)	60	60
number of rubber layers	48	48
rubber thickness per layer (mm)	6	6
number of test specimens	20	20

Table 1. Dimensions of the two types of bearings [EESK 2007].

pressure assuming incompressibility and then reduce this by the factor

$$\sqrt{\frac{24}{\lambda^2 R^2}\left(1 - \frac{4I_2(\lambda R)}{\lambda R I_1(\lambda R)}\right)},$$

using the nominal values of G and K to compute the appropriate values of λR in the reduction factor.

There are two types of isolators in the test program, designated RB2 and RB3 [EESK 2007]. Twenty bearings of each type were tested for horizontal stiffness and vertical stiffness. The rubber compound, individual rubber layer thickness and the number of layers are the same for both types. The differences between the two types are the rubber diameters and the steel shim diameters. Both types have a small central hole of 60 mm diameter that is not filled with rubber. The dimension of the bearings are given in Table 1. The nominal shear modulus of the compound is 0.40 MPa and we assume that the value of K for the purpose of this estimate of the buckling pressure is 2000 MPa. For the dimensions of RB2 and these nominal moduli, the P_{inc} for RB2 is 52 MPa and for RB3 is 46.4 MPa. The value of $\lambda^2 R^2$ for RB2 is 13.5 giving a reduction factor of 0.744 and for RB3 these are 12.0 and 0.7625 respectively. Thus the critical pressures are 38.7 MPa for RB2 and 35.4 MPa for RB3. The interaction between the horizontal stiffness and the vertical load in an elastomeric bearing is given by the expression [Kelly 1997]

$$K_H = \frac{GA_S}{h}\left(1 - \left(\frac{P}{P_{\text{cr}}}\right)^2\right),$$

where P is the actual compressive load; the same equation applies to the pressures, which are 10.29 MPa and 7.61 MPa, respectively, so that the effect is negligible.

5. Test results for the two bearing types

To compute the actual shear modulus of the compound, we use the horizontal stiffness and the full rubber diameter allowing for the area of the hole and for the compression modulus, we use the steel shim area again allowing for the presence of the hole. The shape factors that will be used to determine the bulk modulus through the normalized compression modulus are obtained from the full shim diameter (neglecting the hole) and are 35.83 for RB2 and 33.75 for RB3. The test results for the 20 bearings of each type are given in Table 2. The average horizontal stiffnesses are 0.877 MN/m for RB2 and 0.837 MN/m for RB3. The average vertical stiffnesses are 1947 MN/m for RB2 and 1635 MN/m for RB3. In the case

	RB2		RB3	
specimen	K_v (MN/m)	K_h (MN/m)	K_v (MN/m)	K_h (MN/m)
1	1866.53	0.843	1577.59	0.805
2	2022.69	0.870	1589.76	0.805
3	1902.96	0.870	1666.83	0.794
4	1886.03	0.854	1630.09	0.794
5	1979.11	0.939	1669.56	0.823
6	1955.13	0.868	1584.82	0.847
7	1951.63	0.843	1591.33	0.813
8	1981.81	0.864	1612.79	0.860
9	1951.99	0.835	1632.81	0.823
10	1934.86	0.868	1606.14	0.841
11	1916.89	0.939	1620.63	0.857
12	1877.72	0.854	1563.80	0.836
13	1930.42	0.825	1678.67	0.847
14	1909.77	0.825	1588.89	0.813
15	1950.82	0.864	1675.26	0.893
16	1900.65	0.835	1560.71	0.836
17	2055.71	0.962	1665.53	0.857
18	2034.03	0.962	1726.50	0.893
19	2013.98	0.914	1870.02	0.860
20	1924.18	0.914	1596.16	0.841
Average	1947.35	0.877	1635.39	0.837

Table 2. Test results for the RB2 and RB3 bearings.

of RB2 the average horizontal stiffness using the equation

$$K_h = \frac{GA}{t_r}$$

with the area $A = 0.6334\,\text{m}^2$ and $t_r = 0.288\,\text{m}$, the value of G is $0.399\,\text{MN/m}^2$. The nominal value was $0.40\,\text{MN/m}^2$. From this value of G and the value of the shape factor $S = 35.83$, we have $y = E_c/6GS^2 = 0.3157$ which when equated with (3-7) leads to $x = \lambda R = 3.277$ and this in turn using the definition of $\lambda R = \sqrt{48GS^2/K}$ leads to the value of K as $2361\,\text{MN/m}^2$. The similar computations for RB3 give $G = 0.426\,\text{MN/m}^2$, $y = E_c/6GS^2 = 0.3159$, $x = \lambda R = 3.206$ and $K = 2266\,\text{MN/m}^2$. These results are very consistent with each other and suggest that one might safely assume that the modulus to be used in bearing design might be $2300\,\text{MN/m}^2$. The surprising result is that there is such a large difference in the shear modulus between the two sets of bearings. Without knowing more about the construction process for each set it is difficult to explain why there should be such a difference, but it seems to be systematic rather than random.

6. Conclusion

It has been shown that it is possible to estimate the bulk modulus of natural rubber from test results on bearings used as seismic isolators. The estimated bulk moduli of rubber using the proposed method are reasonably consistent values for the bulk modulus given the difficulty of accurately measuring the vertical stiffness of a bearing in compression test machine. The extremely large stiffness and the relatively low level of the test pressure indicate that the vertical displacement is very small and small errors in this measurement will have a large influence on the calculation of the stiffness and on the estimate of the bulk modulus. Nevertheless it remains one of practical method to determine the bulk modulus from experimental results. What is somewhat surprising in this case is the difference between the shear modulus estimates for the two types of bearings. Given that there is not a great deal of difference between the sizes of each type, it is strange that the compound should give such a large difference in the modulus.

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JAMES M. KELLY: jmkelly@berkeley.edu
Earthquake Engineering Research Center, University of California, Berkeley, 1301 South 46th Street, Richmond CA 94804-4698, United States

JIUN-WEI LAI: adrian-jwlai@berkeley.edu
Department of Civil and Environmental Engineering, University of California, 760 Davis Hall, Berkeley, CA 94720-1710, United States

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